Constraints on new physics from $K \to \pi\nu\bar{\nu}$

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We study constraints on new physics from the recent measurement of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ by the E787 and E949 Collaborations. In our analysis, we consider two models of new physics: (a) extra down-type singlet quark model and (b) R-parity violating minimal supersymmetric standard model. We find that $K^+ \to \pi^+\nu\bar{\nu}$ along with other processes such as $K_L \to \mu^+\mu^-$, ϵ'/ϵ provide useful bounds on the parameter U_{sd}, characterizing the off-diagonal $Z - d - \bar{s}$ coupling of model (a). The bounds on $\mathrm{Re}(U_{sd})$ from $(K_L \to \mu^+\mu^-)_{\mathrm{SD}}$ and $\mathrm{Im}(U_{sd})$ from ϵ'/ϵ are so tight that the branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$ can exceed the standard model value by at most a factor of 2. For model (b), we also obtain stringent bounds on certain combinations of the product of two λ'_{ijk} couplings originating from L number-violating operator $L_iQ_jD_k^c$ using $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \mu^+\mu^-$ processes. Even with the stringent constraints on U_{sd} [in model (a)] and on products of two $\not{R}$ couplings [model (b)], we find that the branching ratio $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ can be substantially different in both of the above models from those predicted in the standard model.

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I. INTRODUCTION

Recently, experiment E949 at Brookhaven National Laboratory detected an event for the rare decay $K^+ \to \pi^+\nu\bar{\nu}$. Combining the previously reported two events by experiment E787, a branching ratio $B(K^+ \to \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ [1] has been obtained. The central value of this branching ratio is about twice that of the standard model (SM) prediction $B(K^+ \to \pi^+\nu\bar{\nu}) = (7.2 \pm 2.1) \times 10^{-11}$ [2–7]. The SM prediction and the combined E787 and E949 results are consistent with each other within 1 standard deviation. Future improvement of the experimental sensitivity will verify whether any meaningful difference emerges. If the experimental result converges to the present central value, it would be an indication of new physics beyond the SM. In extensions of the SM, there are new sources for flavor changing neutral currents, which can affect the branching ratio for $K^+ \to \pi^+\nu\bar{\nu}$ and reproduce the central value obtained by E787 and E949 [5,7–12].

In this work, we analyze the effect of the new E787 and E949 results on two classes of model beyond the SM. The first one is the flavor changing neutral current (FCNC) mediated by a Z boson in the extra down-type quark singlet model. These extra down-type singlet quarks appear naturally in each 27-plet fermion generation of E_6 grand unification theories [13–16]. The mixing of these singlet quarks with the three SM down-type quarks induces tree-level FCNCs by Z exchange. These tree-level FCNC couplings can have significant effects on various kaon decay processes including $K \to \pi\nu\bar{\nu}$.

The second model we consider is the R-parity violating minimal supersymmetric standard model (MSSM). The superpotential of the MSSM contains operators which violate lepton (L) and baryon (B) numbers. The simultaneous presence of both lepton and baryon number violating operators leads to rapid proton decay, which contradicts the experimental bound on proton lifetime [17]. In order to keep the proton lifetime within the experimental limit, one has to impose certain additional symmetry in the model so that the baryon and lepton number violating interactions vanish. In most cases, a discrete multiplicative symmetry called R parity [18] is imposed, where $R = (-1)^{3B+L+2S}$ and S is the spin of the particle. Under this new symmetry, all baryon and lepton number violating operators in the superpotential with mass dimension less than or equal to 4 vanish. This not only forbids rapid proton decay but also predicts a stable lightest supersymmetric particle, which escapes detection providing a unique signature of R-parity conserving MSSM. However, this symmetry is *ad hoc* in nature, with no strong theoretical arguments in support of it. There are many other discrete symmetries, such as baryon parity and lepton parity, both of which can remove the unwanted operators from the superpotential, thus preventing rapid proton decay. Since there is no direct evidence supporting either R-parity conserving or $\not{R}$ MSSM, it is interesting to probe the consequences of the $\not{R}$ model (in such a way that either the B or L number is violated but not both) in light of some recent low energy data. Already, extensive studies

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have been done to look for the direct as well as indirect evidence of R-parity violation from different processes and to put constraints on various $\not{R}$ couplings [19–26]. We consider constraints on the product of two $\not{R}$ couplings that ensue from $K^+ \to \pi^+ \nu\bar{\nu}$ decay as well as other kaon processes, such as $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\mu^+e^-$. In some cases, the bounds obtained in this analysis are stronger than the existing one. Interestingly, even with such a stronger bound, for some combinations of $\not{R}$ couplings, the $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ is well above the standard model prediction.

The rest of this paper is organized in the following way. In Sec. II we discuss the tree-level FCNC effects in the extra down-type singlet quark model. We will then constrain this new FCNC parameter using the latest data on $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, $\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}}$, and ϵ'/ϵ in kaon decay into two pions. After constraining the FCNC parameter space, we look for the prediction for the CP violating processes: $K_L \to \pi^0\nu\bar{\nu}$ and $K_L \to \pi^0 e^+e^-$ in the allowed range of parameter space. In Sec. III we obtain constraints on the product of two $\not{R}$ couplings using the current data on $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}}$, followed by the prediction of $K_L \to \pi^0\nu\bar{\nu}$ and process in the allowed range of the $\not{R}$ couplings. We summarize our results in the last section.

II. Z MEDIATED FCNC WITH EXTRA DOWN-TYPE SINGLET QUARK

FCNC mediated by a Z boson contributing to $K^+ \to \pi^+\nu\bar{\nu}$ can be generated in many ways when going beyond the SM. A simple possibility arises from one new vector-like down-type singlet quark in addition to the SM fermions. In the weak interaction basis, W and Z interactions with quarks (in the current basis) can be written as

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\bar{U}^0_L\gamma^\mu D^0_L W^+,$$
$$\mathcal{L}_Z = -\frac{g}{2c_W}[\bar{U}^0_L\gamma^\mu U^0_L - \bar{D}^0_L\gamma^\mu D^0_L - 2s_W^2(Q_u\bar{U}^0\gamma^\mu U^0 + Q_d\bar{D}^0\gamma^\mu D^0 + Q_d\bar{D}^{0\prime}\gamma^\mu D^{0\prime})]Z_\mu, \tag{1}$$

where $U^0 = (u, c, t)$, $D^0 = (d, s, b)$ are the usual three generations of quarks in the SM, and $D^{0\prime} = d^{0\prime}$ is the additional down-type of quark singlet. One can easily generalize the model to include n generations of vector-like down-type quarks by using $D^{0\prime} = (d_1^{0\prime}, \ldots, d_n^{0\prime})$.

In general, $D^{0\prime}$ can mix with the ordinary quarks in D^0. The down quark mass matrix M_d is diagonalized by 4×4 unitary matrices $V_d^{L\dagger}M_dV_d^R = \mathrm{diag}(m_d, m_s, m_b, m_{d'})$, while the up quark mass matrix M_u is diagonalized by 3×3 unitary matrices $V_u^{L\dagger}M_uV_u^R = \mathrm{diag}(m_u, m_c, m_t)$. We indicate mass diagonal basis $D \equiv (d, s, b, d')$ and in this basis, we have

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\bar{U}_L\gamma^\mu V D_L W^+,$$
$$\mathcal{L}_Z = -\frac{g}{2c_W}[\bar{U}_L\gamma^\mu U_L - \bar{D}_L\gamma^\mu D_L - 2s_W^2(Q_u\bar{U}\gamma^\mu U + Q_d\bar{D}\gamma^\mu D)]Z_\mu - \frac{g}{2c_W}\bar{D}_{Li}\gamma^\mu U_{ij}D_{Lj}Z_\mu, \tag{2}$$

where $Q_{u,d}$ are the electric charges of up and down quarks in a unit of proton charge. $U_{ij} = V_{dil}^{L*}V_{djl}^L$ is a 4×4 matrix, and $V = V_u^{L\dagger}V_d^L$, which is a 3×4 matrix different from the usual Kobayashi-Maskawa (KM) matrix V_{KM}. We shall be concerned with only the U_{sd} element of the U matrix in this paper.

In the absence of d', the theory reduces to the SM. The top left 3×3 block matrix in V corresponds to the usual V_{KM} matrix. The rest of the matrix elements in V are expected to be small since deviations away from the SM are constrained to be small from various experimental data.

A. $K^+ \to \pi^+\nu\bar{\nu}$ process

The FCNC interactions as described by $\mathcal{L}_Z$ in Eq. (2) can contribute to $K^+ \to \pi^+\nu\bar{\nu}$ decay at tree level by exchanging the Z boson as shown in Fig. 1. The relevant Lagrangian is given by

$$\mathcal{L}_Z^\nu = \frac{g^2}{4c_W^2m_Z^2}U_{sd}\bar{s}_L\gamma^\mu d_L\sum_{\ell=e,\mu,\tau}\bar{\nu}_{L_\ell}\gamma_\mu\nu_{L_\ell}. \tag{3}$$

Using $\langle 0|\bar{s}\gamma^\mu\gamma_5 d|K^+\rangle = if_Kp_K^\mu$ and including the SM contribution, one obtains [11]

$$\frac{\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})}{\mathrm{Br}(K^+ \to \pi^0e^+\nu)} = \frac{r_{K^+}\alpha^2}{2\pi^2s_W^4|V_{us}|^2}\sum_{\ell=e,\mu,\tau}|\Delta_K^{\mathrm{SM}} + \Delta_K^Z|^2,$$
$$\Delta_K^{\mathrm{SM}} = \lambda_{sd}^cX_{NL}^\ell + \lambda_{sd}^t\eta_t^XX_0(x_t). \tag{4}$$

The factor $r_{K^+} = 0.901$ accounts for isospin breaking

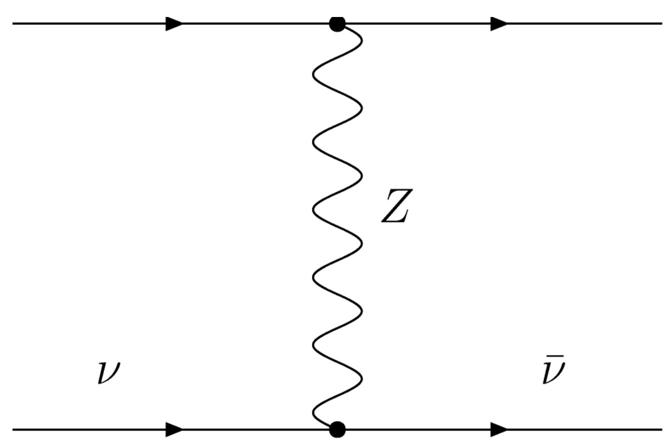

FIG. 1. Feynman diagram for Z exchange tree-level contribution to the $K^+ \to \pi^+\nu\bar{\nu}$ process.

corrections [27], $\alpha = 1/128$, $s_W = \sin\theta_W$. $\Delta_K^{\rm SM}$ is the SM contribution with the charm contributions at next-to-leading order (NLO) found to be $X_{\rm NL}^{e,\mu} = (10.6 \pm 1.5) \times 10^{-4}$, $X_{\rm NL}^{\tau} = (7.1 \pm 1.4) \times 10^{-4}$ [4]. λ_{sd}^i is defined as $V_{is}^* V_{id}$. The top contribution is proportional to the term with λ_{sd}^t. η_t^X is a QCD correction factor which is equal to 0.994. The function X_0 is given by

$$X_0 = \frac{x}{8}\left[\frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2}\ln x\right]; \qquad x = \frac{m_t^2}{m_W^2}. \qquad (5)$$

Using the current best fit values for the Wolfenstein parameters, $\lambda = 0.224$, $A = 0.839$, $\rho = 0.178$, $\eta = 0.341$, and an experimental value of ${\rm Br}(K^+ \to \pi^0 e^+ \nu) = 0.0487$, we obtain the SM value of $(7.28^{+0.13}_{-0.12}) \times 10^{-11}$ for ${\rm Br}(K^+ \to \pi^+ \nu\bar{\nu})$. The theoretical error on the branching ratio is computed by allowing λ_{sd}^t and the charm NLO corrections $(X_{\rm NL}^{e,\mu}, X_{\rm NL}^{\tau})$ to vary within 1σ from their central value. The central value of the SM branching ratio is a factor of 2 smaller than the experimental central value.

The term Δ_K^Z in Eq. (4) characterizes a new contribution from the FCNC Z interaction, which is given by

$$\Delta_K^Z = -\frac{\pi s_W^2}{\alpha} U_{sd}. \qquad (6)$$

With this new contribution, it is possible to reproduce the experimental central value; for example, with ${\rm Re}(U_{sd}) = 0.339 \times 10^{-5}$ and ${\rm Im}(U_{sd}) = 0.055 \times 10^{-5}$, we obtain ${\rm Br}(K^+ \to \pi^+ \nu\bar{\nu}) = 1.47 \times 10^{-10}$ using the central values of λ_{sd}^c, λ_{sd}^t and the charm NLO corrections.

One can also turn the argument around. By using the experimental data on ${\rm Br}(K^+ \to \pi^+ \nu\bar{\nu})$, one can constrain the new FCNC parameter U_{sd}. In Fig. 2 the gray shaded region represents a 90% C.L. allowed region in the ${\rm Re}(U_{sd})$-${\rm Im}(U_{sd})$ plane from ${\rm Br}^{\rm exp}(K^+ \to \pi^+ \nu\bar{\nu})$. The thickness of the band includes the 1σ theoretical error arising from the Cabbibo-Kobayashi-Maskawa (CKM) parameters and the charm NLO contribution. From Fig. 2 one can see that the recent data on ${\rm Br}(K^+ \to \pi^+ \nu\bar{\nu})$ still allows a large part of the parameter space of U_{sd}. However, in the next two subsections we will show that the current information on the short-distance contribution to $K_L \to \mu^+\mu^-$ and the data on ϵ'/ϵ constrain the above allowed parameter space of U_{sd} quite severely.

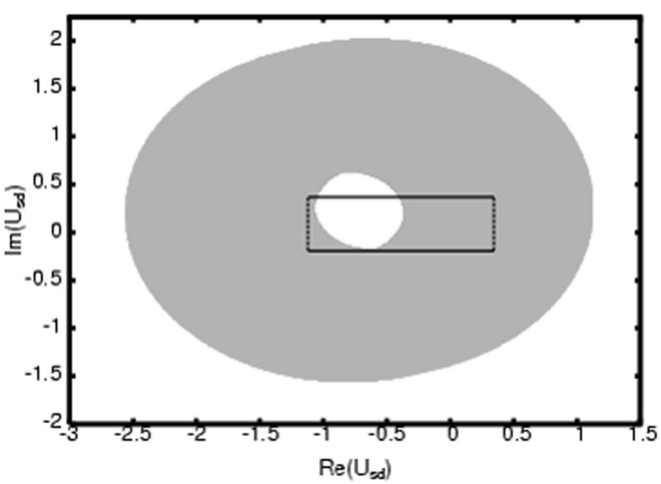

FIG. 2. The shaded region is 90% C.L. allowed parameter space in the ${\rm Re}(U_{sd})$-${\rm Im}(U_{sd})$ plane from ${\rm Br}(K^+ \to \pi^+ \nu\bar{\nu})$ data. The area enclosed by the two solid horizontal and two vertical dotted lines is allowed by ${\rm Br}(K_L \to \mu^+\mu^-)_{\rm SD}$ and ϵ'/ϵ, respectively, at 90% C.L. Both ${\rm Re}(U_{sd})$ and ${\rm Im}(U_{sd})$ are measured in units of 10^{-5}.

B. Constraints from other kaon processes

The same flavor changing Z interaction in Eq. (2) will also have effects on other flavor changing kaon processes. The $\Delta S = 1$ and $\Delta S = 2$ effective Lagrangians relevant for our discussions are given by

$$\mathcal{L}^{\Delta S=1} = \frac{g^2}{4c_W^2 m_Z^2} U_{sd} \bar{s}_L \gamma^\mu d_L \bar{f} \gamma_\mu (2I_3 L - 2Q_f s_W^2) f,$$
$$\mathcal{L}^{\Delta S=2} = \frac{G_F}{\sqrt{2}}\left[U_{sd}^2 - 8\frac{\alpha}{4\pi s_W^2} U_{sd} \sum_{\alpha=c,t} \lambda_{sd}^\alpha Y_0(x_\alpha)\right] \times \bar{s}_L \gamma^\mu d_L \bar{s}_L \gamma_\mu d_L, \qquad (7)$$

where $Y_0(x) = (x/8)[(x-4)/(x-1) + 3x\log x/(x-1)^2]$. The second term in $L^{\Delta S=2}$ is due to a one loop effect, which is important for gauge invariance but is numerically small [11]. Taking $f = \nu, e, \mu$, $L^{\Delta S=1}$ can contribute to processes such as $K^+ \to \pi^+ \bar{\nu}\nu$, $K_L \to \pi^0 \nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, and $K_L \to \pi^0 e^+ e^-$. Taking $f = u, d$, $L^{\Delta S=1}$ can contribute to ϵ'/ϵ, while $L^{\Delta S=2}$ can contribute to ϵ_K and ΔM_K.

1. Constraint from $K_L \to \mu^+\mu^-$

An interesting limit on the FCNC parameter U_{sd} comes from the short-distance contribution to the decay $K_L \to \mu^+\mu^-$. The $K_L \to \mu^+\mu^-$ branching ratio can be decomposed into the dispersive part (ReA) and the absorptive part (ImA). The absorptive can be determined very accurately from the branching ratio ${\rm Br}(K_L \to \gamma\gamma)$ and the resulting $|{\rm Im}A|^2$ alone almost saturates the branching ratio ${\rm Br}(K_L \to \mu^+\mu^-) = (7.07 \pm 0.18) \times 10^{-9}$ [11], leaving very little room for the dispersive contribution ReA. ReA can be further decomposed in ${\rm Re}A_{\rm LD} + {\rm Re}A_{\rm SD}$. Combining results from Refs. [7,28], we have a 90% C.L. bound on ${\rm Br}(K_L \to \mu^+\mu^-)_{\rm SD} \le 2.5 \times 10^{-9}$, with $|{\rm Re}A_{\rm SD}|^2 \equiv {\rm Br}(K_L \to \mu^+\mu^-)_{\rm SD}$. In our analysis we use this value of ${\rm Br}(K_L \to \mu^+\mu^-)_{\rm SD}$ to constrain the FCNC parameter ${\rm Re}(U_{sd})$. The expression for ${\rm Br}(K_L \to \mu^+\mu^-)_{\rm SD}$ is given by [11]

$$\frac{\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}}}{\mathrm{Br}(K^+ \to \mu^+\nu)} = \frac{\tau_{K_L}}{\tau_{K^+}} \frac{\alpha^2}{\pi^2 s_W^4 |V_{us}|^2} [T_{\mathrm{SM}} + \mathrm{Re}(\Delta_K^Z)]^2, \tag{8}$$

where

$$T_{\mathrm{SM}} = Y_{\mathrm{NL}} \mathrm{Re}(\lambda_{sd}^c) + \eta_t^Y Y_0(x_t) \mathrm{Re}(\lambda_{sd}^t). \tag{9}$$

At 90% C.L. we get the following bound on $\mathrm{Re}(U_{sd})$:

$$-1.12 \times 10^{-5} \le \mathrm{Re}(U_{sd}) \le 3.45 \times 10^{-6}. \tag{10}$$

The lower bound is obtained by taking $\mathrm{Re}(\lambda_{sd}^c) = (-0.2204 - 0.0023)$, $\mathrm{Re}(\lambda_{sd}^t) = (-3.04 - 0.31) \times 10^{-4}$, and the charm NLO contribution $Y_{\mathrm{NL}} = (2.94 + 0.28) \times 10^{-4}$. The upper bound is obtained by using $\mathrm{Re}(\lambda_{sd}^c) = (-0.2204 + 0.0022)$, $\mathrm{Re}(\lambda_{sd}^t) = (-3.04 + 0.32) \times 10^{-4}$, and the charm NLO contribution $Y_{\mathrm{NL}} = (2.94 - 0.28) \times 10^{-4}$. In Fig. 2 the above bound is shown by the area enclosed by vertical dotted lines. This shows that a substantial part of the parameter space, which was allowed by $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, is ruled out by $\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}}$.

2. Constraint from ϵ'/ϵ

Following the notation of Ref. [11], we write the expression for ϵ'/ϵ:

$$\epsilon'/\epsilon = F_{\epsilon'}(x_t) \mathrm{Im}(\lambda_{sd}^t) + \Delta_{\epsilon'}, \tag{11}$$

where $F_{\epsilon'}(x_t)$ can be found in Ref. [11] and $\Delta_{\epsilon'} = -\frac{\pi s_W^2}{\alpha} \times (P_X + P_Y + P_Z)\mathrm{Im}(U_{sd})$, with $P_{X,Y,Z}$ given in Ref. [11].

The bound on $\mathrm{Im}(U_{sd})$ from ϵ'/ϵ depends upon the sign of $\mathrm{Im}(U_{sd})$. For $\mathrm{Im}(U_{sd}) > 0$, the upper bound looks like

$$\mathrm{Im}(U_{sd}) \le \frac{(\epsilon'/\epsilon)^{\mathrm{exp}} - (\epsilon'/\epsilon)^{\mathrm{SM}}}{-\frac{\pi s_W^2}{\alpha}(P_X + P_Y + P_Z)}. \tag{12}$$

The experimental value for ϵ'/ϵ is $(1.8 \pm 0.4) \times 10^{-3}$. To maximize the $\mathrm{Im}(U_{sd})$, one should take maximum and minimum values for $(\epsilon'/\epsilon)^{\mathrm{exp}}$ and $(\epsilon'/\epsilon)^{\mathrm{SM}}$, respectively. The minimum value for $(\epsilon'/\epsilon)^{\mathrm{SM}}$ can be obtained by taking the lowest allowed values of $\mathrm{Im}(\lambda_{sd}^t)$, B_6, and B_8 (defined in Ref. [11]). With these choices of input parameters, we found

$$\mathrm{Im}(U_{sd}) \le 3.72 \times 10^{-6}. \tag{13}$$

Now, for $\mathrm{Im}(U_{sd}) < 0$, we have

$$-\mathrm{Im}(U_{sd}) \le \frac{(\epsilon'/\epsilon)^{\mathrm{SM}} - (\epsilon'/\epsilon)^{\mathrm{exp}}}{-\frac{\pi s_W^2}{\alpha}(P_X + P_Y + P_Z)}. \tag{14}$$

In this case, we take $(\epsilon'/\epsilon)^{\mathrm{SM}}_{\mathrm{max}}$ and $(\epsilon'/\epsilon)^{\mathrm{exp}}_{\mathrm{min}}$. This can be achieved by taking the largest allowed values of $\mathrm{Im}(\lambda_{sd}^t)$, B_6, and B_8. This gives us

$$-\mathrm{Im}(U_{sd}) \le 1.91 \times 10^{-6}. \tag{15}$$

Clearly, the bounds on the FCNC parameter $\mathrm{Im}(U_{sd})$ depend upon the experimental values of $(\epsilon'/\epsilon)_{\mathrm{max}}$ and $(\epsilon'/\epsilon)_{\mathrm{min}}$. Finally, combining Eqs. (13) and (15) we get a bound on $\mathrm{Im}(U_{sd})$ at 90% C.L.:

$$-1.91 \times 10^{-6} \le \mathrm{Im}(U_{sd}) \le 3.72 \times 10^{-6}. \tag{16}$$

In Fig. 2 this bound is depicted by the area enclosed by two parallel solid lines. This bound further constrains the parameter space of U_{sd}, which was otherwise allowed by $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}}$.

3. Remark on ΔM_K and ϵ_K

The contributions from the new Z flavor changing neutral current to ΔM_K^Z and ϵ_K^Z are given by

$$M_{12}^K = \frac{G_F^2 M_W^2 f_K^2 B_K M_K}{12\pi^2} \Big\{ -8U_{ds}[\lambda_{ds}^c \eta_c Y_0(x_c) + \lambda_{ds}^t \eta_t Y_0(x_t)] + \frac{4\pi s_W^2}{\alpha} \eta_c U_{sd}^2 \Big\}, \tag{17}$$

$$\Delta M_K^Z = 2\mathrm{Re}(M_{12}^K), \qquad \epsilon_K^Z = e^{i\pi/4} \frac{\mathrm{Im}(M_{12}^K)}{\sqrt{2}\Delta M_K}.$$

We observe that ΔM_K and ϵ_K evaluated from the above are close to the standard model predictions after constraints from $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \mu^+\mu^-$ are taken into account. Uncertainties in long distance contributions to ΔM_K make this process unsuitable for obtaining strong constraints.

4. Predictions for $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, and $K_L \to \pi^0 e^+ e^-$

In the previous discussions, we have shown how recent data on $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$, $\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}}$, and ϵ'/ϵ could be used to constrain the tree-level FCNC parameter U_{sd} in the extra down-type singlet quark model. In view of the tight constraints on U_{sd} obtained, we now examine the range of allowed branching ratios for $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, and $K_L \to \pi^0 e^+ e^-$ within the allowed parameter space of U_{sd}.

In Fig. 3 we show the variation of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ (in units of 10^{-10}) in the allowed $\mathrm{Re}(U_{sd})$-$\mathrm{Im}(U_{sd})$ plane. It turns out that the branching ratio can reach up to 1.5×10^{-10} at the edge of the allowed parameter $\mathrm{Re}(U_{ds})$ and $\mathrm{Im}(U_{ds})$. Another point to note is that the branching ratio depends very weakly on the FCNC parameter $\mathrm{Im}(U_{ds})$. In the computation of this branching ratio, we consider only the central values for all theoretical input parameters.

The channel $K_L \to \pi^0\nu\bar{\nu}$ in the SM and in the model under consideration is dominantly *CP* violating. In the

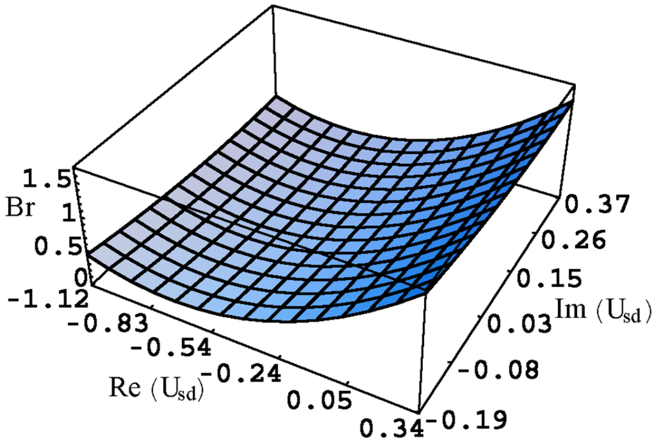

FIG. 3 (color online). Variation of $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})$ (in units of 10^{-10}) in the allowed parameter space of $\mathrm{Re}(U_{sd})$ and $\mathrm{Im}(U_{sd})$ (both in units of 10^{-5}) as shown by the rectangular box in Fig. 2.

standard model this decay mode is determined mostly by the intermediate top quark state and the uncertainty due to the charm contribution is negligible compared to $K^+ \to \pi^+\nu\bar{\nu}$ channel. We have [11]

$$\frac{\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})}{\mathrm{Br}(K^+ \to \pi^0 e^+\nu)} = r_{K_L}\frac{\tau_{K_L}}{\tau_{K^+}}\frac{\alpha^2}{2\pi^2 s_W^4 |V_{us}|^2} \times \sum_{\ell=e,\mu,\tau} |\mathrm{Im}(\Delta_K^{\mathrm{SM}} + \Delta_K^Z)|^2, \quad (18)$$

where Δ_K^{SM} and Δ_K^Z are defined in Eqs. (4) and (6), respectively. The standard model branching fraction for the process $K_L \to \pi^0\nu\bar{\nu}$ is $2.93^{+0.84}_{-0.67} \times 10^{-11}$ in the same ballpark as other estimates [7,29,30]. The errors correspond to the 1σ error in the CKM elements. In Fig. 4, we show the variation of $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ (in units of 10^{-10}) in the allowed $\mathrm{Re}(U_{sd})$-$\mathrm{Im}(U_{sd})$ parameter space. As a *CP* violating process, the branching ratio is solely dependent upon the $\mathrm{Im}(U_{sd})$ part of the FCNC parameter U_{sd} and it can reach as high as 10×10^{-11} at the edge of the allowed region. Presently, there is an upper bound on the branching ratio of 5.9×10^{-7} at 90% C.L. [17].

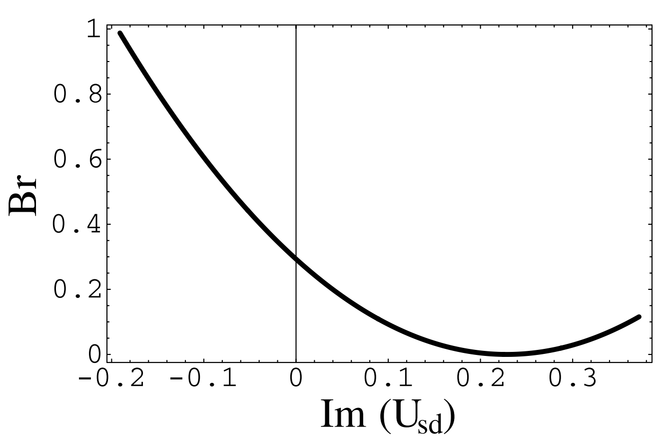

FIG. 4. Variation of $\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})$ (in units of 10^{-10}) in the allowed range of $\mathrm{Im}(U_{sd})$ (in units of 10^{-5}).

We now discuss the *CP* violating mode $K_L \to \pi^0 e^+ e^-$. It has recently been shown [31–33] that there are two main contributions to this process: the amplitude arising from the short-distance physics and a mixing contribution arising from conversion of K_L to K_S and subsequent decay of K_S into $\pi^0 e^+ e^-$. Using the experimental input on $\mathrm{Br}(K_S \to \pi^0 e^+ e^-)$, it has been shown that this mixing contribution dominates over the short-distance contribution in the rate by a factor of 5–6. As a result of this, the new physics contribution to the short-distance amplitude through Z exchange, using constraints on the parameter U_{sd} in Fig. 2, hardly affects the total rate. The experimental bound on $K_L \to \pi^0 e^+ e^-$ is larger than theory prediction by an order of magnitude, so no stronger bounds on U_{sd} can be obtained.

III. FCNC FROM *R*-PARITY VIOLATING INTERACTIONS IN MSSM

The most general superpotential of the MSSM can contain *R*-parity violating interaction terms:

$$\mathcal{W}_{\not R} = \lambda_{ijk}\hat{L}_i\hat{L}_j\hat{E}_k^c + \lambda'_{ijk}\hat{L}_i\hat{Q}_j\hat{D}_k^c + \lambda''_{ijk}\hat{U}_i^c\hat{D}_j^c\hat{D}_k^c, \quad (19)$$

where $\hat{E}_i^c$, $\hat{U}_i^c$, and $\hat{D}_i^c$ are ith type singlet lepton, up-type, and down-type quark superfields, respectively, and $\hat{L}_i$ and $\hat{Q}_i$ are SU(2) doublet lepton and quark superfields, respectively. The symmetry of the superpotential requires $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$. It is clear from Eq. (19) that the first two terms violate the lepton number, whereas the last one violates the baryon number. The simultaneous presence of both lepton and baryon number violating terms in the superpotential will lead to rapid proton decay. To prevent this, we can have either lepton or baryon number violating terms but not both in the superpotential. The $\not R$ interactions, in general, can have 27 λ'-type and 9 each of λ and λ''-type new couplings, which in general can be complex. The phase of a single coupling can be absorbed in the definition of the sfermion field, but the product of couplings can have a nontrivial phase. In our analysis we assume that only the λ'_{ijk} type of lepton number violating couplings are present. Furthermore, we constrain the product of two λ'-type couplings at a time and assume that there are no accidental cancellations and that only one product dominates at a time.

From the $\not R$ superpotential [Eq. (19)] one can write down the effective Lagrangian relevant for our purpose generated by the exchanging of different sfermions:

$$\mathcal{L}_{\not R} = \frac{\lambda'_{ijk}\lambda'^{*}_{i'j'k}}{2m^2_{\tilde{d}^k_R}}[\bar{\nu}^{i'}_L\gamma^\mu\nu^i_L\bar{d}^{j'}_L\gamma_\mu d^j_L + \bar{e}^{i'}_L\gamma^\mu e^i_L\bar{u}^{j'}_L\gamma_\mu u^j_L - \nu^{i'}_L\gamma^\mu e^i_L\bar{d}^{j'}_L\gamma_\mu u^j_L - \bar{e}^{i'}_L\gamma^\mu\nu^i_L\bar{u}^{j'}_L\gamma_\mu d^j_L]$$
$$- \frac{\lambda'_{ijk}\lambda'^{*}_{i'jk'}}{2m^2_{\tilde{d}^j_L}}\bar{\nu}^{i'}_L\gamma^\mu\nu^i_L\bar{d}^k_R\gamma_\mu d^{k'}_R - \frac{\lambda'_{ijk}\lambda'^{*}_{i'jk'}}{2m^2_{\tilde{u}^j_L}}\bar{e}^{i'}_L\gamma^\mu e^i_L\bar{d}^k_R\gamma_\mu d^{k'}_R - \frac{\lambda'_{ijk}\lambda'^{*}_{ij'k'}}{2m^2_{\tilde{e}^i_L}}\bar{u}^{j'}_{L\beta}\gamma^\mu u^j_{L\alpha}\bar{d}^k_{R\alpha}\gamma_\mu d^{k'}_{R\beta}$$
$$- \frac{\lambda'_{ijk}\lambda'^{*}_{ij'k'}}{2m^2_{\tilde{\nu}^i_L}}\bar{d}^{j'}_{L\beta}\gamma^\mu d^j_{L\alpha}\bar{d}^k_{R\alpha}\gamma_\mu d^{k'}_{R\beta}. \tag{20}$$

In the above, α and β are color indices.

A. Constraints from $K^+ \to \pi^+\nu\bar{\nu}$ process

In $\not R$ MSSM, the $K^+ \to \pi^+\nu\bar{\nu}$ process receives two nonzero contributions from the exchange of $\tilde{d}^k_R$ and $\tilde{d}^j_L$ squarks. Keeping the SM contribution, one obtains

$$\frac{\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})}{\mathrm{Br}(K^+ \to \pi^0 e^+\nu)} = \frac{r_{K^+}\alpha^2}{2\pi^2 s^4_W|V_{us}|^2}\left[\sum_{\ell=e,\mu,\tau}|\Delta^{\mathrm{SM}}_K + \Delta^{\not R}_{K\ell\ell}|^2 + \sum_{i\neq i'}|\Delta^{\not R}_{Kii'}|^2\right], \tag{21}$$

where the contribution from the lepton number violating operator $\hat{L}_i\hat{Q}_j\hat{D}^c_k$ is given by

$$\Delta^{\not R}_{Kii'} = \frac{\pi s^2_W}{\sqrt{2}G_F\alpha}\left[\frac{\lambda'_{i'j2}\lambda'^{*}_{ij1}}{2m^2_{\tilde{d}^j_L}} - \frac{\lambda'_{i'1k}\lambda'^{*}_{i2k}}{2m^2_{\tilde{d}^k_R}}\right]. \tag{22}$$

There are two contributions arising from the product of two types of λ' coupling; in one case ($\lambda'_{i'j2}\lambda'^{*}_{ij1}$), the propagator squark comes from left-handed doublet quark superfield $\hat{Q}_j$, whereas in the other case ($\lambda'_{i'1k}\lambda'^{*}_{i2k}$), the propagator squark comes from right-handed singlet quark superfield $\hat{D}^c_k$. In our analysis we take one combination of couplings to be nonzero at a time by setting all others to zero. The limits on $\not R$ couplings are usually quoted for $m_{\tilde{f}} = 100$ GeV. Following this general practice, throughout our analysis we assume sfermion masses to be degenerate with $m_{\tilde{f}} = 100$ GeV, and limits for higher $m_{\tilde{f}}$ can be obtained easily by scaling.

From Eqs. (21) and (22) we see that only in the case of same flavor neutrinos in the final state ($i = i'$) will the $\not R$ contributions interfere with the standard model one. In Fig. 5(a), the area within the circle represents the 90% C.L. allowed region in the Re($\lambda'_{ij2}\lambda'^{*}_{ij1}$)-Im($\lambda'_{ij2}\lambda'^{*}_{ij1}$) [first combination of $\not R$ couplings in Eq. (22)] plane. Similarly, the 90% C.L. allowed regions for the second combination of $\not R$ couplings is shown in Fig. 5(b) in the Re($\lambda'_{i1k}\lambda'^{*}_{i2k}$)-Im($\lambda'_{i1k}\lambda'^{*}_{i2k}$) plane. The solid contour represents the $\pi^+\nu_e\nu_e$ and $\pi^+\nu_\mu\nu_\mu$ (with $i = 1, 2$) final state, while the dashed one corresponds to the $\pi^+\nu_\tau\nu_\tau$ (with $i = 3$) final state. The product of $\not R$ couplings are in units of 10^{-5}. The marginal difference between the solid and the dashed contours arises due the difference in the charm contribution at NLO for e, μ, and τ. The relative shifts of the bounds between Figs. 5(a) and 5(b) can traced to the relative sign difference between two combinations of $\not R$ couplings in Eq. (22).

In the scenario with $i \neq i'$, the $\not R$ operators contribute to the amplitude $\mathcal{M}(K^+ \to \pi^+\nu\bar{\nu})$ incoherently, without interfering with the standard model. Moreover, both combinations of $\not R$ couplings in Eq. (22) will have the same contribution to the $K^+ \to \pi^+\nu\bar{\nu}$ process. From the experimentally observed branching ratio of the $K^+ \to \pi^+\nu\bar{\nu}$ process, we obtain $|\lambda'_{i'j2}\lambda'_{ij1}| \leq 0.89 \times 10^{-5}$ at

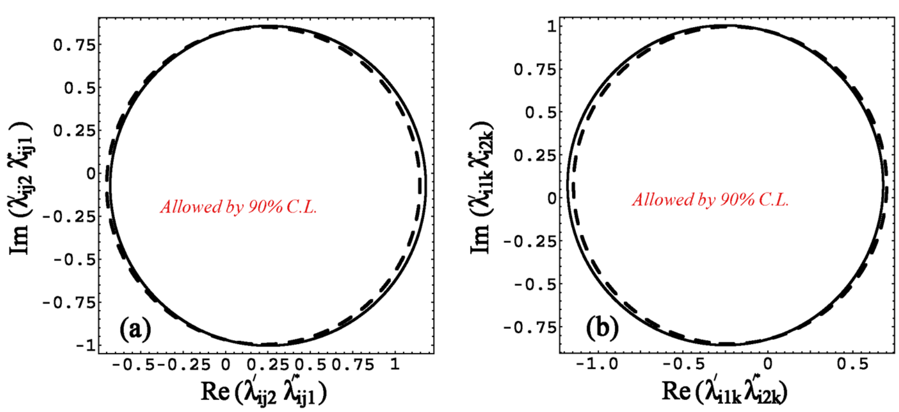

FIG. 5 (color online). 90% C.L. allowed regions for the product of $\not R$ couplings (with $i = i'$) (in units of 10^{-5}), which interfere with the SM from Br($K^+ \to \pi^+\nu\bar{\nu}$) data. The solid and dashed contours correspond to the ($i = e, \mu$) and ($i = \tau$) cases, respectively. (a) and (b) correspond to the first and second combinations of $\not R$ couplings in Eq. (22), respectively.

90% C.L. for $m_{\tilde{d}^j_L} = 100$ GeV. The same bound will also apply for the other combination of $\not{R}$ couplings $\lambda'_{i'1k}\lambda'^*_{i2k}$ in Eq. (22).

1. Constraints from $K_L \to \mu^+\mu^-$ process

The $K_L \to \mu^+\mu^-$ process bounds the real part of the product $\lambda'_{2i1}\lambda'^*_{2i2}$. The expression of the branching ratio $\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}}$ has been given in Eq. (8). The $\not{R}$ contribution can be easily included by replacing $\Delta^Z_{K_L}$ in Eq. (9) by $-(\pi s^2_W/\sqrt{2}G_F\alpha)\lambda'_{2j1}\lambda'^*_{2j2}/2m^2_{\tilde{u}^j_L}$. Using the 90% C.L. upper bound on the $\mathrm{Br}(K_L \to \mu^+\mu^-)_{\mathrm{SD}} \leq 2.5 \times 10^{-9}$ (as mentioned before), we obtain

$$-3.719 \times 10^{-6} < \mathrm{Re}(\lambda'_{2j1}\lambda'^*_{2j2}) < 1.14 \times 10^{-6} \quad (23)$$

for $m_{\tilde{u}^j_L} = 100$ GeV.

2. Constraints from $K^+ \to \pi^+\mu^+e^-$ process

This lepton number violating process has no contribution from the standard model. However, in the $\not{R}$ model, $\lambda'_{1j1}\lambda'^*_{2j2}$ coupling can induce such a decay process mediated by virtual $\tilde{u}_L$. The branching ratio is given by

$$\frac{\mathrm{Br}(K^+ \to \pi^+\mu^+e^-)}{\mathrm{Br}(K^+ \to \pi^0e^+\bar{\nu})} = \frac{r_{K^+}\alpha^2}{2\pi^2 s^4_W|V_{us}|^2}|\Delta^{\not{R}}_{K^+}|^2, \quad (24)$$

$$\Delta^{\not{R}}_{K^+} = \frac{\pi s^2_W}{\sqrt{2}G_F\alpha}\left(\frac{\lambda'_{1j1}\lambda'^*_{2j2}}{2m^2_{\tilde{u}^j_L}}\right). \quad (25)$$

Experimentally, we have only an upper bound on the branching ratio, $\mathrm{Br}(K^+ \to \pi^+\mu^+e^-) < 2.8 \times 10^{-11}$ at 90% C.L. Using this bound we obtain at 90% C.L.

$$|\lambda'_{1j1}\lambda'^*_{2j2}| \leq 2.684 \times 10^{-6} \quad (26)$$

for $m_{\tilde{u}^j_L} = 100$ GeV. However, in Ref. [34] the authors have obtained a stronger bound of $|\lambda'_{1j1}\lambda'^*_{2j2}| \leq 8.0 \times 10^{-7}$ from the $K_L \to \mu e$ process.

We now display in Table I the best current upper bounds on the product of two $\not{R}$ couplings with the processes which provide the limit. We present bounds obtained by us from $K^+ \to \pi^+\nu\bar{\nu}$ as well as some earlier bounds from other processes such as ΔM_K and $\mu \to e$ conversion in the nuclei which are sometimes stronger.

From our analysis, it is clear that the bounds obtained on the different combinations of $\not{R}$ couplings from the $K^+ \to \pi^+\nu\bar{\nu}$ process are of the order of 10^{-5}. In the case where the pair of $\not{R}$ couplings interfere with the standard model, we obtain bounds on the real and the imaginary part of the pair separately from Fig. 5, while in the noninterfering case, we obtain a bound on the magnitude of the pair of $\not{R}$ couplings involved. Note that earlier bounds on the above combination were $\sim\mathcal{O}(10^{-3})$, allow-

TABLE I. Current relevant upper bounds on the values of products of two $\not{R}$ couplings. Bounds corresponding to Refs. [35,36] for certain combinations are stronger than the one obtained here.

Couplings	Bounds	Source
$\lambda'_{i1k}\lambda'^*_{i2k}$	$-1.168 \times 10^{-5} \leq \mathrm{Re}(\lambda'_{i1k}\lambda'^*_{i2k}) \leq 0.67 \times 10^{-5}$	
	$-0.85 \times 10^{-5} \leq \mathrm{Im}(\lambda'_{i1k}\lambda'^*_{i2k}) \leq 1.0 \times 10^{-5}$	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{112}\lambda'_{111}\vert$	4.8×10^{-7}	ΔM_K [35]
$\vert\lambda'_{122}\lambda'_{121}\vert$	4.6×10^{-7}	ΔM_K [35]
$\lambda'_{132}\lambda'^*_{131}$	$-0.67 \times 10^{-5} \leq \mathrm{Re}(\lambda'_{132}\lambda'^*_{131}) \leq 1.17 \times 10^{-5}$	
	$-1.0 \times 10^{-5} \leq \mathrm{Im}(\lambda'_{132}\lambda'^*_{131}) \leq 0.85 \times 10^{-5}$	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{212}\lambda'_{211}\vert$	4.8×10^{-7}	ΔM_K [35]
$\vert\lambda'_{222}\lambda'_{221}\vert$	4.6×10^{-7}	ΔM_K [35]
$\lambda'_{232}\lambda'^*_{231}$	$-3.719 \times 10^{-6} \leq \mathrm{Re}(\lambda'_{232}\lambda'^*_{231}) \leq 1.14 \times 10^{-6}$	$K_L \to \mu^+\mu^-$
	$-1.0 \times 10^{-5} \leq \mathrm{Im}(\lambda'_{232}\lambda'^*_{231}) \leq 0.85 \times 10^{-5}$	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{312}\lambda'_{311}\vert$	4.8×10^{-7}	ΔM_K [35]
$\lambda'_{3j2}\lambda'^*_{3j1}$	$-0.67 \times 10^{-5} \leq \mathrm{Re}(\lambda'_{3j2}\lambda'^*_{3j1}) \leq 1.168 \times 10^{-5}$	
$(j = 2, 3)$	$-1.0 \times 10^{-5} \leq \mathrm{Im}(\lambda'_{3j2}\lambda'^*_{3j1}) \leq 0.85 \times 10^{-5}$	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{11k}\lambda'_{22k}\vert$	4×10^{-7}	μTi $\to e$Ti [36]
$\vert\lambda'_{21k}\lambda'_{12k}\vert$	4.3×10^{-7}	μTi $\to e$Ti [36]
$\vert\lambda'_{11k}\lambda'_{32k}\vert$	0.89×10^{-5}	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{21k}\lambda'_{32k}\vert$	0.89×10^{-5}	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{31k}\lambda'_{12k}\vert$	0.89×10^{-5}	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{31k}\lambda'_{22k}\vert$	0.89×10^{-5}	$K^+ \to \pi^+\nu\bar{\nu}$
$\vert\lambda'_{i'j2}\lambda'_{ij1}\vert$[a]	0.89×10^{-5}	$K^+ \to \pi^+\nu\bar{\nu}$

[a]The $K_L \to \mu e$ process puts stronger limits (8×10^{-7}) on the combinations $|\lambda'_{212}\lambda'_{111}|$, $|\lambda'_{222}\lambda'_{121}|$, and $|\lambda'_{232}\lambda'_{131}|$ [34].

ing significant room for enhancement of the process $K^+ \to \pi^+ \nu \bar{\nu}$. The other process $K_L \to \mu^+ \mu^-$ puts a limit on $\mathrm{Re}(\lambda'_{232}\lambda'^*_{231})$, which is of the order of 10^{-6}. The ΔM_K and $\mu \to e$ conversion in the nuclei provide bounds of the order of 10^{-7} on some combinations of $\not{R}$ couplings which are otherwise weakly constrained by our analysis.[1]

3. Prediction for $K_L \to \pi^0 \nu \bar{\nu}$ process

In the presence of the L number violating $\lambda'_{ijk} L_i Q_j D^c_k$ operator, several products of two $\not{R}$ couplings can contribute to the CP violating process $K_L \to \pi^0 \nu \bar{\nu}$. We have

$$\frac{\mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu})}{\mathrm{Br}(K^+ \to \pi^0 e^+ \nu)} = \kappa \Bigg[\sum_{\ell = e,\mu,\tau} |\mathrm{Im}(\Delta_K^{\mathrm{SM}} + \Delta^{\not{R}}_{K_L \ell\ell})|^2 + \sum_{i \neq i'} |\Delta^{\not{R}}_{K_L ii'}|^2 \Bigg], \qquad (27)$$

$$\kappa = r_{K_L} \frac{\tau_{K_L}}{\tau_{K^+}} \frac{\alpha^2}{2\pi^2 s_W^4 |V_{us}|^2},$$

$$\Delta^{\not{R}}_{K_L ii'} = \frac{\pi s_W^2}{\sqrt{2} G_F \alpha} \Bigg[\frac{\lambda'_{i'j1}\lambda'^*_{ij2}}{2m^2_{\tilde{d}^j_L}} - \frac{\lambda'_{i'2k}\lambda'^*_{i1k}}{2m^2_{\tilde{d}^k_R}} - \frac{\lambda'_{i'j2}\lambda'^*_{ij1}}{2m^2_{\tilde{d}^j_L}} + \frac{\lambda'_{i'1k}\lambda'^*_{i2k}}{2m^2_{\tilde{d}^k_R}} \Bigg]. \qquad (28)$$

One notes that for $i = i'$, the decays are CP violating, but for $i \neq i'$, the decays are not necessarily CP violating, which is very different from the SM [38].

As we see from Eq. (28), several combinations of two $\not{R}$ couplings are involved in this case. In our numerical calculation we consider each combination of $\not{R}$ couplings one by one. For $i = i'$ these $\not{R}$ contributions interfere with the standard model one, while for $i \neq i'$, these new operators create a neutrino pair which is not a CP eigenstate. We treat these two cases separately. First, we assume $i = i'$; in this case the branching ratio $\mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu})$ can reach as large as 2.0×10^{-9} for the allowed values of $\lambda'_{131}\lambda'^*_{132}$, $\lambda'_{232}\lambda'^*_{231}$, $\lambda'_{322}\lambda'^*_{321}$, $\lambda'_{332}\lambda'^*_{331}$ couplings. In fact, we find that for all the relevant pairs of $\not{R}$ couplings (with $i = i'$) whose bound is $\sim\mathcal{O}(10^{-5})$, the maximum value of the branching ratio is 2.0×10^{-9}, which is almost 2 orders of magnitude larger than the standard model prediction: $2.93^{+0.84}_{-0.67} \times 10^{-11}$. Experimentally, we have only an upper limit for this branching ratio, which is 5.9×10^{-7} at 90% C.L. In the second scenario, where $i \neq i'$, $K_L \to \mu e$ and $\mu\mathrm{Ti} \to e\mathrm{Ti}$ set bounds of the order of $\mathcal{O}(10^{-7})$ on the magnitude of the following combination of $\not{R}$ couplings: $|\lambda'_{11k}\lambda'_{22k}|$, $|\lambda'_{21k}\lambda'_{12k}|$, $|\lambda'_{212}\lambda'_{111}|$, $|\lambda'_{222}\lambda'_{121}|$, and $|\lambda'_{232}\lambda'_{131}|$. As can be seen from Table I, the bound on the magnitude of other combinations of $\not{R}$ couplings is 0.89×10^{-5} obtained from $K^+ \to \pi^+ \nu \bar{\nu}$. We find that the branching ratio can reach up to 1.39×10^{-9} for the pair of $\not{R}$ couplings in Eq. (28) whose magnitude satisfies the above mentioned limit.

IV. CONCLUSIONS

In this paper, we have examined the effects of new physics originated from two different kind of models on several rare flavor changing processes involving the K meson: $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \mu^+ \mu^-$, ϵ'/ϵ, $K_L \to \pi^0 e^+ e^-$, $K_L \to \pi^0 \nu \bar{\nu}$. In the first model, in addition to the SM quarks we also have one extra down-type singlet quark. The presence of such an additional singlet quark leads to a new off-diagonal Z mediated FCNC coupling U_{ij}, between the SM quarks of flavor i and j. In this paper we have considered off-diagonal Z coupling between first two generation down-type SM quarks, denoted by a complex parameter U_{sd}. We then obtained a 90% C.L. bound on this mixing parameter using known experimental data on $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \mu^+ \mu^-$, and ϵ'/ϵ. It turned out that the allowed parameter space of U_{sd} from the $K^+ \to \pi^+ \nu \bar{\nu}$ process is severely constrained from ϵ'/ϵ (imaginary part of U_{sd}): $-1.91 \times 10^{-6} \le \mathrm{Im}(U_{sd}) \le 3.72 \times 10^{-6}$, while $(K_L \to \mu^+ \mu^-)_{\mathrm{SD}}$ constrains the real part of U_{sd}: $-1.12 \times 10^{-5} \le \mathrm{Re}(U_{sd}) \le 3.45 \times 10^{-6}$. We did not find any significant deviation from the SM prediction of $K^0 - \bar{K}^0$ oscillation. The value of $\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ can reach up to 1.5×10^{-10} at the edge of the allowed parameter space of U_{sd}. Moreover, we can also reproduce the central value of experimentally measured $\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})$. We have also studied other CP violating kaon processes $K_L \to \pi^0 \nu \bar{\nu}$ and $K_L \to \pi^0 e^+ e^-$ in the allowed parameter space of U_{sd}. The value of $\mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu})$ can reach as high as 10×10^{-11} at the edge of the allowed parameter space. At present from experiment we have an upper limit on this branching ratio 5.9×10^{-7} at 90% C.L. [17]. We found that the process $K_L \to \pi^0 e^+ e^-$ is very weakly dependent on the new physics parameter U_{sd}, because of the fact that the dominant contribution to this decay amplitude arises from the mixing of K_L and K_S, followed by $K_S \to \pi^0 e^+ e^-$ decay.

The second model we have considered is the $\not{R}$ MSSM. We have computed the bounds on the product of two $\not{R}$ couplings of the type $\lambda'\lambda'$ using $K^+ \to \pi^+ \nu \bar{\nu}$, $K_L \to \mu^+ \mu^-$, and $K^+ \to \pi^+ \mu^+ e^-$ processes. We have assumed that the product of two $\not{R}$ couplings are, in general, complex and all the sfermion masses are degenerate with mass of 100 GeV in order to compare with earlier bounds obtained in literature. One can obtain bounds for any other sfermion mass by scaling. In deriving the bounds, full standard model amplitudes have been taken into account. One should note that, in several cases, our

[1]Note that we do not use ϵ'/ϵ to put bounds on products of λ' couplings because the theoretical expression for this quantity involves $\not{R}$ scalar couplings, involving more model dependence [37].

bounds are complementary to the bounds obtained from other processes such as ΔM_K, $K_L \to \mu e$, and μTi $\to e$Ti. We have found that processes such as $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \nu\bar{\nu}$ can be significantly enhanced compared to their standard model predictions. The constraints on the product of $\not{R}$ couplings $\lambda'_{131}\lambda^{*\prime}_{132}$ from the decay mode $K_L \to \pi^0 e^+ e^-$ is $\sim\mathcal{O}(10^{-4})$, which is 1 order of magnitude weaker than the bound obtained from $K^+ \to \pi^+ \nu\bar{\nu}$. As we have explained before, the dominant contribution to the $K_L \to \pi^0 e^+ e^-$ process arises from the mixing between K_L and K_S. After taking into account this mixing contribution, the standard model prediction for $\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = (3.2^{+1.2}_{-0.8}) \times 10^{-11}$ [31], whereas the experimental upper bound in 5.1×10^{-10} at 90% C.L., leaving very little room for the new physics contribution.

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