Heavy and light pentaquark effective chiral Lagrangian

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(Received 15 April 2004; revised manuscript received 13 September 2004; published 29 November 2004)

Using the SU(3) flavor symmetry, we construct the effective chiral Lagrangians for the light and heavy pentaquarks. With the correction from the nonzero current quark mass, we derive the Gell-Mann–Okubo type relations for various pentaquark multiplet masses. We also derive Coleman-Glashow relations for antisextet heavy pentaquark magnetic moments. We study possible decays of pentaquarks into conventional hadrons and interactions between and within various pentaquark multiplets and derive their coupling constants in the SU(3) flavor symmetry limit. Possible kinematically allowed pionic decay modes are pointed out. Finally we discuss the possible mixing between different pentaquark multiplets induced by the quark mass which breaks SU(3) symmetry. The pentaquark decay patterns receive correction from this breaking effect.

DOI: 10.1103/PhysRevD.70.094045 PACS numbers: 12.39.Mk

I. INTRODUCTION

Since LEPS Collaboration announced the discovery of the exotic baryon $\Theta^+(1540)$ with very narrow width [1], many other groups have claimed the observation of this pentaquark state [2–12]. NA49 observed a new pentaquark $\Xi^{--}(1862)$ [13], which needs confirmation from other groups [14]. Recently, H1 Collaboration claimed the discovery of a heavy pentaquark around 3099 MeV with the quark content $udud\bar{c}$ [15]. It is interesting to note that several groups reported negative results [16–18].

There is preliminary evidence that Θ^+ is an isoscalar because no enhancement was observed in the pK^+ invariant mass distribution [4,6,7,12]. Most of the theoretical models assume that Θ^+ is in $SU(3)_F$ $\bar{\mathbf{10}}$ representation.

The parity of Θ^+ pentaquark remains unknown. Theoretical approaches advocating positive parity include the chiral soliton model (CSM) [19], the diquark model [20], some quark models [21–25], and a lattice calculation [26]. On the other hand, some other theoretical approaches tend to favor negative parity such as two lattice QCD simulations [27,28], QCD sum rule approaches [29,30], several quark model studies [31–33], and proposing a stable diamond structure for Θ^+ [34].

The narrow width of the Θ^+ pentaquark is another puzzle. All the experiments can only determine the upper bound of the pentaquark width up to the detector resolution. The reanalysis of previous kaon nucleus scattering data indicates the decay width of Θ^+ should be 1 or 2 MeV or less [35], which makes the theoretical interpretation very difficult.

There have appeared several attempts to explain the narrow width. One possibility is the mismatch between the spin-flavor wave functions of the initial and final states when the Θ^+ pentaquark decays through the fall-apart mechanism [23,36–38].

Another possible interpretation of the narrow width puzzle is the possible mismatch between the spatial wave functions of final and initial states [34]. The reason is simple. The Θ^+ pentaquark with the stable diamond structure bound by nonplanar flux tubes is hard to decay to hadrons bound by planar flux tubes [34]. But this scheme has not been studied quantitatively.

In the chiral soliton model, the narrowness of Θ^+ results from the cancellation of the coupling constants at different N_c orders [39]. It is suggested that one of two nearly degenerate pentaquarks sharing the same dominant decay mode can be arranged to decouple from the decay channel after diagonalizing the mixing mass matrix via a kaon nucleon loop [40].

Recently heavy pentaquarks have received much attention [20,21,28,41–51]. In the heavy quark limit, the heavy antiquark decouples and acts as a spectator. The pentaquark system simplifies significantly. In fact, the heavy pentaquark system can be used as a test ground of the various models developed for the light pentaquarks.

Model calculation has shown that the even-parity pentaquark antidecuplet and octet lie close to each other and ideal mixing occurs if quantum numbers allow [20]. The odd-parity pentaquark octet and singlet are several hundred MeV lower than the antidecuplet and even-parity octet. Strong transitions between different pentaquark multiplets may occur [52].

At present the underlying dynamics which binds four quarks and one antiquark into a narrow resonance above threshold is still a mystery. We will explore the strong interactions between pentaquark multiplets using the SU(3) flavor symmetry as the guide. Effective chiral

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Lagrangians have been used to study the strong decay modes of pentaquarks [49,52–55].

In Section II, we construct the effective chiral Lagrangian involving light and heavy pentaquark multiplets. Then we discuss the mass splitting from the current quark mass correction within the same multiplet. In Section IV, we derive the coupling constants of the pentaquark interactions and discuss possible strong decay modes. In Section V, we discuss pentaquark mixing between different multiplets. Then we discuss how the pentaquark decay patterns are modified by the nonzero quark mass correction. The final section is a short discussion.

II. EFFECTIVE CHIRAL LAGRANGIANS

A. Notation

The approximate chiral symmetry and its spontaneous breaking have played an important role in hadron physics. Through the nonlinear realization of spontaneous chiral symmetry breaking, we may study the interaction between the chiral field and hadrons, which always involves the derivative of the chiral field. The nonzero current quark mass breaks the chiral symmetry explicitly. These corrections are taken into account perturbatively together with the chiral loop correction. Generally speaking, chiral symmetry provides a natural framework to organize the hadronic strong interaction associated with the light quarks.

However, we want to emphasize in the beginning that we will not make a systematic analysis based on the strict power-counting scheme of chiral perturbation theory in the present work. For example, we will not consider chiral loop corrections. Instead we focus on the tree-level effects and build the effective chiral Lagrangian for pentaquarks in order to discuss their decay patterns and their mass splitting induced by the quark mass correction.

In writing down the pentaquark effective chiral Lagrangians, we follow the standard notation in the chiral perturbation theory. First the eight Goldstone bosons are introduced exponentially. We use the short-hand notation π to denote the matrix of them.

$$\Sigma \equiv \xi^2 \equiv \exp\left(\frac{2i\pi}{F_\pi}\right), \tag{1}$$

$$\pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \tag{2}$$

where $F_\pi = 92.4$ MeV is the pion decay constant.

Under the $SU(3)_L \times SU(3)_R$ chiral transformation, $\Sigma(x)$ and $\xi(x)$ transform as

$$\Sigma(x) \to L\Sigma(x)R^\dagger, \qquad \xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger \tag{3}$$

where $L \in SU(3)_L$, $R \in SU(3)_R$, $U(x)$ is a nonlinear function of $\pi(x)$ and L, R.

The chiral connection V_μ and the axial-vector field A_μ are defined as

$$\begin{aligned} V_\mu &= \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\ A_\mu &= \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger). \end{aligned} \tag{4}$$

The vector V_μ and axial-vector A_μ transform under chiral SU(3) as

$$V_\mu \to UV_\mu U^\dagger + U\partial_\mu U^\dagger, \qquad A_\mu \to UA_\mu U^\dagger. \tag{5}$$

With the chiral connection, we can construct the chirally covariant derivative $\mathcal{D}_\mu$. For the matter field ϕ, which is in the fundamental representation, we have

$$\mathcal{D}_\mu \phi = (\partial_\mu + V_\mu)\phi, \qquad \mathcal{D}_\mu \to U\mathcal{D}_\mu U^\dagger. \tag{6}$$

For the matter field in the adjoint representation like the nucleon octet B, we have

$$\mathcal{D}_\mu B = \partial_\mu B + [V_\mu, B] \tag{7}$$

where the octet baryon field reads

$$(B^i_j) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}. \tag{8}$$

B. Matter fields

In Jaffe and Wilczek's (JW's) diquark model [20], the color wave function of the two diquarks within the pentaquark must be antisymmetric $\mathbf{3_C}$. In order to get an exotic antidecuplet, the two scalar diquarks combine into the symmetric SU(3) $\bar{\mathbf{6}}_\mathbf{F}$: $[ud]^2$, $[ud][ds]_+$, $[su]^2$, $[su] \times [ds]_+$, $[ds]^2$, and $[ds][ud]_+$. Bose statistics demands symmetric total wave function of the diquark-diquark system, which leads to the antisymmetric spatial wave function with one orbital excitation. The resulting pentaquark antidecuplet P_{ijk} and octet O^i_{1j} have $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. We will use the $J = \frac{1}{2}$ case to illustrate the formalism. One can perform a similar discussion on $J = \frac{3}{2}$ pentaquarks with some replacements, e.g., $P \to P_\mu$ where μ is the Lorentz index for the Rarita-Schwinger spinor.

The pentaquark octet has similar tensor representation with the nucleon octet B^i_j. The members of pentaquark antidecuplet are $P_{333} = \Theta^+$, $P_{133} = \frac{1}{\sqrt{3}}N^0_{\bar{10}}$, $P_{233} = -\frac{1}{\sqrt{3}}N^+_{\bar{10}}$, $P_{113} = \frac{1}{\sqrt{3}}\Sigma^-_{\bar{10}}$, $P_{123} = -\frac{1}{\sqrt{6}}\Sigma^0_{\bar{10}}$, $P_{223} = \frac{1}{\sqrt{3}}\Sigma^+_{\bar{10}}$, $P_{111} = \Xi^{--}_{\bar{10}}$, $P_{112} = -\frac{1}{\sqrt{3}}\Xi^-_{\bar{10}}$, $P_{122} = \frac{1}{\sqrt{3}}\Xi^0_{\bar{10}}$, and $P_{222} = -\Xi^+_{\bar{10}}$.

Other lighter pentaquarks can be formed if the two scalar diquarks are in the antisymmetric $SU(3)_F$ **3** representation [52]: $[ud][su]_-$, $[ud][ds]_-$, and $[su][ds]_-$, where $[q_1q_2][q_3q_4]_- = \sqrt{\frac{1}{2}}([q_1q_2][q_3q_4] - [q_3q_4][q_1q_2])$. No orbital excitation is needed to ensure the symmetric total wave function of two diquarks since the spin-flavor-color part is symmetric. The two diquarks combine with the antiquark to form a $SU(3)_F$ pentaquark octet O_{2j}^i and singlet Λ_1. The total angular momentum of these pentaquarks is $\frac{1}{2}$ and the parity is negative. There are no accompanying $J = \frac{3}{2}$ multiplets.

Replacing the light antiquark with one anticharm or antibottom quark in Jaffe and Wilczek's model leads to one even-parity antisextet S_{ij} [20,47] and one odd-parity triplet T^i [46,48]. In Karliner and Lipkin's diquark triquark model, the heavy pentaquarks in antisextet and triplet are all even-parity baryons [47]. The heavy pentaquark multiplets are

$$(S_{ij}^c) = \begin{pmatrix} \Xi_{5c}^{--} & -\frac{1}{\sqrt{2}}\Xi_{5c}^- & \frac{1}{\sqrt{2}}\Sigma_{5c}^- \\ -\frac{1}{\sqrt{2}}\Xi_{5c}^- & \Xi_{5c}^0 & -\frac{1}{\sqrt{2}}\Sigma_{5c}^0 \\ \frac{1}{\sqrt{2}}\Sigma_{5c}^- & -\frac{1}{\sqrt{2}}\Sigma_{5c}^0 & \Theta_{5c}^0 \end{pmatrix}, \qquad (S_{ij}^b) = \begin{pmatrix} \Xi_{5b}^{-} & -\frac{1}{\sqrt{2}}\Xi_{5b}^0 & \frac{1}{\sqrt{2}}\Sigma_{5b}^0 \\ -\frac{1}{\sqrt{2}}\Xi_{5b}^0 & \Xi_{5b}^+ & -\frac{1}{\sqrt{2}}\Sigma_{5b}^+ \\ \frac{1}{\sqrt{2}}\Sigma_{5b}^0 & -\frac{1}{\sqrt{2}}\Sigma_{5b}^+ & \Theta_{5b}^+ \end{pmatrix},$$
$$(T_c^i) = \begin{pmatrix} \Sigma_{5c}^{\prime 0} \\ \Sigma_{5c}^{\prime -} \\ \Xi_{5c}^{\prime -} \end{pmatrix}, \qquad (T_b^i) = \begin{pmatrix} \Sigma_{5b}^{\prime +} \\ \Sigma_{5b}^{\prime 0} \\ \Xi_{5b}^{\prime 0} \end{pmatrix}. \tag{9}$$

In writing down the effective chiral Lagrangians, we need the pseudoscalar heavy mesons $\bar{Q}^i$ in $SU(3)_F$ fundamental representation:

$$(Q_i) = (\, Q\bar{u}, \quad Q\bar{d}, \quad Q\bar{s}\,). \tag{10}$$

Under chiral transformation, the matter fields transform as

$$B_j^i \to U_a^i B_b^a U_j^{\dagger b}, \qquad D_\mu^{ijk} \to U_a^i U_b^j U_c^k D_\mu^{abc}, \qquad O_{1j}^i \to U_a^i O_{1b}^a U_j^{\dagger b}, \qquad O_{2j}^i \to U_a^i O_{2b}^a U_j^{\dagger b}, \qquad \Lambda_1 \to \Lambda_1,$$
$$P_{ijk} \to P_{abc} U_i^{\dagger a} U_j^{\dagger b} U_k^{\dagger c}, \qquad \bar{Q}^i \to U_a^i \bar{Q}^a, \qquad S_{ij} \to S_{ab} U_i^{\dagger a} U_j^{\dagger b}, \qquad T^i \to U_a^i T^a, \tag{11}$$

where D_μ^{ijk} represents the Δ decuplet.

The chirally covariant derivatives of these matter fields have the same transformation as the matter fields. They are

$$\begin{gathered} \mathcal{D}_\mu B_j^i = \partial_\mu B_j^i + V_{\mu,a}^i B_j^a - B_a^i V_{\mu,j}^a, \qquad \mathcal{D}_\mu D^{ijk} = \partial_\mu D^{ijk} + V_{\mu,a}^i D^{ajk} + V_{\mu,a}^j D^{iak} + V_{\mu,a}^k D^{ija}, \\ \mathcal{D}_\mu O_{1j}^i = \partial_\mu O_{1j}^i + V_{\mu,a}^i O_{1j}^a - O_{1a}^i V_{\mu,j}^a, \qquad \mathcal{D}_\mu O_{2j}^i = \partial_\mu O_{2j}^i + V_{\mu,a}^i O_{2j}^a - O_{2a}^i V_{\mu,j}^a, \\ \mathcal{D}_\mu P_{ijk} = \partial_\mu P_{ijk} + P_{ija} V_{\mu,k}^{\dagger a} + P_{iak} V_{\mu,j}^{\dagger a} + P_{ajk} V_{\mu,i}^{\dagger a}, \qquad \mathcal{D}_\mu \bar{Q}^i = \partial_\mu \bar{Q}^i + V_{\mu,j}^i \bar{Q}^j, \\ \mathcal{D}_\mu S_{ij} = \partial_\mu S_{ij} + S_{ia} V_{\mu,j}^{\dagger a} + S_{aj} V_{\mu,i}^{\dagger a}, \qquad \mathcal{D}_\mu T^i = \partial_\mu T^i + V_{\mu,j}^i T^j. \end{gathered} \tag{12}$$

The current quark mass matrix $m = \mathrm{diag}(\hat{m}, \hat{m}, m_s)$ breaks SU(3) flavor symmetry and induces pentaquark mass splittings and pentaquark mixings. It transforms as $m \to LmR^\dagger = RmL^\dagger$ under $SU(3)_L \times SU(3)_R$ chiral transformation. We have ignored the isospin breaking effect and adopted $m_u = m_d = \hat{m}$. Hence, the following combination of m and ξ transforms as the matter field:

$$(\xi m \xi + \xi^\dagger m \xi^\dagger) \to U(\xi m \xi + \xi^\dagger m \xi^\dagger) U^\dagger \tag{13}$$

which is denoted by m^c.

C. Mass, kinematic, and interaction terms with the chiral field

Pentaquark multiplets in the diquark model are: $\overline{\mathbf{10}}$ and $\mathbf{8_1}$ with $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$, and **1** and $\mathbf{8_2}$ with $J^P = \frac{1}{2}^-$. In the following, we write down the chiral Lagrangian involving all these multiplets with $J = \frac{1}{2}$. With these matter field multiplets and their corresponding chiral transformations, we first write down the chiral Lagrangian involving mass terms, kinematic terms, and interaction terms between the matter fields and the chiral field. The Lagrangian has the form

$$\mathcal{L} = \mathcal{L}_\Sigma + \mathcal{L}_B + \mathcal{L}_D + \mathcal{L}_P + \mathcal{L}_{O_1} + \mathcal{L}_{O_2} + \mathcal{L}_{\Lambda_1} + \mathcal{L}_Q + \mathcal{L}_S + \mathcal{L}_T + \mathcal{L}_{\mathrm{int}}, \tag{14}$$

where

$$\mathcal{L}_\Sigma = \frac{F_\pi^2}{4}\mathrm{Tr}[\partial_\mu\Sigma^\dagger\partial^\mu\Sigma - 2\mu m(\Sigma + \Sigma^\dagger)], \qquad \mathcal{L}_B = \mathrm{Tr}\overline{B}(i\not{\mathcal{D}} - m_B)B - D_B\mathrm{Tr}\overline{B}\gamma^\mu\gamma_5\{A_\mu, B\} - F_B\mathrm{Tr}\overline{B}\gamma^\mu\gamma_5[A_\mu, B],$$
$$\mathcal{L}_D = \bar{D}(i\not{\mathcal{D}} - m_D)D + \mathcal{G}_D\overline{D}\gamma_5\not{A}D, \qquad \mathcal{L}_P = \overline{P}(i\not{\mathcal{D}} - m_P)P + \mathcal{G}_P\overline{P}\gamma_5\not{A}P,$$
$$\mathcal{L}_{O_1} = \mathrm{Tr}\overline{O}_1(i\not{\mathcal{D}} - m_{O_1})O_1 - D_{O_1}\mathrm{Tr}\overline{O}_1\gamma^\mu\gamma_5\{A_\mu, O_1\} - F_{O_1}\mathrm{Tr}\overline{O}_1\gamma^\mu\gamma_5[A_\mu, O_1],$$
$$\mathcal{L}_{O_2} = \mathrm{Tr}\overline{O}_2(i\not{\mathcal{D}} - m_{O_2})O_2 - D_{O_2}\mathrm{Tr}\overline{O}_2\gamma^\mu\gamma_5\{A_\mu, O_2\} - F_{O_2}\mathrm{Tr}\overline{O}_2\gamma^\mu\gamma_5[A_\mu, O_2], \qquad \mathcal{L}_{\Lambda_1} = \overline{\Lambda}_1 i\not{\partial}\Lambda_1 + m_{\Lambda_1}\overline{\Lambda}_1\Lambda_1,$$
$$\mathcal{L}_Q = (\mathcal{D}_\mu Q)(\mathcal{D}^\mu\bar{Q}) - m_Q^2 Q\bar{Q}, \qquad \mathcal{L}_S = \overline{S}(i\not{\mathcal{D}} - m_S)S + \mathcal{G}_S\overline{S}\gamma_5\not{A}S, \qquad \mathcal{L}_T = \overline{T}(i\not{\mathcal{D}} - m_T)T + \mathcal{G}_T\overline{T}\gamma_5\not{A}T. \qquad (15)$$

In the above equations, m_B, m_P etc. are hadron masses in the chiral limit.

D. Interaction between different matter fields

The interaction parts of the chiral Lagrangian between different matter fields read

$$\mathcal{L}_{PAB} = C_{PAB}\overline{P}\Gamma_P\not{A}B + h.c., \qquad \mathcal{L}_{O_1AD} = C_{O_1AD}\overline{O}_1A^\mu D_\mu + h.c., \qquad \mathcal{L}_{O_2AD} = C_{O_2AD}\overline{O}_2 i\gamma_5 A^\mu D_\mu + h.c.,$$
$$\mathcal{L}_{O_1AB} = C_{O_1AB}\mathrm{Tr}\overline{O}_1\gamma_5\gamma^\mu\{A_\mu, B\} + \mathcal{H}_{O_1AB}\mathrm{Tr}\overline{O}_1\gamma_5\gamma^\mu[A_\mu, B] + h.c.,$$
$$\mathcal{L}_{O_1AP} = C_{O_1AP}\overline{O}_1\Gamma_P\not{A}P + h.c., \mathcal{L}_{O_2AB} = C_{O_2AB}\mathrm{Tr}(\overline{O}_2\gamma^\mu\{A_\mu, B\}) + \mathcal{H}_{O_2AB}\mathrm{Tr}(\overline{O}_2\gamma^\mu[A_\mu, B]) + h.c.,$$
$$\mathcal{L}_{O_2AP} = C_{O_2AP}\overline{O}_2\Gamma_P\gamma_5\not{A}P + h.c., \qquad \mathcal{L}_{O_2AO_1} = C_{O_2AO_1}\mathrm{Tr}(\overline{O}_2\gamma^\mu\{A_\mu, O_1\}) + \mathcal{H}_{O_2AO_1}\mathrm{Tr}(\overline{O}_2\gamma^\mu[A_\mu, O_1]) + h.c.,$$
$$\mathcal{L}_{\Lambda_1AB} = C_{\Lambda_1AB}\mathrm{Tr}(\overline{\Lambda}_1\Gamma_{\Lambda_1}\not{A}B) + h.c., \qquad \mathcal{L}_{\Lambda_1AO_1} = C_{\Lambda_1AO_1}\mathrm{Tr}(\overline{\Lambda}_1\Gamma_{\Lambda_1}A/O_1) + h.c.,$$
$$\mathcal{L}_{\Lambda_1AO_2} = C_{\Lambda_1AO_2}\mathrm{Tr}(\overline{\Lambda}_1\Gamma_{\Lambda_1}\gamma_5\not{A}O_2) + h.c., \qquad \mathcal{L}_{SQB} = C_{SQB}\overline{S}\Gamma_S\bar{Q}B + h.c, \qquad \mathcal{L}_{SQP} = C_{SQP}\overline{S}\Gamma_{SP}\bar{Q}P + h.c,$$
$$\mathcal{L}_{SQO_1} = C_{SQO_1}\overline{S}\Gamma_S\bar{Q}O_1 + h.c, \qquad \mathcal{L}_{SQO_2} = C_{SQO_2}\overline{S}\Gamma_S\gamma_5\bar{Q}O_2 + h.c, \qquad \mathcal{L}_{TQB} = C_{TQB}\overline{T}\Gamma_T B\bar{Q} + h.c,$$
$$\mathcal{L}_{TQO_1} = C_{TQO_1}\overline{T}\Gamma_T O_1\bar{Q} + h.c, \qquad \mathcal{L}_{TQO_2} = C_{TQO_2}\overline{T}\Gamma_T\gamma_5 O_2\bar{Q} + h.c, \qquad \mathcal{L}_{TQ\Lambda_1} = C_{TQ\Lambda_1}\overline{T}\Gamma_{T\Lambda_1}\bar{Q}\Lambda_1 + h.c,$$
$$\mathcal{L}_{SAT} = C_{SAT}\overline{S}\Gamma_{ST}\not{A}T + h.c., \qquad (16)$$

where D^μ is the Rarita-Schwinger spinor for the Δ decuplet, the subscripts P, S, and T are the parities of pentaquarks (antidecuplet, antisextet, triplet, respectively), and the subscript SP is the product of S and P. $\Gamma_+ = \gamma_5$ and $\Gamma_- = 1$.

III. MASS AND MAGNETIC MOMENT RELATIONS

A. Mass relations

We now include the nonzero current quark mass corrections, which induce mass splittings in the pentaquark multiplet. These symmetry-breaking terms for various pentaquark multiplets are

$$\mathcal{L}_P^m = \alpha_P\overline{P}m^cP + \beta_P\overline{P}P\mathrm{Tr}(m\Sigma + \Sigma^\dagger m), \qquad (17)$$

$$\mathcal{L}_{O_1}^m = \alpha_{d1}\mathrm{Tr}(\overline{O}_1\{m^c, O_1\}) + \alpha_{f1}\mathrm{Tr}(\overline{O}_1[m^c, O_1]) + \beta_{O_1}\mathrm{Tr}(\overline{O}_1O_1)\mathrm{Tr}(m\Sigma + \Sigma^\dagger m), \qquad (18)$$

$$\mathcal{L}_{O_2}^m = \alpha_{d2}\mathrm{Tr}(\overline{O}_2\{m^c, O_2\}) + \alpha_{f2}\mathrm{Tr}(\overline{O}_2[m^c, O_2]) + \beta_{O_2}\mathrm{Tr}(\overline{O}_2O_2)\mathrm{Tr}(m\Sigma + \Sigma^\dagger m), \qquad (19)$$

$$\mathcal{L}_S^m = \alpha_S\overline{S}m^cS + \beta_S\overline{S}S\mathrm{Tr}(m\Sigma + \Sigma^\dagger m). \qquad (20)$$

The β terms do not induce mass splittings. We ignore them in the following.

Expanding Eq. (17), we get the mass splittings $\Delta m_i \equiv m_i - m_{\mathrm{penta}}$ for pentaquark antidecuplet P,

$$\Delta m_\Theta = 2\alpha_P m_s, \qquad \Delta m_{N_{\bar{10}}} = \frac{2}{3}\alpha_P(\hat{m} + 2m_s),$$
$$\Delta m_{\Sigma_{\bar{10}}} = \frac{2}{3}\alpha_P(2\hat{m} + m_s), \qquad \Delta m_{\Xi_{\bar{10}}} = 2\alpha_P\hat{m}. \qquad (21)$$

From the above mass splittings, we can derive the following mass relations:

$$m_{N_{\bar{10}}} - m_{\Sigma_{\bar{10}}} = m_\Theta - m_{N_{\bar{10}}},$$
$$m_{\Sigma_{\bar{10}}} - m_{\Xi_{\bar{10}}} = m_{N_{\bar{10}}} - m_{\Sigma_{\bar{10}}}. \qquad (22)$$

These relations have already been derived using the chiral soliton model [19] and chiral Lagrangian approach [53,54]. The equal splitting for antidecuplet pentaquark was also discussed in Ref. [32].

Similarly, for the pentaquark octet O_2

$$\Delta m_{N_{8_2}} = \alpha_{d2}(2\hat{m} + 2m_s) + \alpha_{f2}(2\hat{m} - 2m_s),$$
$$\Delta m_{\Sigma_{8_2}} = \alpha_{d2}(4\hat{m}),$$
$$\Delta m_{\Xi_{8_2}} = \alpha_{d2}(2\hat{m} + 2m_s) - \alpha_{f2}(2\hat{m} - 2m_s),$$
$$\Delta m_{\Lambda_{8_2}} = \frac{1}{3}\alpha_{d2}(4\hat{m} + 8m_s). \qquad (23)$$

Hence we have the mass relation

$$2m_{N_8} + 2m_{\Xi_8} = 3m_{\Lambda_8} + m_{\Sigma_8}, \qquad (24)$$

which was first derived in Ref. [52]. The pentaquark octet O_1 has a similar expression. The mass relations for ideally mixed pentaquark antidecuplet P and pentaquark octet O_1 have been discussed in Ref. [53].

For the heavy pentaquark antisextet S_c and S_b we get

$$\Delta m_{\Xi_{5Q}} = 2\alpha_{S_Q}\hat{m}, \qquad \Delta m_{\Sigma_{5Q}} = \alpha_{S_Q}(\hat{m} + m_s), \qquad \Delta m_{\Theta_{5Q}} = 2\alpha_{S_Q}m_s. \tag{25}$$

$$m_{\Xi_{5Q}} - m_{\Sigma_{5Q}} = m_{\Sigma_{5Q}} - m_{\Theta_{5Q}}. \tag{26}$$

The heavy pentaquark mass splittings have been discussed in Refs. [20,21,47,49]. Especially in the diquark model it is very simple to derive these mass relations with the Hamiltonian $H_s = M + n_s(m_s + \alpha)$.

B. Heavy pentaquark magnetic moment relations

As in Refs. [52,56,57], we derive the magnetic moment relations of the heavy pentaquark antisextet [47] in Jaffe and Wilczek's model for the first time. Interested readers are referred to Refs. [47,52,58–60] for the magnetic moments of light pentaquarks. Here we list the results only:

$$\begin{aligned} \mu_{\Xi^0_{5c}} + \mu_{\Xi^{--}_{5c}} &= 2\mu_{\Xi^-_{5c}}, \\ 3\mu_{\Theta^0_c} - \mu_{\Sigma^0_{5c}} - 2\mu_{\Sigma^-_{5c}} &= \mu_{\Xi^0_{5c}} - \mu_{\Xi^{--}_{5c}}, \\ \mu_{\Xi^+_{5b}} + \mu_{\Xi^-_{5b}} &= 2\mu_{\Xi^0_{5b}}, \\ 3\mu_{\Theta^+_b} - \mu_{\Sigma^+_{5b}} - 2\mu_{\Sigma^0_{5b}} &= \mu_{\Xi^+_{5b}} - \mu_{\Xi^-_{5b}}. \end{aligned} \tag{27}$$

These relations hold for both $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ antisextet in JW's model.

The magnetic moments of $J^P = \frac{1}{2}^-$ heavy pentaquarks in the diquark model are all identical because they come from heavy antiquark only.

IV. POSSIBLE STRONG DECAYS AND COUPLING CONSTANTS IN THE SU(3) SYMMETRY LIMIT

Besides the possible decays of pentaquarks into conventional hadrons, we also consider the strong interactions and possible transitions between pentaquark multiplets. If pentaquarks are bound by flux tubes and have the nonplanar diamond structure as suggested in [34], then the possible transitions between pentaquarks might get enhanced because of the special stable structure, although the decay phase space is smaller. Expanding the interaction terms in the previous section we obtain the coupling constants for different decay modes. We present the results up to one pseudoscalar meson field. Since some interaction terms have similar flavor structure, it is enough to consider the following pieces: $\mathcal{L}_{O_1AP}$, $\mathcal{L}_{O_1AD}$, $\mathcal{L}_{O_2AO_1}$, $\mathcal{L}_{\Lambda_1AO_1}$, $\mathcal{L}_{SAT}$, $\mathcal{L}_{SQO_1}$, $\mathcal{L}_{SQP}$, and $\mathcal{L}_{TQO_1}$.

A. Possible strong decays of antidecuplet pentaquark P_{ijk}

SU(3) symmetry forbids the antidecuplet to decay to the Δ decuplet and pion octet or the $\Lambda_1\pi$. The chiral Lagrangian and couplings of the antidecuplet with pseudoscalar meson octet π and nucleon octet B can be found in Refs. [53,55].

The antidecuplet pentaquarks P_{ijk} and the octet O_1 pentaquarks lie close to each other [20]. Some states especially are nearly degenerate and mix ideally. So the strong interaction between these two multiplets is very important. One example is the identification of $N(1440)$ and $N(1710)$ as nucleonlike pentaquarks in the diquark model [20]. Such a big mass splitting after the diagonalization of the mixing mass matrix will allow the pionic transition to occur kinematically. We collect the couplings of the pentaquark antidecuplet with the even-parity pentaquark octet and pseudoscalar octet in Table I.

We want to emphasize that the odd-parity pentaquark octet O_2 lies much lower than the antidecuplet. Pionic decay modes $P \to O_2\pi$ are allowed kinematically in

TABLE I. Couplings of the pentaquark antidecuplet P_{ijk} with the pentaquark octet O^i_{1j} and pseudoscalar meson octet π^i_j. The universal coupling constant $-(1/F_\pi)C_{O_1AP}$ is omitted.

Θ^+		$N^+_{\bar{10}}$		$N^0_{\bar{10}}$		$\Sigma^+_{\bar{10}}$	
$K^+n_{8,1}$	1	$\pi^+n_{8,1}$	$-\frac{1}{\sqrt{3}}$	$\pi^0n_{8,1}$	$\frac{1}{\sqrt{6}}$	$\pi^+\Lambda_{8,1}$	$\frac{1}{\sqrt{2}}$
$K^0p_{8,1}$	-1	$\pi^0p_{8,1}$	$-\frac{1}{\sqrt{6}}$	$\pi^-p_{8,1}$	$-\frac{1}{\sqrt{3}}$	$\pi^+\Sigma^0_{8,1}$	$-\frac{1}{\sqrt{6}}$
		$\eta_8p_{8,1}$	$\frac{1}{\sqrt{2}}$	$\eta_8n_{8,1}$	$\frac{1}{\sqrt{2}}$	$\pi^0\Sigma^+_{8,1}$	$\frac{1}{\sqrt{6}}$
		$K^+\Lambda_{8,1}$	$-\frac{1}{\sqrt{2}}$	$K^+\Sigma^-_{8,1}$	$\frac{1}{\sqrt{3}}$	$\eta_8\Sigma^+_{8,1}$	$-\frac{1}{\sqrt{2}}$
		$K^+\Sigma^0_{8,1}$	$\frac{1}{\sqrt{6}}$	$K^0\Lambda_{8,1}$	$-\frac{1}{\sqrt{2}}$	$K^+\Xi^0_{8,1}$	$-\frac{1}{\sqrt{3}}$
		$K^0\Sigma^+_{8,1}$	$\frac{1}{\sqrt{3}}$	$K^0\Sigma^0_{8,1}$	$-\frac{1}{\sqrt{6}}$	$\bar{K}^0p_{8,1}$	$\frac{1}{\sqrt{3}}$
$\Sigma^0_{\bar{10}}$		$\Sigma^-_{\bar{10}}$		$\Xi^+_{\bar{10}}$		$\Xi^0_{\bar{10}}$	
$\pi^+\Sigma^-_{8,1}$	$-\frac{1}{\sqrt{6}}$	$\pi^0\Sigma^-_{8,1}$	$\frac{1}{\sqrt{6}}$	$\pi^+\Xi^0_{8,1}$	1	$\pi^+\Xi^-_{8,1}$	$-\frac{1}{\sqrt{3}}$
$\pi^0\Lambda_{8,1}$	$-\frac{1}{\sqrt{2}}$	$\pi^-\Lambda_{8,1}$	$-\frac{1}{\sqrt{2}}$	$\bar{K}^0\Sigma^+_{8,1}$	-1	$\pi^0\Xi^0_{8,1}$	$-\sqrt{\frac{2}{3}}$
$\pi^-\Sigma^+_{8,1}$	$\frac{1}{\sqrt{6}}$	$\pi^-\Sigma^0_{8,1}$	$-\frac{1}{\sqrt{6}}$			$\bar{K}^0\Sigma^0_{8,1}$	$\sqrt{\frac{2}{3}}$
$\eta_8\Sigma^0_{8,1}$	$\frac{1}{\sqrt{2}}$	$\eta_8\Sigma^-_{8,1}$	$\frac{1}{\sqrt{2}}$			$K^-\Sigma^+_{8,1}$	$\frac{1}{\sqrt{3}}$
$K^+\Xi^-_{8,1}$	$\frac{1}{\sqrt{6}}$	$K^0\Xi^-_{8,1}$	$\frac{1}{\sqrt{3}}$				
$K^0\Xi^0_{8,1}$	$-\frac{1}{\sqrt{6}}$	$K^-n_{8,1}$	$-\frac{1}{\sqrt{3}}$				
$\bar{K}^0n_{8,1}$	$\frac{1}{\sqrt{6}}$						
$K^-p_{8,1}$	$-\frac{1}{\sqrt{6}}$						
$\Xi^-_{\bar{10}}$		$\Xi^{--}_{\bar{10}}$					
$\pi^0\Xi^-_{8,1}$	$\sqrt{\frac{2}{3}}$	$\pi^-\Xi^-_{8,1}$	1				
$\pi^-\Xi^0_{8,1}$	$-\frac{1}{\sqrt{3}}$	$K^-\Sigma^-_{8,1}$	-1				
$\bar{K}^0\Sigma^-_{8,1}$	$\frac{1}{\sqrt{3}}$						
$K^-\Sigma^0_{8,1}$	$-\sqrt{\frac{2}{3}}$						

TABLE II. Couplings of the pentaquark octet O^i_{1j} with the baryon decuplet D^{ijk} and pseudoscalar meson octet π^i_j. The universal coupling constant $-(1/F_\pi)C_{O_1AD}$ is omitted. Except the universal coupling constant, the couplings of O_2 are the same.

$\Xi^-_{8,1}$		$\Xi^0_{8,1}$		$p_{8,1}$		$n_{8,1}$	
$\pi^-\Xi^{*0}$	$\frac{1}{\sqrt{3}}$	$\pi^+\Xi^{*-}$	$-\frac{1}{\sqrt{3}}$	$\pi^0\Delta^+$	$\sqrt{\frac{2}{3}}$	$\pi^0\Delta^0$	$\sqrt{\frac{2}{3}}$
$\pi^0\Xi^{*-}$	$-\frac{1}{\sqrt{6}}$	$\pi^0\Xi^{*0}$	$-\frac{1}{\sqrt{6}}$	$\pi^+\Delta^0$	$\frac{1}{\sqrt{3}}$	$\pi^-\Delta^+$	$-\frac{1}{\sqrt{3}}$
$\eta_8\Xi^{*-}$	$\frac{1}{\sqrt{2}}$	$\eta_8\Xi^{*0}$	$-\frac{1}{\sqrt{2}}$	$\pi^-\Delta^{++}$	-1	$\pi^+\Delta^-$	1
$\bar{K}^0\Sigma^{*-}$	$-\frac{1}{\sqrt{3}}$	$K^-\Sigma^{*+}$	$\frac{1}{\sqrt{3}}$	$K^+\Sigma^{*0}$	$\frac{1}{\sqrt{6}}$	$K^0\Sigma^{*0}$	$-\frac{1}{\sqrt{6}}$
$K^-\Sigma^{*0}$	$-\frac{1}{\sqrt{6}}$	$\bar{K}^0\Sigma^{*0}$	$\frac{1}{\sqrt{6}}$	$K^0\Sigma^{*+}$	$-\frac{1}{\sqrt{3}}$	$K^+\Sigma^{*-}$	$\frac{1}{\sqrt{3}}$
$K^0\Omega^-$	1	$K^+\Omega^-$	-1				
$\Sigma^0_{8,1}$		$\Sigma^+_{8,1}$		$\Sigma^-_{8,1}$		$\Lambda_{8,1}$	
$\pi^+\Sigma^{*-}$	$\frac{1}{\sqrt{6}}$	$\pi^+\Sigma^{*0}$	$-\frac{1}{\sqrt{6}}$	$\pi^-\Sigma^{*0}$	$\frac{1}{\sqrt{6}}$	$\pi^+\Sigma^{*-}$	$-\frac{1}{\sqrt{2}}$
$\pi^-\Sigma^{*+}$	$\frac{1}{\sqrt{6}}$	$\pi^0\Sigma^{*+}$	$-\frac{1}{\sqrt{6}}$	$\pi^0\Sigma^{*-}$	$-\frac{1}{\sqrt{6}}$	$\pi^0\Sigma^{*0}$	$-\frac{1}{\sqrt{2}}$
$\eta_8\Sigma^{*0}$	$\frac{1}{\sqrt{2}}$	$\eta_8\Sigma^{*+}$	$-\frac{1}{\sqrt{2}}$	$\eta_8\Sigma^{*-}$	$\frac{1}{\sqrt{2}}$	$\pi^-\Sigma^{*+}$	$\frac{1}{\sqrt{2}}$
$K^0\Xi^{*0}$	$\frac{1}{\sqrt{6}}$	$K^+\Xi^{*0}$	$-\frac{1}{\sqrt{3}}$	$K^-\Delta^0$	$-\frac{1}{\sqrt{3}}$	$K^0\Xi^{*0}$	$\frac{1}{\sqrt{2}}$
$K^-\Delta^+$	$-\sqrt{\frac{2}{3}}$	$K^-\Delta^{++}$	1	$K^0\Xi^{*-}$	$\frac{1}{\sqrt{3}}$	$K^+\Xi^{*-}$	$-\frac{1}{\sqrt{2}}$
$\bar{K}^0\Delta^0$	$-\sqrt{\frac{2}{3}}$	$\bar{K}^0\Delta^+$	$\frac{1}{\sqrt{3}}$	$\bar{K}^0\Delta^-$	-1		
$K^+\Xi^{*-}$	$\frac{1}{\sqrt{6}}$						

many channels. The coupling constants can also be found from Table I.

Replacing the octet O^i_{1j} with corresponding B^i_j in Table I, one gets the coupling constants of the pentaquark antidecuplet with the nucleon octet and pseudoscalar meson octet.

B. Possible strong decays of light pentaquark octet $O_{1,2}$

The couplings of the light pentaquark octet $O_{1,2}$ with the pseudoscalar meson octet π and nucleon octet B can be found in Ref. [52,53].

The even-parity and odd-parity octet pentaquarks can also decay into the Δ decuplet and the pseudoscalar meson octet. Jaffe and Wilczek pointed out that the decay mode $\Xi^-_5 \to \Xi^{*0}\pi^-$ observed by NA49 Collaboration may indicate the possible existence of the even-parity octet [20]. Since these decay modes can be measured in the near future, we present the couplings of the octet pentaquarks $O_{1,2}$ with the decuplet baryon and the pion octet in Table II.

Similarly, since the odd-parity octet O_2 is lower than the even-parity octet O_1, the pionic decay mode $O_1 \to O_2\pi$ is allowed kinematically in some channels. The couplings are collected in Table III. One can also get the coupling constants of the pentaquark octet with the nucleon octet and pseudoscalar meson octet from Table III with special b and proper replacement [52,53].

It is straightforward to derive the coupling of the pentaquark singlet Λ_1 with the pentaquark octet O^i_{1j} and the pseudoscalar meson octet π^i_j:

$$\mathcal{L}_{\Lambda_1 AO_1} = -\frac{1}{F_\pi} C_{\Lambda_1 AO_1} \overline{\Lambda}_1 \Gamma_{\Lambda_1} (\partial\!\!\!/ \pi^0 \Sigma^0_{8,1} + \partial\!\!\!/ \pi^+ \Sigma^-_{8,1} + \partial\!\!\!/ \pi^- \Sigma^+_{8,1} + \partial\!\!\!/ K^+ \Xi^-_{8,1} + \partial\!\!\!/ K^0 \Xi^0_{8,1} + \partial\!\!\!/ K^- p_{8,1} + \partial\!\!\!/ \bar{K}^0 n_{8,1}) + h.c.. \qquad (28)$$

C. Possible strong decays of heavy pentaquarks

The interaction of heavy pentaquarks with the heavy vector mesons and nucleon octet has the same flavor structure as in the case of heavy pseudoscalar mesons. It is interesting to note that the heavy pentaquark observed by H1 Collaboration sits right on the threshold of Δ and D meson. One may wonder whether this resonance is affected largely by the threshold behavior. However, in the $SU(3)_F$ symmetry limit one heavy pentaquark in antisextet cannot decay to a Δ decuplet member and a heavy pseudoscalar meson. In other words, this state cannot be explained as a coupled channel effect between $D^{*-}p$ and $D\Delta$ through t-channel pion exchange. The antisextet will not decay to Λ_1 plus a heavy meson. Similarly, the heavy triplet will not decay to the Δ decuplet plus a heavy meson or the antidecuplet plus a heavy meson.

The interaction between heavy pentaquarks, the nucleon octet B, and the pseudoscalar meson octet π was discussed in [48,49]. In Jaffe and Wilczek's diquark model, the odd-parity heavy pentaquark triplet is much lower than the even-parity heavy antisextet. Pionic decay $S \to T\pi$ may happen in many channels. It will be very interesting to explore this kind of decay process experimentally. Now the heavy quark acts as a spectator. We collect the relevant coupling constants in Table IV.

We list the couplings of the heavy pentaquark antisextet with the light pentaquark octet $O_{1,2}$ and the heavy pseudoscalar mesons in Table V, and those of the antisextet with the anitdecuplet and heavy meson triplet in Table VI. The couplings of the heavy pentaquark triplet with the light pentaquark octet $O_{1,2}$ and heavy meson triplet are presented in Table VII. All these processes might be forbidden by kinematics.

D. Interaction of pentaquarks within the same multiplet

For completeness, we also consider the interactions within the same pentaquark multiplet arising from these terms: $\mathcal{G}_P \bar{P}\gamma_5 A\!\!\!/ P$, $(D_O + F_O)\mathrm{Tr}(\overline{O}\gamma_5\gamma^\mu A_\mu O) + (D_O - F_O)\mathrm{Tr}(\overline{O}\gamma_5\gamma^\mu O A_\mu)$, $\mathcal{G}_S \bar{S}\gamma_5 A\!\!\!/ S$, and $\mathcal{G}_T \bar{T}\gamma_5 A\!\!\!/ T$. The coupling constants for the pentaquark octet O_1 or O_2 can be found in Table III through simple replacements. We collect other couplings in Tables VIII, IX, and X.

TABLE III. Couplings of the pentaquark octet O^i_{1j} with the pentaquark octet O^i_{2j} and pseudoscalar meson octet π^i_j. The universal coupling constant $-(1/F_\pi)C_{O_2AO_1}$ is omitted. The constant $b = \mathcal{H}_{O_2AO_1}/C_{O_2AO_1}$.

$\Xi^-_{8,1}$		$\Xi^0_{8,1}$		$p_{8,1}$		$n_{8,1}$	
$\pi^-\Xi^0_{8,2}$	$1-b$	$\pi^+\Xi^-_{8,2}$	$1-b$	$\pi^0 p_{8,2}$	$\frac{1}{\sqrt{2}}(1+b)$	$\pi^0 n_{8,2}$	$-\frac{1}{\sqrt{2}}(1+b)$
$\pi^0\Xi^-_{8,2}$	$\frac{1}{\sqrt{2}}(1-b)$	$\pi^0\Xi^0_{8,2}$	$-\frac{1}{\sqrt{2}}(1-b)$	$\pi^+ n_{8,2}$	$1+b$	$\pi^- p_{8,2}$	$1+b$
$\eta_8\Xi^-_{8,2}$	$-\frac{1}{\sqrt{6}}(3b+1)$	$\eta_8\Xi^0_{8,2}$	$-\frac{1}{\sqrt{6}}(3b+1)$	$\eta_8 p_{8,2}$	$\frac{1}{\sqrt{6}}(3b-1)$	$\eta_8 n_{8,2}$	$\frac{1}{\sqrt{6}}(3b-1)$
$\bar{K}^0\Sigma^-_{8,2}$	$1+b$	$K^-\Sigma^+_{8,2}$	$1+b$	$K^+\Sigma^0_{8,2}$	$\frac{1}{\sqrt{2}}(1-b)$	$K^0\Sigma^0_{8,2}$	$-\frac{1}{\sqrt{2}}(1-b)$
$K^-\Sigma^0_{8,2}$	$\frac{1}{\sqrt{2}}(1+b)$	$\bar{K}^0\Sigma^0_{8,2}$	$-\frac{1}{\sqrt{2}}(1+b)$	$K^0\Sigma^+_{8,2}$	$1-b$	$K^+\Sigma^-_{8,2}$	$1-b$
$K^-\Lambda_{8,2}$	$\frac{1}{\sqrt{6}}(3b-1)$	$\bar{K}^0\Lambda_{8,2}$	$\frac{1}{\sqrt{6}}(3b-1)$	$K^+\Lambda_{8,2}$	$-\frac{1}{\sqrt{6}}(3b+1)$	$K^0\Lambda_{8,2}$	$-\frac{1}{\sqrt{6}}(3b+1)$
$\Sigma^0_{8,1}$		$\Sigma^+_{8,1}$		$\Sigma^-_{8,1}$		$\Lambda_{8,1}$	
$\pi^+\Sigma^-_{8,2}$	$\sqrt{2}b$	$\pi^+\Sigma^0_{8,2}$	$-\sqrt{2}b$	$\pi^-\Sigma^0_{8,2}$	$\sqrt{2}b$	$\pi^-\Sigma^+_{8,2}$	$\sqrt{\frac{2}{3}}$
$\pi^-\Sigma^+_{8,2}$	$-\sqrt{2}b$	$\pi^0\Sigma^+_{8,2}$	$\sqrt{2}b$	$\pi^0\Sigma^-_{8,2}$	$-\sqrt{2}b$	$\pi^+\Sigma^-_{8,2}$	$\sqrt{\frac{2}{3}}$
$\pi^0\Lambda_{8,2}$	$\sqrt{\frac{2}{3}}$	$\eta_8\Sigma^+_{8,2}$	$\sqrt{\frac{2}{3}}$	$\eta_8\Sigma^-_{8,2}$	$\sqrt{\frac{2}{3}}$	$\pi^0\Sigma^0_{8,2}$	$\sqrt{\frac{2}{3}}$
$\eta_8\Sigma^0_{8,2}$	$\sqrt{\frac{2}{3}}$	$\pi^+\Lambda_{8,2}$	$\sqrt{\frac{2}{3}}$	$\pi^-\Lambda_{8,2}$	$\sqrt{\frac{2}{3}}$	$\eta_8\Lambda_{8,2}$	$-\sqrt{\frac{2}{3}}$
$K^- p_{8,2}$	$\frac{1}{\sqrt{2}}(1-b)$	$K^+\Xi^0_{8,2}$	$1+b$	$K^0\Xi^-_{8,2}$	$1+b$	$K^+\Xi^-_{8,2}$	$\frac{1}{\sqrt{6}}(3b-1)$
$\bar{K}^0 n_{8,2}$	$-\frac{1}{\sqrt{2}}(1-b)$	$\bar{K}^0 p_{8,2}$	$1-b$	$K^- n_{8,2}$	$1-b$	$K^0\Xi^0_{8,2}$	$\frac{1}{\sqrt{6}}(3b-1)$
$K^+\Xi^-_{8,2}$	$\frac{1}{\sqrt{2}}(1+b)$					$K^- p_{8,2}$	$-\frac{1}{\sqrt{6}}(3b+1)$
$K^0\Xi^0_{8,2}$	$-\frac{1}{\sqrt{2}}(1+b)$					$\bar{K}^0 n_{8,2}$	$-\frac{1}{\sqrt{6}}(3b+1)$

E. Decay widths

There are four types of interaction terms corresponding to four kinds of Lorentz structures depending on the parities of the pentaquarks:

$$a_1\bar{F}_1\gamma_5\gamma^\mu\partial_\mu M F_2, \qquad a_2\bar{F}_1\gamma^\mu\partial_\mu M F_2, \qquad a_3\bar{F}_1\gamma^5\bar{Q}F_2, \qquad a_4\bar{F}_1\,\bar{Q}\,F_2, \tag{29}$$

where F_1 and F_2 denote initial and final fermions, respectively, and M and $\bar{Q}$ are the pseudoscalar mesons. The corresponding decay widths are

$$\begin{aligned}
\Gamma_1 &= \frac{|\mathbf{p}^*|}{8\pi m_1^2}a_1^2(m_1+m_2)^2[(m_1-m_2)^2-m_M^2],\\
\Gamma_2 &= \frac{|\mathbf{p}^*|}{8\pi m_1^2}a_2^2(m_1-m_2)^2[(m_1+m_2)^2-m_M^2],\\
\Gamma_3 &= \frac{|\mathbf{p}^*|}{8\pi m_1^2}a_3^2[(m_1-m_2)^2-m_Q^2],\\
\Gamma_4 &= \frac{|\mathbf{p}^*|}{8\pi m_1^2}a_4^2[(m_1+m_2)^2-m_Q^2],
\end{aligned} \tag{30}$$

where $\mathbf{p}^*$ is the meson momentum in the parent particle

TABLE IV. Couplings of the heavy pentaquark antisextet S_{ij} with the heavy pentaquark triplet T^i and pseudoscalar meson octet π^i_j. The universal coupling constant $-(1/F_\pi)C_{SAT}$ is omitted.

$\Xi^{--}_{5c}(\Xi^-_{5b})$		$\Xi^-_{5c}(\Xi^0_{5b})$		$\Xi^0_{5c}(\Xi^+_{5b})$	
$K^-\Sigma'^-_{5c}(\Sigma'^0_{5b})$	1	$K^-\Sigma'^0_{5c}(\Sigma'^+_{5b})$	$\frac{1}{\sqrt{2}}$	$\pi^+\Xi'^-_{5c}(\Xi'^0_{5b})$	1
$\pi^-\Xi'^-_{5c}(\Xi'^0_{5b})$	-1	$\bar{K}^0\Sigma'^-_{5c}(\Sigma'^0_{5b})$	$-\frac{1}{\sqrt{2}}$	$\bar{K}^0\Sigma'^0_{5c}(\Sigma'^+_{5b})$	-1
		$\pi^0\Xi'^-_{5c}(\Xi'^0_{5b})$	-1		
$\Sigma^-_{5c}(\Sigma^0_{5b})$		$\Sigma^0_{5c}(\Sigma^+_{5b})$		$\Theta^0_{5c}(\Theta^+_{5b})$	
$\pi^-\Sigma'^0_{5c}(\Sigma'^+_{5b})$	$\frac{1}{\sqrt{2}}$	$\pi^0\Sigma'^0_{5c}(\Sigma'^+_{5b})$	$\frac{1}{2}$	$K^0\Sigma'^0_{5c}(\Sigma'^+_{5b})$	1
$\pi^0\Sigma'^-_{5c}(\Sigma'^0_{5b})$	$-\frac{1}{2}$	$\eta_8\Sigma'^0_{5c}(\Sigma'^+_{5b})$	$-\frac{\sqrt{3}}{2}$	$K^+\Sigma'^-_{5c}(\Sigma'^0_{5b})$	-1
$\eta_8\Sigma'^-_{5c}(\Sigma'^0_{5b})$	$-\frac{\sqrt{3}}{2}$	$\pi^+\Sigma'^-_{5c}(\Sigma'^0_{5b})$	$\frac{1}{\sqrt{2}}$		
$K^0\Xi'^-_{5c}(\Xi'^0_{5b})$	$-\frac{1}{\sqrt{2}}$	$K^+\Xi'^-_{5c}(\Xi'^0_{5b})$	$-\frac{1}{\sqrt{2}}$		

TABLE V. Couplings of the heavy pentaquark antisextet S_{ij} with the light pentaquark octet O^i_{1j} and heavy flavor pseudoscalar meson triplet $\bar{Q}^i$. The universal coupling constant C_{SQO_1} is omitted.

$\Xi^{--}_{5c}(\Xi^-_{5b})$		$\Xi^-_{5c}(\Xi^0_{5b})$		$\Xi^0_{5c}(\Xi^+_{5b})$	
$D^-(B^0)\Xi^-_{8,1}$	1	$\bar{D}^0(B^+)\Xi^-_{8,1}$	$\frac{1}{\sqrt{2}}$	$\bar{D}^0(B^+)\Xi^0_{8,1}$	-1
$D^-_s(B^0_s)\Sigma^-_{8,1}$	-1	$D^-(B^0)\Xi^0_{8,1}$	$-\frac{1}{\sqrt{2}}$	$D^-_s(B^0_s)\Sigma^+_{8,1}$	1
		$D^-_s(B^0_s)\Xi^0_{8,1}$	-1		
$\Sigma^-_{5c}(\Sigma^0_{5b})$		$\Sigma^0_{5c}(\Sigma^+_{5b})$		$\Theta^0_{5c}(\Theta^+_{5b})$	
$\bar{D}^0(B^+)\Sigma^-_{8,1}$	$\frac{1}{\sqrt{2}}$	$\bar{D}^0(B^+)\Sigma^0_{8,1}$	$\frac{1}{2}$	$\bar{D}^0(B^+)n_{8,1}$	1
$D^-(B^0)\Sigma^0_{8,1}$	$-\frac{1}{2}$	$\bar{D}^0(B^+)\Lambda_{8,1}$	$-\frac{\sqrt{3}}{2}$	$D^-(B^0)p_{8,1}$	-1
$D^-(B^0)\Lambda_{8,1}$	$-\frac{\sqrt{3}}{2}$	$D^-(B^0)\Sigma^+_{8,1}$	$\frac{1}{\sqrt{2}}$		
$D^-_s(B^0_s)n_{8,1}$	$-\frac{1}{\sqrt{2}}$	$D^-_s(B^0_s)p_{8,1}$	$-\frac{1}{\sqrt{2}}$		

TABLE VI. Couplings of the heavy pentaquark antisextet S_{ij} with the pentaquark antidecuplet P_{ijk} and heavy flavor pseudoscalar meson triplet $\bar{Q}^i$. The universal coupling constant C_{SQP} is omitted.

$\Xi_{5c}^{--}(\Xi_{5b}^{-})$		$\Xi_{5c}^{-}(\Xi_{5b}^{0})$		$\Xi_{5c}^{0}(\Xi_{5b}^{+})$	
$\bar{D}^0(B^+)\Xi_{\overline{10}}^{--}$	1	$\bar{D}^0(B^+)\Xi_{\overline{10}}^{-}$	$\sqrt{\frac{2}{3}}$	$\bar{D}^0(B^+)\Xi_{\overline{10}}^{0}$	$\frac{1}{\sqrt{3}}$
$D^-(B^0)\Xi_{\overline{10}}^{-}$	$-\frac{1}{\sqrt{3}}$	$D^-(B^0)\Xi_{\overline{10}}^{0}$	$-\sqrt{\frac{2}{3}}$	$D^-(B^0)\Xi_{\overline{10}}^{+}$	-1
$D_s^-(B_s^0)\Sigma_{\overline{10}}^{-}$	$\frac{1}{\sqrt{3}}$	$D_s^-(B_s^0)\Sigma_{\overline{10}}^{0}$	$\frac{1}{\sqrt{3}}$	$D_s^-(B_s^0)\Sigma_{\overline{10}}^{+}$	$\frac{1}{\sqrt{3}}$
$\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$		$\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$		$\Theta_{5c}^{0}(\Theta_{5b}^{+})$	
$\bar{D}^0(B^+)\Sigma_{\overline{10}}^{-}$	$\sqrt{\frac{2}{3}}$	$\bar{D}^0(B^+)\Sigma_{\overline{10}}^{0}$	$\frac{1}{\sqrt{3}}$	$\bar{D}^0(B^+)N_{\overline{10}}^{0}$	$\frac{1}{\sqrt{3}}$
$D^-(B^0)\Sigma_{\overline{10}}^{0}$	$-\frac{1}{\sqrt{3}}$	$D^-(B^0)\Sigma_{\overline{10}}^{+}$	$-\sqrt{\frac{2}{3}}$	$D^-(B^0)N_{\overline{10}}^{+}$	$-\frac{1}{\sqrt{3}}$
$D_s^-(B_s^0)N_{\overline{10}}^{0}$	$\sqrt{\frac{2}{3}}$	$D_s^-(B_s^0)N_{\overline{10}}^{+}$	$\sqrt{\frac{2}{3}}$	$D_s^-(B_s^0)\Theta^+$	1

F_1 rest frame.

$$|\mathbf{p}^*|^2 = \frac{1}{4m_1^2}[m_1^2 - (m_2 + m_M)^2][m_1^2 - (m_2 - m_M)^2]. \tag{31}$$

For example, Θ^+ width reads

$$\Gamma_{\Theta^+} = 2\Gamma_{\Theta^+\to K^+ n} = 2\Gamma_{\Theta^+\to K^0 p} = \frac{C_{PAB}^2|\mathbf{p}_1|}{4\pi F_\pi^2 m_\Theta^2}(m_\Theta + m_N)^2[(m_\Theta - m_N)^2 - m_K^2]. \tag{32}$$

If the mass of Λ_1 is around 1405 MeV [52], it decays to $\pi\Sigma$ only. Its width is

$$\Gamma_{\Lambda_1} = 3\Gamma_{\Lambda_1\to\pi^+\Sigma^-} = \frac{3C_{\Lambda_1 AB}^2|\mathbf{p}_2|}{8\pi F_\pi^2 m_{\Lambda_1}^2}(m_{\Lambda_1} - m_\Sigma)^2[(m_{\Lambda_1} + m_\Sigma)^2 - m_\pi^2]. \tag{33}$$

Even with $|C_{\Lambda_1 AB}| = 10|C_{PAB}|$, hence $\Gamma_{\Lambda_1}/\Gamma_{\Theta^+} \approx 43$, Λ_1 is still not a broad resonance assuming the current experimental upper bound of Θ^+ width.

TABLE VII. Couplings of the heavy pentaquark triplet T^i with the pentaquark octet O_{1j}^i and heavy flavor pseudoscalar meson triplet $\bar{Q}^i$. The universal coupling constant C_{TQO_1} is omitted.

$\Sigma_{5c}^{\prime 0}(\Sigma_{5b}^{\prime +})$		$\Sigma_{5c}^{\prime -}(\Sigma_{5b}^{\prime 0})$		$\Xi_{5c}^{\prime -}(\Xi_{5b}^{\prime 0})$	
$\bar{D}^0(B^+)\Sigma_{8,1}^{0}$	$\frac{1}{\sqrt{2}}$	$\bar{D}^0(B^+)\Sigma_{8,1}^{-}$	1	$\bar{D}^0(B^+)\Xi_{8,1}^{-}$	1
$\bar{D}^0(B^+)\Lambda_{8,1}$	$\frac{1}{\sqrt{6}}$	$D^-(B^0)\Sigma_{8,1}^{0}$	$-\frac{1}{\sqrt{2}}$	$D^-(B^0)\Xi_{8,1}^{0}$	1
$D^-(B^0)\Sigma_{8,1}^{+}$	1	$D^-(B^0)\Lambda_{8,1}$	$\frac{1}{\sqrt{6}}$	$D_s^-(B_s^0)\Lambda_{8,1}$	$-\sqrt{\frac{2}{3}}$
$D_s^-(B_s^0)p_{8,1}$	1	$D_s^-(B_s^0)n_{8,1}$	1		

TABLE VIII. Couplings of the pentaquark antidecuplet P_{ijk} with the pseudoscalar meson octet π_j^i. The universal coupling constant $-(1/F_\pi)\mathcal{G}_P$ is omitted.

Θ^+		$N_{\overline{10}}^{+}$		$N_{\overline{10}}^{0}$		$\Sigma_{\overline{10}}^{+}$	
$K^+N_{\overline{10}}^0$	$\frac{1}{\sqrt{3}}$	$\pi^+N_{\overline{10}}^0$	$-\frac{1}{3}$	$\pi^0N_{\overline{10}}^0$	$\frac{1}{3\sqrt{2}}$	$\pi^+\Sigma_{\overline{10}}^0$	$-\frac{\sqrt{2}}{3}$
$K^0N_{\overline{10}}^+$	$-\frac{1}{\sqrt{3}}$	$\pi^0N_{\overline{10}}^+$	$-\frac{1}{3\sqrt{2}}$	$\pi^-N_{\overline{10}}^+$	$-\frac{1}{3}$	$\pi^0\Sigma_{\overline{10}}^+$	$-\frac{\sqrt{2}}{3}$
$\eta_8\Theta^+$	$-\frac{2}{\sqrt{6}}$	$\eta_8N_{\overline{10}}^+$	$-\frac{1}{\sqrt{6}}$	$\eta_8N_{\overline{10}}^0$	$-\frac{1}{\sqrt{6}}$	$K^+\Xi_{\overline{10}}^0$	$\frac{1}{3}$
		$K^+\Sigma_{\overline{10}}^0$	$\frac{\sqrt{2}}{3}$	$K^+\Sigma_{\overline{10}}^-$	$\frac{2}{3}$	$K^0\Xi_{\overline{10}}^+$	$-\frac{1}{\sqrt{3}}$
		$K^0\Sigma_{\overline{10}}^+$	$-\frac{2}{3}$	$K^-\Theta^+$	$\frac{1}{\sqrt{3}}$	$\bar{K}^0N_{\overline{10}}^+$	$-\frac{2}{3}$
		$\bar{K}^0\Theta^+$	$-\frac{1}{\sqrt{3}}$	$K^0\Sigma_{\overline{10}}^0$	$-\frac{\sqrt{2}}{3}$		
$\Sigma_{\overline{10}}^{0}$		$\Sigma_{\overline{10}}^{-}$		$\Xi_{\overline{10}}^{+}$		$\Xi_{\overline{10}}^{0}$	
$\pi^+\Sigma_{\overline{10}}^-$	$-\frac{\sqrt{2}}{3}$	$\pi^0\Sigma_{\overline{10}}^-$	$\frac{\sqrt{2}}{3}$	$\pi^+\Xi_{\overline{10}}^0$	$-\frac{1}{\sqrt{3}}$	$\pi^+\Xi_{\overline{10}}^-$	$-\frac{2}{3}$
$\pi^-\Sigma_{\overline{10}}^+$	$-\frac{\sqrt{2}}{3}$	$\pi^-\Sigma_{\overline{10}}^0$	$-\frac{\sqrt{2}}{3}$	$\pi^0\Xi_{\overline{10}}^+$	$-\frac{1}{\sqrt{2}}$	$\pi^0\Xi_{\overline{10}}^0$	$-\frac{1}{3\sqrt{2}}$
$K^+\Xi_{\overline{10}}^-$	$\frac{\sqrt{2}}{3}$	$K^+\Xi_{\overline{10}}^{--}$	$\frac{1}{\sqrt{3}}$	$\eta_8\Xi_{\overline{10}}^+$	$\frac{1}{\sqrt{6}}$	$\eta_8\Xi_{\overline{10}}^0$	$\frac{1}{\sqrt{6}}$
$K^-N_{\overline{10}}^+$	$\frac{\sqrt{2}}{3}$	$K^0\Xi_{\overline{10}}^-$	$-\frac{1}{3}$	$\bar{K}^0\Sigma_{\overline{10}}^+$	$-\frac{1}{\sqrt{3}}$	$\pi^-\Xi_{\overline{10}}^+$	$-\frac{1}{\sqrt{3}}$
$K^0\Xi_{\overline{10}}^0$	$-\frac{\sqrt{2}}{3}$	$K^-N_{\overline{10}}^0$	$\frac{2}{3}$			$\bar{K}^0\Sigma_{\overline{10}}^0$	$-\frac{\sqrt{2}}{3}$
$\bar{K}^0N_{\overline{10}}^0$	$-\frac{\sqrt{2}}{3}$					$K^-\Sigma_{\overline{10}}^+$	$\frac{1}{3}$
$\Xi_{\overline{10}}^{-}$		$\Xi_{\overline{10}}^{--}$					
$\pi^+\Xi_{\overline{10}}^{--}$	$-\frac{1}{\sqrt{3}}$	$\pi^0\Xi_{\overline{10}}^{--}$	$\frac{1}{\sqrt{2}}$				
$\pi^0\Xi_{\overline{10}}^{-}$	$\frac{1}{3\sqrt{2}}$	$\pi^-\Xi_{\overline{10}}^{-}$	$-\frac{1}{\sqrt{3}}$				
$\pi^-\Xi_{\overline{10}}^{0}$	$-\frac{2}{3}$	$\eta_8\Xi_{\overline{10}}^{--}$	$\frac{1}{\sqrt{6}}$				
$\eta_8\Xi_{\overline{10}}^{-}$	$\frac{1}{\sqrt{6}}$	$K^-\Sigma_{\overline{10}}^{-}$	$\frac{1}{\sqrt{3}}$				
$\bar{K}^0\Sigma_{\overline{10}}^{-}$	$-\frac{1}{3}$						
$K^-\Sigma_{\overline{10}}^{0}$	$\frac{\sqrt{2}}{3}$						

TABLE IX. Couplings of the heavy pentaquark antisextet S_{ij} with the pseudoscalar meson octet π_j^i. The universal coupling constant $-(1/F_\pi)\mathcal{G}_S$ is omitted.

$\Xi_{5c}^{--}(\Xi_{5b}^{-})$		$\Xi_{5c}^{-}(\Xi_{5b}^{0})$		$\Xi_{5c}^{0}(\Xi_{5b}^{+})$	
$\pi^0\Xi_{5c}^{--}(\Xi_{5b}^{-})$	$\frac{1}{\sqrt{2}}$	$\pi^+\Xi_{5c}^{--}(\Xi_{5b}^{-})$	$-\frac{1}{\sqrt{2}}$	$\pi^+\Xi_{5c}^{-}(\Xi_{5b}^{0})$	$-\frac{1}{\sqrt{2}}$
$\pi^-\Xi_{5c}^{-}(\Xi_{5b}^{0})$	$-\frac{1}{\sqrt{2}}$	$\pi^-\Xi_{5c}^{0}(\Xi_{5b}^{+})$	$-\frac{1}{\sqrt{2}}$	$\pi^0\Xi_{5c}^{0}(\Xi_{5b}^{+})$	$-\frac{1}{\sqrt{2}}$
$\eta_8\Xi_{5c}^{--}(\Xi_{5b}^{-})$	$\frac{1}{\sqrt{6}}$	$\eta_8\Xi_{5c}^{-}(\Xi_{5b}^{0})$	$\frac{1}{\sqrt{6}}$	$\eta_8\Xi_{5c}^{0}(\Xi_{5b}^{+})$	$\frac{1}{\sqrt{6}}$
$K^-\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$	$\frac{1}{\sqrt{2}}$	$\bar{K}^0\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$	$-\frac{1}{2}$	$\bar{K}^0\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$	$-\frac{1}{\sqrt{2}}$
		$K^-\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$	$\frac{1}{2}$		
$\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$		$\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$		$\Theta_{5c}^{0}(\Theta_{5b}^{+})$	
$\pi^0\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$	$\frac{1}{2\sqrt{2}}$	$\pi^+\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$	$-\frac{1}{2}$	$K^+\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$	$\frac{1}{\sqrt{2}}$
$\pi^-\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$	$-\frac{1}{2}$	$\pi^0\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$	$-\frac{1}{2\sqrt{2}}$	$K^0\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$	$-\frac{1}{\sqrt{2}}$
$\eta_8\Sigma_{5c}^{-}(\Sigma_{5b}^{0})$	$-\frac{1}{2\sqrt{6}}$	$\eta_8\Sigma_{5c}^{0}(\Sigma_{5b}^{+})$	$-\frac{1}{2\sqrt{6}}$	$\eta_8\Theta_{5c}^{0}(\Theta_{5b}^{+})$	$-\frac{2}{\sqrt{6}}$
$K^+\Xi_{5c}^{--}(\Xi_{5b}^{-})$	$\frac{1}{\sqrt{2}}$	$K^+\Xi_{5c}^{-}(\Xi_{5b}^{0})$	$\frac{1}{2}$		
$K^0\Xi_{5c}^{-}(\Xi_{5b}^{0})$	$-\frac{1}{2}$	$K^0\Xi_{5c}^{0}(\Xi_{5b}^{+})$	$-\frac{1}{\sqrt{2}}$		
$K^-\Theta_{5c}^{0}(\Theta_{5b}^{+})$	$\frac{1}{\sqrt{2}}$	$\bar{K}^0\Theta_{5c}^{0}(\Theta_{5b}^{+})$	$-\frac{1}{\sqrt{2}}$		

TABLE X. Couplings of the heavy pentaquark triplet T^i with the pseudoscalar meson octet π^i_j. The universal coupling constant $\mathcal{G}_{TAT}$ is omitted.

$\Sigma'^0_{5c}(\Sigma'^+_{5b})$		$\Sigma'^-_{5c}(\Sigma'^0_{5b})$		$\Xi'^-_{5c}(\Xi'^0_{5b})$	
$\pi^+\Sigma'^-_{5c}(\Sigma'^0_{5b})$	1	$\pi^0\Sigma'^-_{5c}(\Sigma'^0_{5b})$	$-\frac{1}{\sqrt{2}}$	$\eta_8\Xi'^-_{5c}(\Xi'^0_{5b})$	$-\frac{2}{\sqrt{6}}$
$\pi^0\Sigma'^0_{5c}(\Sigma'^+_{5b})$	$\frac{1}{\sqrt{2}}$	$\eta_8\Sigma'^-_{5c}(\Sigma'^0_{5b})$	$\frac{1}{\sqrt{6}}$	$\bar{K}^0\Sigma'^-_{5c}(\Sigma'^0_{5b})$	1
$\eta_8\Sigma'^0_{5c}(\Sigma'^+_{5b})$	$\frac{1}{\sqrt{6}}$	$\pi^-\Sigma'^0_{5c}(\Sigma'^+_{5b})$	1	$K^-\Sigma'^0_{5c}(\Sigma'^+_{5b})$	1
$K^0\Xi'^-_{5c}(\Xi'^0_{5b})$	1	$K^0\Xi'^-_{5c}(\Xi'^0_{5b})$	1		

The ratio $BR[\Xi^{--}_{\bar{10}} \to \Sigma^- K^-]/BR[\Xi^{--}_{\bar{10}} \to \Xi^-\pi^-]$ is different for positive and negative parity pentaquark, which is independent of models. This property has been proposed to determine the parity of the pentaquark antidecuplet in Ref. [55]. If we assume $\Gamma(\Xi^{--}_{\bar{10}} \to \Xi^-_{8,2}\pi^-)$ is significantly smaller than $\Gamma(\Xi^{--}_{\bar{10}} \to \Sigma^- K^-)$ and $\Gamma(\Xi^{--}_{\bar{10}} \to \Xi^-\pi^-)$ because of phase space suppression, then the ratio $\Gamma_{\Xi^{--}_{\bar{10}}}/\Gamma_{\Theta^+}$ is about 4.1 for positive parity and 2.0 for negative parity with $m_{\Xi^{--}_{\bar{10}}} = 1862$ MeV. If their widths can be measured accurately, the parity of the antidecuplet can be determined [55].

V. PENTAQUARK MIXING BETWEEN DIFFERENT MULTIPLETS

Generally speaking, the quark mass correction may induce possible mixing of pentaquarks between different $SU(3)_F$ multiplets. By replacing the axial-vector field in Eq. (16) by the mass matrix m^c, we can find the following mixing Lagrangians. We use the following three cases for illustration.

$$\mathcal{L}^m_{O_1P} = C_{O_1P}\bar{O}_1 m^c P + h.c., \qquad \mathcal{L}^m_{\Lambda_1 O_2} = C_{\Lambda_1 O_2}\mathrm{Tr}\bar{\Lambda}_1 m^c O_2 + h.c., \qquad \mathcal{L}^m_{ST} = C_{ST}\bar{S} m^c T + h.c. \tag{34}$$

Such a mixing occurs only between multiplets with the same parity. Expanding $\mathcal{L}^m_{O_1P}$ $\mathcal{L}^m_{\Lambda_1 O_2}$, and $\mathcal{L}^m_{ST}$, we have

$$\begin{aligned}\mathcal{L}^m_{O_1P} &= \frac{2}{\sqrt{3}}C_{O_1P}(m_s - \hat{m})[\bar{\Sigma}^-_{8,1}\Sigma^-_{\bar{10}} + \bar{\Sigma}^0_{8,1}\Sigma^0_{\bar{10}} - \bar{\Sigma}^+_{8,1}\Sigma^+_{\bar{10}} + \bar{p}_{8,1}N^+_{\bar{10}} + \bar{n}_{8,1}N^0_{\bar{10}}] + h.c.,\\ \mathcal{L}^m_{\Lambda_1 O_2} &= \frac{4}{\sqrt{6}}C_{\Lambda_1 O_2}(\hat{m} - m_s)\bar{\Lambda}_1\Lambda_{8,2} + h.c., \qquad \mathcal{L}^m_{ST} = \sqrt{2}C_{ST}(m_s - \hat{m})[\bar{\Sigma}^-_{5c}\Sigma'^-_{5c} - \bar{\Sigma}^0_{5c}\Sigma'^0_{5c}] + h.c.,\end{aligned} \tag{35}$$

where the heavy quark is taken to be the charm quark.

Because of the mixing of pentaquarks between different multiplets, the mass eigenstates differ from the flavor eigenstates. If the mixing parameters C_{O_1P} etc. are determined from future experiments, one may diagonalize the pentaquark mixing mass matrix to get the mass eigenstates. We note that the mixing angles may deviate significantly from that in the ideal mixing case as assumed in Refs. [20,53].

VI. MODIFICATION OF PENTAQUARK DECAY PATTERNS BY SU(3)-BREAKING QUARK MASS MATRIX

The SU(3)-breaking quark mass matrix modifies the decay patterns of pentaquarks. Firstly the coupling constant in the SU(3) flavor symmetry limit receives $\mathcal{O}[m^2_{\pi,K,\eta}/(4\pi F_\pi)^2]$ corrections. Secondly some decay modes which are forbidden in the symmetry limit are allowed now.

When pentaquarks decay into the pseudoscalar meson octet and nucleon octet, one may need to consider the following corrections in addition to the SU(3) flavor symmetric Lagrangian:

$$\begin{aligned}\mathcal{L}^m_{PAB} &= \alpha_{PAB}\bar{P}m^c\Gamma_P\gamma^\mu(G_{8_1})_\mu + \beta_{PAB}\bar{P}m^c\Gamma_P\gamma^\mu(G_{8_2})_\mu + \gamma_{PAB}\bar{P}m^c\Gamma_P\gamma^\mu(G_{\bar{10}})_\mu + \delta_{PAB}\bar{P}m^c\Gamma_P\gamma^\mu(G_{27})_\mu + h.c.,\\ \mathcal{L}^m_{O_1AB} &= \alpha_{O_1AB}\mathrm{Tr}[\bar{O}_1 m^c]\gamma_5\gamma^\mu(G_1)_\mu + \beta^1_{O_1AB}\mathrm{Tr}[\bar{O}_1 m^c\gamma_5\gamma^\mu(G_{8_1})_\mu] + \beta^2_{O_1AB}\mathrm{Tr}[\bar{O}_1 m^c\gamma_5\gamma^\mu(G_{8_2})_\mu]\\ &\quad + \gamma^1_{O_1AB}\mathrm{Tr}[\bar{O}_1\gamma_5\gamma^\mu(G_{8_1})_\mu m^c] + \gamma^2_{O_1AB}\mathrm{Tr}[\bar{O}_1\gamma_5\gamma^\mu(G_{8_2})_\mu m^c] + \delta_{O_1AB}\bar{O}_1 m^c\gamma_5\gamma^\mu(G_{10})_\mu\\ &\quad + \eta_{O_1AB}\bar{O}_1 m^c\gamma_5\gamma^\mu(G_{\bar{10}})_\mu + \kappa_{O_1AB}\bar{O}_1 m^c\gamma_5\gamma^\mu(G_{27})_\mu + h.c.,\\ \mathcal{L}^m_{\Lambda_1AB} &= \alpha_{\Lambda_1AB}\bar{\Lambda}_1\mathrm{Tr}[m^c\Gamma_{\Lambda_1}\gamma^\mu(G_{8_1})_\mu] + \beta_{\Lambda_1AB}\bar{\Lambda}_1\mathrm{Tr}[m^c\Gamma_{\Lambda_1}\gamma^\mu(G_{8_2})_\mu] + h.c.,\end{aligned} \tag{36}$$

where

$$
\begin{aligned}
(G_1)_\mu &= A^i_{\mu,j}B^j_i,\\
(G_{8_1})^i_{\mu,j} &= A^i_{\mu,m}B^m_j - \frac{1}{3}\delta^i_j A^m_{\mu,n}B^n_m,\\
(G_{8_2})^i_{\mu,j} &= A^m_{\mu,j}B^i_m - \frac{1}{3}\delta^i_j A^m_{\mu,n}B^n_m,\\
(G_{10})^{ijk}_\mu &= \epsilon^{iab}(A^j_{\mu,a}B^k_b + A^k_{\mu,a}B^j_b) + \epsilon^{jab}(A^i_{\mu,a}B^k_b + A^k_{\mu,a}B^i_b) + \epsilon^{kab}(A^i_{\mu,a}B^j_b + A^j_{\mu,a}B^i_b),\\
(G_{\bar{10}})_{\mu,ijk} &= \epsilon_{iab}(A^a_{\mu,j}B^b_k + A^a_{\mu,k}B^b_j) + \epsilon_{jab}(A^a_{\mu,i}B^b_k + A^a_{\mu,k}B^b_i) + \epsilon_{kab}(A^a_{\mu,i}B^b_j + A^a_{\mu,j}B^b_i),\\
(G_{27})^{ij}_{\mu,kl} &= (A^i_{\mu,k}B^j_l + A^j_{\mu,k}B^i_l + A^i_{\mu,l}B^j_k + A^j_{\mu,l}B^i_k) - \frac{1}{5}\{\delta^i_k(A^j_{\mu,m}B^m_l + A^m_{\mu,l}B^j_m) + \delta^i_l(A^j_{\mu,m}B^m_k + A^m_{\mu,k}B^j_m)\\
&\quad + \delta^j_k(A^i_{\mu,m}B^m_l + A^m_{\mu,l}B^i_m) + \delta^j_l(A^i_{\mu,m}B^m_k + A^m_{\mu,k}B^i_m)\} + \frac{1}{10}(\delta^i_k\delta^j_k + \delta^i_l\delta^j_k)A^m_{\mu,n}B^n_m.
\end{aligned}
$$

For the antisextet heavy pentaquark state decaying to a triplet heavy pentaquark and one pseudoscalar meson, we have

$$
\begin{aligned}
\mathcal{L}^m_{SAT} &= \alpha_{SAT}\bar{S}m^c\Gamma_{ST}\gamma^\mu(H_3)_\mu + \beta_{SAT}\bar{S}m^c\Gamma_{ST}\gamma^\mu(H_{\bar{6}})_\mu\\
&\quad + \gamma_{SAT}\bar{S}m^c\Gamma_{ST}\gamma^\mu(H_{15})_\mu + h.c.,
\end{aligned}
\tag{37}
$$

where

$$
\begin{aligned}
(H_3)^i_\mu &= A^i_{\mu,a}T^a,\\
(H_{\bar{6}})_{\mu,ij} &= A^a_{\mu,j}T^b\epsilon_{abi} + A^a_{\mu,i}T^b\epsilon_{abj},\\
(H_{15})^{ij}_{\mu,k} &= A^i_{\mu,k}T^j + A^j_{\mu,k}T^i - \frac{1}{4}(\delta^i_k A^j_{\mu,a}T^a + \delta^j_k A^i_{\mu,a}T^a).
\end{aligned}
$$

Similarly, if heavy pentaquarks decay to heavy flavor mesons and octet baryons, the Lagrangians including the breaking effects are

$$
\begin{aligned}
\mathcal{L}^m_{SQB} &= \alpha_{SQB}\bar{S}m^c\Gamma_S(J_3) + \beta_{SQB}\bar{S}m^c\Gamma_S(J_{\bar{6}})\\
&\quad + \gamma_{SQB}\bar{S}m^c\Gamma_S(J_{15}) + h.c.,\\
\mathcal{L}^m_{TQB} &= \alpha_{TQB}\bar{T}m^c\Gamma_T(J_3) + \beta_{TQB}\bar{T}m^c\Gamma_T(J_{\bar{6}})\\
&\quad + \gamma_{TQB}\bar{T}m^c\Gamma_T(J_{15}) + h.c.,
\end{aligned}
\tag{38}
$$

where

$$
\begin{aligned}
&(J_3)^i = \bar{Q}^a B^i_a, \qquad (J_{\bar{6}})_{ij} = \bar{Q}^a B^b_j\epsilon_{abi} + \bar{Q}^a B^b_i\epsilon_{abj},\\
&(J_{15})^{ij}_k = \bar{Q}^j B^i_k + \bar{Q}^i B^j_k - \frac{1}{4}(\delta^i_k\bar{Q}^a B^j_a + \delta^j_k\bar{Q}^a B^i_a).
\end{aligned}
$$

In the case of the antisextet pentaquark decaying to the antidecuplet, we have

$$
\mathcal{L}^m_{SQP} = \alpha_{SQP}\bar{S}m^c\Gamma_{SP}(W_{\bar{6}}) + \beta_{SQP}\bar{S}m^c\Gamma_{SP}(W_{\bar{24}}) + h.c.,
\tag{39}
$$

where

$$
(W_{\bar{6}})_{ij} = \bar{Q}^k P_{ijk}, \qquad (W_{\bar{24}})^i_{jkl} = \bar{Q}^i P_{jkl}.
$$

The $SU(3)_F$-breaking effects will also modify transitions between different states in the same pentaquark multiplet.

$$
\begin{aligned}
\mathcal{L}^m_{PAP} &= \alpha_P\bar{P}m^c\gamma_5\gamma^\mu(X_8)_\mu + \beta_P\bar{P}m^c\gamma_5\gamma^\mu(X_{\bar{10}})_\mu\\
&\quad + \gamma_P\bar{P}m^c\gamma_5\gamma^\mu(X_{27})_\mu + \delta_P\bar{P}m^c\gamma_5\gamma^\mu(X_{\bar{35}})_\mu,
\end{aligned}
\tag{40}
$$

where

$$
\begin{aligned}
(X_8)^i_{\mu,j} &= A^c_{\mu,a}P_{bcj}\epsilon^{abi},\\
(X_{\bar{10}})_{\mu,ijk} &= A^a_{\mu,i}P_{ajk} + A^a_{\mu,j}P_{aik} + A^a_{\mu,k}P_{aij},\\
(X_{27})^{ij}_{\mu,kl} &= (A^i_{\mu,a}P_{bkl}\epsilon^{abj} + A^j_{\mu,a}P_{bkl}\epsilon^{abi}) - \frac{1}{5}(\delta^i_k A^c_{\mu,a}P_{bcl}\epsilon^{abj} + \delta^i_l A^c_{\mu,a}P_{bck}\epsilon^{abj} + \delta^j_k A^c_{\mu,a}P_{bcl}\epsilon^{abi} + \delta^j_l A^c_{\mu,a}P_{bck}\epsilon^{abi}),\\
(X_{\bar{35}})^i_{\mu,jklm} &= (A^i_{\mu,j}P_{klm} + A^i_{\mu,k}P_{jlm} + A^i_{\mu,l}P_{jkm} + A^i_{\mu,m}P_{jkl}) - \frac{1}{6}\{\delta^i_j(A^n_{\mu,k}P_{nlm} + A^n_{\mu,l}P_{nkm} + A^n_{\mu,m}P_{nkl})\\
&\quad + \delta^i_k(A^n_{\mu,j}P_{nlm} + A^n_{\mu,l}P_{njm} + A^n_{\mu,m}P_{njl}) + \delta^i_l(A^n_{\mu,j}P_{nkm} + A^n_{\mu,k}P_{njm} + A^n_{\mu,m}P_{njk})\\
&\quad + \delta^i_m(A^n_{\mu,j}P_{nkl} + A^n_{\mu,k}P_{njl} + A^n_{\mu,l}P_{njk})\}.
\end{aligned}
$$

$$
\mathcal{L}^m_{SAS} = \alpha_S\bar{S}m^c\gamma_5\gamma^\mu(Z_3)_\mu + \beta_S\bar{S}m^c\gamma_5\gamma^\mu(Z_{\bar{6}})_\mu + \gamma_S\bar{S}m^c\gamma_5\gamma^\mu(Z_{15})_\mu + \delta_S\bar{S}m^c\gamma_5\gamma^\mu(Z_{\bar{24}})_\mu,
\tag{41}
$$

where

$$
\begin{aligned}
(Z_3)^i_\mu &= A^l_{\mu,j} S_{lk} \epsilon^{ijk},\\
(Z_{\bar{6}})_{\mu,ij} &= A^k_{\mu,i} S_{jk} + A^k_{\mu,i} S_{ik},\\
(Z_{15})^{ij}_{\mu,k} &= A^i_{\mu,a} S_{bk} \epsilon^{abj} + A^j_{\mu,a} S_{bk} \epsilon^{abi} - \frac{1}{4}(\delta^i_k A^c_{\mu,a} S_{bc} \epsilon^{abj} + \delta^j_k A^c_{\mu,a} S_{bc} \epsilon^{abi}),\\
(Z_{\bar{24}})^i_{\mu,jkl} &= A^i_{\mu,j} S_{kl} + A^i_{\mu,k} S_{jl} + A^i_{\mu,l} S_{jk} - \frac{1}{5}\{\delta^i_j (A^a_{\mu,k} S_{al} + A^a_{\mu,l} S_{ak}) + \delta^i_k (A^a_{\mu,j} S_{al} + A^a_{\mu,l} S_{aj}) + \delta^i_l (A^a_{\mu,j} S_{ak} + A^a_{\mu,k} S_{aj})\}.
\end{aligned}
$$

TABLE XI. Strong decay modes of the three observed pentaquarks and other exotic pentaquarks with corresponding coupling constants. Y or N represents the decay mode which is kinematically allowed or forbidden in JW's model with the masses estimated in Ref. [20,47,48,52]. Y or N in the parentheses corresponds to the case of the heavy pseudoscalar meson being replaced by the heavy vector meson. Whenever the pentaquark lies very close to the threshold of the final state, we indicate this case with $*$.

Θ^+			$\Xi^{--}_{\bar{10}}$			$\Theta^0_c(\Theta^+_b)$		
$K^+ n$	$-\frac{1}{F_\pi} C_{PAB}$	Y	$\pi^- \Xi^-$	$-\frac{1}{F_\pi} C_{PAB}$	Y	$D^-(B^0) p$	$-C_{SQB}$	Y(Y)
$K^0 p$	$\frac{1}{F_\pi} C_{PAB}$	Y	$K^- \Sigma^-$	$\frac{1}{F_\pi} C_{PAB}$	Y	$\bar{D}^0(B^+) n$	C_{SQB}	Y(Y)
$K^+ N^0_{\bar{10}}$	$-\frac{1}{\sqrt{3}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$\pi^- \Xi^-_{\bar{10}}$	$\frac{1}{\sqrt{3}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$D^-(B^0) N^+_{\bar{10}}$	$-\frac{1}{\sqrt{3}} C_{SQP}$	N(N)
$K^0 N^+_{\bar{10}}$	$\frac{1}{\sqrt{3}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$\pi^0 \Xi^{--}_{\bar{10}}$	$-\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$\bar{D}^0(B^+) N^0_{\bar{10}}$	$\frac{1}{\sqrt{3}} C_{SQP}$	N(N)
$\eta_8 \Theta^+$	$\frac{2}{\sqrt{6}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$\eta_8 \Xi^{--}_{\bar{10}}$	$-\frac{1}{\sqrt{6}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$D^-_s(B^0_s) \Theta^+$	C_{SQP}	N(N)
$K^+ n_{8,1}$	$-\frac{1}{F_\pi} C_{O_1AP}$	N	$K^- \Sigma^-_{\bar{10}}$	$-\frac{1}{\sqrt{3}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$D^-(B^0) p_{8,1}$	$-C_{SQO_1}$	N(N)
$K^0 p_{8,1}$	$\frac{1}{F_\pi} C_{O_1AP}$	N	$\pi^- \Xi^-_{8,1}$	$-\frac{1}{F_\pi} C_{O_1AP}$	$*$	$\bar{D}^0(B^+) n_{8,1}$	C_{SQO_1}	N(N)
$K^+ n_{8,2}$	$-\frac{1}{F_\pi} C_{O_2AP}$	N	$K^- \Sigma^-_{8,1}$	$\frac{1}{F_\pi} C_{O_1AP}$	N	$D^-(B^0) p_{8,2}$	$-C_{SQO_2}$	N(N)
$K^0 p_{8,2}$	$\frac{1}{F_\pi} C_{O_2AP}$	N	$\pi^- \Xi^-_{8,2}$	$-\frac{1}{F_\pi} C_{O_2AP}$	Y	$\bar{D}^0(B^+) n_{8,2}$	C_{SQO_2}	N(N)
			$K^- \Sigma^-_{8,2}$	$\frac{1}{F_\pi} C_{O_2AP}$	$*$	$K^+ \Sigma^-_{5c}$	$-\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_S)$	N
						$K^0 \Sigma^0_{5c}$	$\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_S)$	N
						$\eta_8 \Theta^0_c$	$\frac{2}{\sqrt{6}}(\frac{1}{F_\pi} \mathcal{G}_S)$	N
						$K^0 \Sigma'^0_{5c}$	$-\frac{1}{F_\pi} C_{SAT}$	$*$
						$K^+ \Sigma'^-_{5c}$	$\frac{1}{F_\pi} C_{SAT}$	$*$
$\Xi^+_{\bar{10}}$			$\Xi^{--}_{5c}(\Xi^-_{5b})$			$\Xi^0_{5c}(\Xi^+_{5b})$		
$\pi^+ \Xi^0$	$-\frac{1}{F_\pi} C_{PAB}$	Y	$D^-(B^0) \Xi^-$	C_{SQB}	Y(Y)	$\bar{D}^0(B^+) \Xi^0$	$-C_{SQB}$	Y(Y)
$\bar{K}^0 \Sigma^+$	$\frac{1}{F_\pi} C_{PAB}$	Y	$D^-_s(B^0_s) \Sigma^-$	$-C_{SQB}$	Y(Y)	$D^-_s(B^0_s) \Sigma^+$	C_{SQB}	Y(Y)
$\pi^+ \Xi^0_{\bar{10}}$	$\frac{1}{\sqrt{3}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$\bar{D}^0(B^+) \Xi^{--}_{\bar{10}}$	C_{SQP}	N(N)	$\bar{D}^0(B^+) \Xi^0_{\bar{10}}$	$\frac{1}{\sqrt{3}} C_{SQP}$	N(N)
$\pi^0 \Xi^+_{\bar{10}}$	$\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$D^-(B^0) \Xi^-_{\bar{10}}$	$-\frac{1}{\sqrt{3}} C_{SQP}$	N(N)	$D^-(B^0) \Xi^+_{\bar{10}}$	$-C_{SQP}$	N(N)
$\eta_8 \Xi^+_{\bar{10}}$	$-\frac{1}{\sqrt{6}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$D^-_s(B^0_s) \Sigma^-_{\bar{10}}$	$\frac{1}{\sqrt{3}} C_{SQP}$	N(N)	$D^-_s(B^0_s) \Sigma^+_{\bar{10}}$	$\frac{1}{\sqrt{3}} C_{SQP}$	N(N)
$\bar{K}^0 \Sigma^+_{\bar{10}}$	$\frac{1}{\sqrt{3}}(\frac{1}{F_\pi} \mathcal{G}_P)$	N	$D^-(B^0) \Xi^-_{8,1}$	C_{SQO_1}	N(N)	$\bar{D}^0(B^+) \Xi^0_{8,1}$	$-C_{SQO_1}$	N(N)
$\pi^+ \Xi^0_{8,1}$	$-\frac{1}{F_\pi} C_{O_1AP}$	$*$	$D^-_s(B^0_s) \Sigma^-_{8,1}$	$-C_{SQO_1}$	N(N)	$D^-_s(B^0_s) \Sigma^+_{8,1}$	C_{SQO_1}	N(N)
$\bar{K}^0 \Sigma^+_{8,1}$	$\frac{1}{F_\pi} C_{O_1AP}$	N	$D^-(B^0) \Xi^-_{8,2}$	C_{SQO_2}	Y($*$)	$\bar{D}^0(B^+) \Xi^0_{8,2}$	$-C_{SQO_2}$	Y($*$)
$\pi^+ \Xi^0_{8,2}$	$-\frac{1}{F_\pi} C_{O_2AP}$	Y	$D^-_s(B^0_s) \Sigma^-_{8,2}$	$-C_{SQO_2}$	Y(Y)	$D^-_s(B^0_s) \Sigma^+_{8,2}$	C_{SQO_2}	Y(Y)
$\bar{K}^0 \Sigma^+_{8,2}$	$\frac{1}{F_\pi} C_{O_2AP}$	$*$	$\pi^0 \Xi^{--}_{5c}(\Xi^-_{5b})$	$-\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_S)$	N	$\pi^+ \Xi^-_{5c}(\Xi^0_{5b})$	$\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_s)$	N
			$\pi^- \Xi^-_{5c}(\Xi^0_{5b})$	$\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_S)$	N	$\pi^0 \Xi^0_{5c}(\Xi^+_{5b})$	$\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_s)$	N
			$\eta_8 \Xi^{--}_{5c}(\Xi^-_{5b})$	$-\frac{1}{\sqrt{6}}(\frac{1}{F_\pi} \mathcal{G}_S)$	N	$\eta_8 \Xi^0_{5c}(\Xi^+_{5b})$	$-\frac{1}{\sqrt{6}}(\frac{1}{F_\pi} \mathcal{G}_s)$	N
			$K^- \Sigma^-_{5c}(\Sigma^0_{5b})$	$-\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_S)$	Y	$\bar{K}^0 \Sigma^0_{5c}(\Sigma^+_{5b})$	$\frac{1}{\sqrt{2}}(\frac{1}{F_\pi} \mathcal{G}_s)$	Y
			$K^- \Sigma'^-_{5c}(\Sigma'^0_{5b})$	$-\frac{1}{F_\pi} C_{SAT}$	Y	$\bar{K}^0 \Sigma'^0_{5c}(\Sigma'^+_{5b})$	$\frac{1}{F_\pi} C_{SAT}$	Y
			$\pi^- \Xi'^-_{5c}(\Xi'^0_{5b})$	$\frac{1}{F_\pi} C_{SAT}$	Y	$\pi^+ \Xi'^-_{5c}(\Xi'^0_{5b})$	$-\frac{1}{F_\pi} C_{SAT}$	Y

$$\mathcal{L}^{m}_{TAT} = \alpha_T \bar{T} m^c \gamma_5 \gamma^\mu (H_3)_\mu + \beta_T \bar{T} m^c \gamma_5 \gamma^\mu (H_{\bar{6}})_\mu + \gamma_T \bar{T} m^c \gamma_5 \gamma^\mu (H_{15})_\mu, \tag{42}$$

where $(H_3)^i_\mu$, $(H_{\bar{6}})_{\mu,ij}$, and $(H_{15})^{ij}_{\mu,k}$ are just those in $\mathcal{L}^m_{SAT}$.

The Clebsch-Gordan coefficients for the different decay modes in the above interaction Lagrangians can be derived using the useful tables collected in Ref. [61].

VII. SUMMARY AND DISCUSSIONS

In the framework of Jaffe and Wilczek's diquark model, these pentaquark multiplets include one even-parity antidecuplet, one even-parity octet, one odd-parity octet, one odd-parity singlet, one even-parity heavy antisextet, and one odd-parity heavy triplet. We have constructed the effective chiral Lagrangians involving the above six $SU(3)_F$ pentaquark multiplets.

After taking into account the symmetry-breaking corrections from the nonzero quark mass matrix, we have derived the Gell-Mann–Okubo mass relations for different pentaquark multiplets. We have also given the Coleman-Glashow relations for heavy pentaquark magnetic moments.

We have discussed the couplings of pentaquarks with other pentaquarks and pseudoscalar mesons in the SU(3) flavor symmetry limit. We have also investigated the possible decays of pentaquarks to the Δ decuplet and pseudoscalar mesons. The SU(3) breaking quark mass matrix induces mixing of pentaquarks between different multiplets and modifies their decay patterns.

If symmetry and kinematics allow, the most efficient decay mechanism of pentaquarks is for the four quarks and one antiquark to regroup with each other into a three-quark baryon and a meson. This is in contrast to the $^3P_0$ decay model for the ordinary hadrons. This regrouping is coined as the "fall-apart" mechanism, which leads to selection rules in the octet pentaquark decays. This fall-apart decay mechanism can be taken care of in the chiral Lagrangian formalism through keeping the flavor indices explicitly [52,53]. The couplings of two octet baryons with a pseudoscalar meson with the general F/D flavor structure are presented in Table III. It is pointed out that the fall-apart mechanism requires $b = \frac{1}{3}$ for the even-parity pentaquark octet decay to the nucleon octet and pseudoscalar meson octet [53]. In contrast, this mechanism requires $b = -1$ for the odd-parity pentaquark octet decay to the nucleon octet and pseudoscalar meson octet [52].

We collect all the possible decay modes of Θ^+, $\Xi^{--}_{\bar{10}}$, and Θ^0_c in Table XI with corresponding coupling constants in the chiral limit. We find that $\Xi^{--}_{\bar{10}}$ can also decay into $\Xi^-_{8,2}$ via the emission of a π^-. The heavy pentaquark Θ^0_c has four decay channels, $D^- p$, $\bar{D}^0 n$, $D^{*-} p$, and $\bar{D}^{*0} n$. The decay modes and corresponding couplings of the other exotic members in the antidecuplet and antisextet are also included in the table. Using the mass of Θ^0_c from the H1 experiment as a constraint, we have updated our old mass estimates of heavy pentaquarks in Ref. [47] and use the new values to analyze the possible decay modes in Table XI. Hopefully our present study may help the future experimental discovery of those missing pentaquarks.

ACKNOWLEDGMENTS

S. L. Z. thanks Professor W.-Y. P. Hwang and COSPA center at National Taiwan University for the hospitality. This project was supported by the National Natural Science Foundation of China under Grant No. 10375003, Ministry of Education of China, FANEDD and SRF for ROCS, SEM. We also thank one of the referees for informing us of Ref. [61], which provides a cross-check of the results presented in this work.

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