(p,d) and (p,t) Reactions on B^{11}, C^{14}, O^{16}, and O^{18}†

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Pickup (p,d) reactions on B^{11}, C^{14}, O^{16}, and O^{18} have been observed at proton energies ranging from 17.6 to 20 MeV. Also, (p,t) reactions on C^{14} and O^{18} were observed. Angular distributions were taken and absolute differential cross sections were obtained for all reactions except $C^{14}(p,d)C^{13}$ and $C^{14}(p,t)C^{12}$. The deuteron results were analyzed using the plane-wave Butler formalism and reduced widths and spectroscopic factors were calculated. The results indicate that the single-particle reduced width is a function of the binding energy of the transferred nucleon, which may indicate that the plane-wave approximation is not completely adequate for analyzing stripping and pickup experiments on light nuclei. The (p,t) results, when compared to those for the inverse (t,p) reactions, indicate that such experiments may possibly be described using a simple stripping formalism.

I. INTRODUCTION

IN the past ten years many stripping and pickup experiments have been performed. Much of the work has been done using (d,p) stripping experiments. Recently, the need for information about nuclei unavailable to stripping experiments has led to increasing interest in pickup experiments. Most of the pickup experiments reported have been (d,t) reactions, although a growing number of (p,d) experiments have been performed and reported on. [For example, Standing, Reynolds, and Bennett have performed (p,d) experiments on many light nuclei.[1–3]] This is rather unfortunate since in the analysis of the results of (p,d) experiments the stripping transform is rather well known, whereas, in the analysis of (d,t) experiments a somewhat arbitrary evaluation of the normalization of the triton's asymptotic wave function leads to an uncertainty in the results obtained.

A study of the Q values for these reactions reveals the reason for this situation. To get meaningful, easily analyzed data, 10- to 25-MeV deuterons are required for (d,t) reactions, while 15- to 30-MeV protons are necessary for (p,d) reactions. The number of deuteron accelerators in the required energy range is relatively large in comparison to the number of proton accelerators in the necessary range. However, advances in accelerator technology give reason to believe that (p,d) experiments will become a much more useful tool in the near future.

The experiments described here consist of observations of deuterons—and tritons, where possible—resulting from protons incident on B^{11}, C^{14}, O^{16}, and O^{18}. C^{14} and O^{18} were picked because it was felt that it would be useful to check the results of (d,t) experiments on these nuclei; also, it was felt that these nuclei would be the light nuclei likely to have an observable (p,t) cross section. O^{16} was chosen because a check on the surprising $O^{16}(d,t)O^{15}$ results[4]—which indicated that the reduced width for this reaction is an order of magnitude smaller than it theoretically should be—was considered to be of great interest. B^{11} was chosen because it was felt that more experimental information about this theoretically difficult mass region would be useful. The analysis of experimental results was performed using the formulas and terminology used in the review article by MacFarlane and French.[4]

II. EXPERIMENTAL METHODS

These experiments were performed in a 60-in. scattering chamber using 17.6- to 20-MeV protons from the Princeton variable energy cyclotron. Beam collimation was done at the entrance to the scattering chamber with either a $\frac{1}{4}$- or $\frac{1}{8}$-in. diameter carbon collimator. After passing through the target the undeflected protons were collected in a Faraday cage and the current was integrated. The charge collected was used to normalize data taken at different angles.

The proton beam energy was calibrated and stabilized by making use of an end point ionization detector described by Schrank.[5] A feedback from this device to the magnet control of the cyclotron kept the mean beam energy within approximately 50 keV of the selected energy. This was adequate stability for these experiments, since the energy spread of the beam passing through the $\frac{1}{8}$-in. collimator was 140–180 keV and the energy spread of the beam after passing through the $\frac{1}{4}$-in. collimator was 200–250 keV.

A ΔE, E counter telescope was used for particle detection. The ΔE counter used was a double-celled proportional counter. It was filled with a gas consisting of 90% argon, 10% methane, and the path length in gas was 3 in. This counter was operated at pressures ranging from 20 to 45 in. of mercury. The E counter was usually a diffused junction solid-state detector, although both NaI(Tl) and CsI(Tl) E counters were also used. Collimation was done at the entrance to the

† This work was supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.
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[1] K. G. Standing, Phys. Rev. **101**, 152 (1956).
[2] J. B. Reynolds and K. G. Standing, Phys. Rev. **101**, 158 (1956).
[3] E. F. Bennett, Phys. Rev. **122**, 595 (1961).
[4] M. H. MacFarlane and J. B. French, Revs. Mod. Phys. **32**, 567 (1960).
[5] G. Schrank, Rev. Sci. Instr. **26**, 677 (1955).

ΔE proportional counter and there was negligible gas-scattering loss of particles in the ΔE counter.

A block diagram of the electronics used is shown in Fig. 1. The ΔE and E pulses were multiplied by a transistor multiplier designed by R. L. Chase whose output was proportional to $\Delta E(E+k\Delta E+E_0)$ which is in turn proportional to MZ^2; the output of the multiplier was analyzed by a 20-channel analyzer. A typical multiplied pulse spectrum is shown in Fig. 2. The pulses falling in the channels of the multiplied pulse spectrum corresponding to deuterons and tritons were then used to gate a 200-channel analyzer which recorded the spectra of the E pulses of these particles. Deuteron and triton spectra were stored in separate subgroups of the analyzer.

The particle selection system was adjusted at the start of each run to give good separation of deuterons from the $F^{19}(p,d)F^{18}$ reaction. Separation of deuterons was usually very good except at angles less than 15° in lab system. At these angles it was necessary to reduce the beam current drastically in order to minimize pile-up. Except at these angles, the probable errors in the angular distributions are almost entirely due to the statistics of counting. No attempt, other than choosing carefully the lower level of the deuteron gate, was made to compensate for the imperfect resolution of the multiplied pulse spectrum. It was estimated, however, that corrections for the loss of deuteron counts below the lower level of the gate would raise the measured cross sections by less than 5%.

The level structures of the nuclei observed in these experiments are well known, so the Q values and l values for the deuterons observed were known before the experiments. Energy calibrations using initially $F^{19}(p,d)F^{18}$ deuterons and later also $O^{18}(p,d)O^{17}$ deuterons as standards were done before the observation of each new reaction. Then, using observed energies and l values, deuterons leaving the final nucleus in the various final states were identified.

The probable errors in θ^2 listed in the discussion of experimental results are due primarily to statistics. In the cases where the cross section was determined by reference to a standard cross section, an additional source of error was the quoted probable error in the standard cross section. Finally, in certain cases—the most obvious example being the 3.68-MeV and 3.86-MeV states in the $C^{14}(p,d)C^{13}$ experiment—an additional source of error was the difficulty in resolving two closely spaced peaks of deuterons.

III. TARGETS

B^{11}

B^{11} was available in the form of powdered boron, enriched to 89% in B^{11}. A target was prepared from this by floating a suspension of boron in polystyrene and benzene on water. The benzene evaporated leaving a foil containing about 1 mg/cm² polystyrene and about 1.5 mg/cm² boron. In order to secure the differential cross section a target of natural B_4C was used. This was prepared by pouring a slurry of B_4C in distilled water onto 0.1-mil Pt foil and letting the water evaporate. Very acceptable, though fragile, targets could be made in this manner.

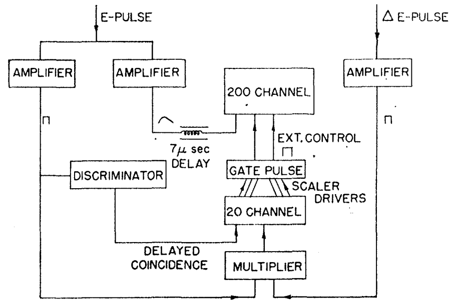

FIG. 1. Block diagram of electronics used in this experiment. Pulses from counter telescope are amplified by two delay line clipped amplifiers and fed into pulse multiplier. The output of the multiplier is analyzed by 20-channel analyzer which is gated by an energy discriminator. Pulses falling in the appropriate channels of the 20-channel analyzer are used to gate a 200-channel analyzer. The input to the 200-channel analyzer is the energy pulses which have been amplified by a low-noise amplifier.

C^{14}

The C^{14} foil consisted of elemental carbon, enriched to 80% in C^{14} deposited on a gold foil of about 1.5 mg/cm². The C^{14} target contained a large contamination of hydrogen, oxygen, and natural carbon. This target contained 0.27 ± 0.1 mg/cm² of C^{14}.[6]

O^{16}

Foils of mylar and boric acid (H_3BO_3) were used. The boric acid was prepared by depositing a slurry of benzene and boric acid on painter's foil (approximately 0.2 mg/cm² Al), letting the slurry almost dry with

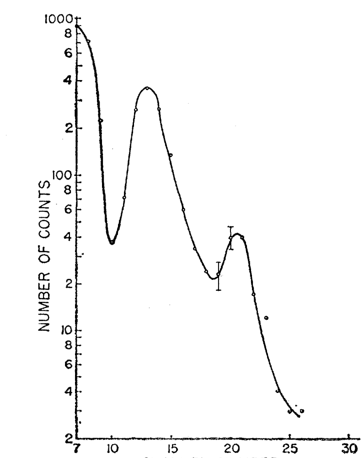

FIG. 2. Multiplied pulse spectrum for O^{18} at 20° lab. This is a spectrum displayed on 20-channel analyzer as it was used to select deuterons and tritons. Part of the proton peak falls below the lower level of the 20 channels analyzed.

[6] This target was obtained from J. N. McGruer, University of Pittsburgh. Details of the target's composition were furnished by E. K. Warburton.

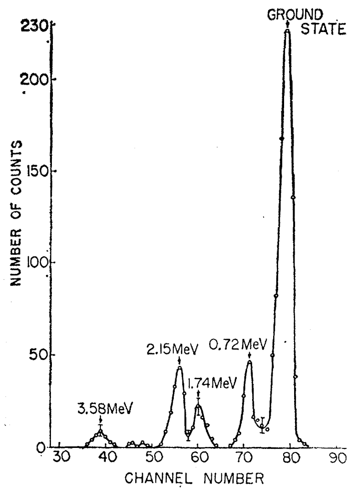

FIG. 3. Energy spectrum of deuterons from $B^{11}(p,d)B^{10}$. This spectrum was obtained using a solid-state detector placed at 20° lab. Proton energy was 19.0 MeV.

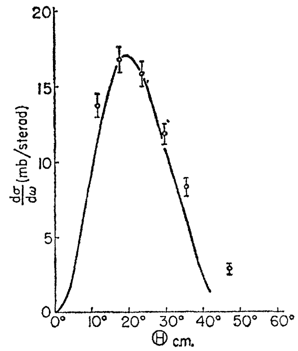

FIG. 5. Angular distribution of deuterons from $B^{11}(p,d)B^{10}$ ground state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=5.5$ F. Proton energy was 19.0 MeV.

uniform distribution of boric acid, then applying a few drops of polystyrene benzene as a binder. It was found that, by applying the binder while the boric acid deposit was still slightly damp, the polystyrene spread uniformly without disturbing the boric acid.

O^{18}

O^{18} was obtained in the form of Al_2O_3, enriched to 90.2% in O^{18}. Foils were produced by evaporating suspensions of Al_2O_3 in polystyrene and benzene on both water and glass plates. It was felt that the foils produced on water were definitely superior, containing approximately 1.2 mg/cm² Al_2O_3 and approximately 1 mg/cm² polystyrene.

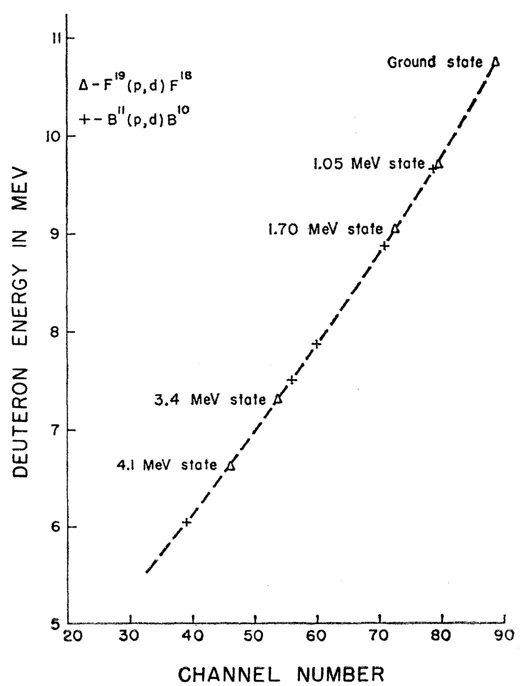

FIG. 4. Energy calibration for $B^{11}(p,d)B^{10}$. This is an example of the calibration curves used in these experiments. The $F^{19}(p,d)F^{18}$ reaction was used as an energy standard.

IV. EXPERIMENTAL RESULTS

Energy spectra were obtained for all targets at 4° or 5° intervals between 10° lab and 50° lab and at 10° intervals between 50° and 90° lab. The (p,d) reactions were also observed at 140° and 160° lab for all targets. No evidence for back-angle peaking was found for any target.

The angular distributions of deuterons in all cases in these experiments are shown with a plane-wave Butler curve whose l and r_0 were chosen to yield the best fit to the experimental data. A change in l would in all cases force one to choose a value for r_0 outside of the generally accepted limits of physical significance for light nuclei—that is, a value less than 4 F or greater than 8 F. A change in r_0 would in most cases not have such a drastic effect on the fit—one could vary r_0 by ± 0.1 F without producing a significant shift of the theoretical curve. However, a small shift in r_0 (0.1 F) in all cases would change the value of θ^2 by less than 5%.

The angular distributions of tritons are shown with a $j_0^2(qr_0)$ curve. These curves are picked to give best fit with the experimental points to the eye, but it was felt that no great significance could be attached to these curves. A plane-wave curve is most significant

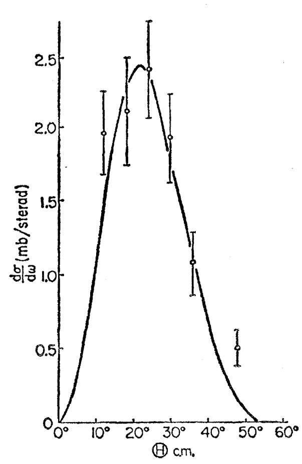

FIG. 6. Angular distribution of deuterons from $B^{11}(p,d)B^{10*}$ 0.72-MeV state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=5.1$ F. Proton energy was 19.0 MeV.

TABLE I. Parameters used in fitting experimental results.

Reactions and incident energy	Final-state excitation (MeV)	J_π	l	r_0 (F)	θ^2	$\theta^2/\theta_{g.s.}^2$	Probable error in θ^2
$B^{11}(p,d)B^{10}$,							
19.0 MeV,	0	3+	1	5.5	0.74	1	15%
(J=3/2−)	0.72	1+	1	5.1	0.011	0.15	20%
	1.74	0+	1	5.9	0.029	0.39	20%
	2.15	1+	1	6.0	0.011	0.15	20%
	3.58	2+	1	6.0	0.0031	0.042	30%
$C^{14}(p,d)C^{13}$,							
18.5 MeV,	0	1/2−	1	5.3	0.063[a]	1	
(J=0+)	3.09	1/2+	0	5.5	0.017[a]	0.027	50%[b]
	3.68	3/2−	1	5.9	0.051[a]	0.813	10%[b]
	3.86	5/2+	2	4.6	0.026[a]	0.413	+25%[b] −50%
$O^{16}(p,d)O^{15}$, (J=0+)							
18.0 MeV	0	1/2−	1	5.2	0.0096	...	25%
19.0 MeV	0	1/2−	1	5.2	0.013	...	20%
20.0 MeV	0	1/2−	1	5.2	0.017	...	15%
$O^{18}(p,d)O^{17}$,							
17.6 MeV,	0	5/2+	2	5.7	0.40	1	10%
(J=0+)	0.87	1/2+	0	6.0	0.011	0.27	15%
	3.06	1/2−	1	6.0	0.013	0.33	20%
$C^{14}(p,t)C^{12}$, 18.5 MeV, (J=0+)	0	0+	0	5.1			
$O^{18}(p,t)O^{16}$, 17.6 MeV, (J=0+)	0	0+	0	6.2			

[a] These values are normalized to the corrected value of McGruer *et al.*, of 0.063, for the $C^{13}(d,p)C^{14}$ reduced width given by MacFarlane and French.
[b] These probable errors do not include the probable error of the measurement of the ground-state reduced width as quoted in the literature (25%) and, in fact, are just the probable errors in $\theta^2/\theta_{g.s.}^2$.

when it fits the first maximum of the angular distribution, but most of this first maximum was not included in the experimental points because the momentum transfer at 0° was large.

$B^{11}(p,d)B^{10}$

A typical energy spectrum of deuterons is shown in Fig. 3. The E counter which produced this spectrum was a solid-state detector. The small number of counts between the 2.15-MeV state and the 3.58-MeV state were not reproducible. Because of this, it was decided that these counts were probably due to protons and not to deuterons leaving B^{10} in the 2.86-MeV state proposed by Galloway and Sillitto.[7] In Fig. 4 a sample energy calibration curve is displayed to illustrate the method of assignment of states used in all of these experiments. In this case the deuterons from the $F^{19}(p,d)F^{18}$ reaction observed by Bennett are used for the calibration.[3] The 1-MeV state observed by Bennett has been arbitrarily assigned an excitation of 1.05 MeV, with the feeling that this assignment is not in error by more than 0.05 MeV on the basis of present knowledge about the level structure of F^{18}. All $F^{19}(p,d)F^{18}$ calibration spectra were taken at a lab angle of 15°. As may be seen from the figure, the deuteron groups from $B^{11}(p,d)B^{10}$ may be assigned lab energies of 9.64, 8.87, 7.86, 7.50, and 6.03 MeV; all of these assignments have a probable error of

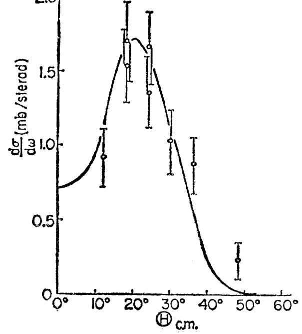

FIG. 7. Angular distribution of deuterons from $B^{11}(p,d)B^{10*}$ 1.74-MeV state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=5.9$ F. Proton energy was 19.0 MeV.

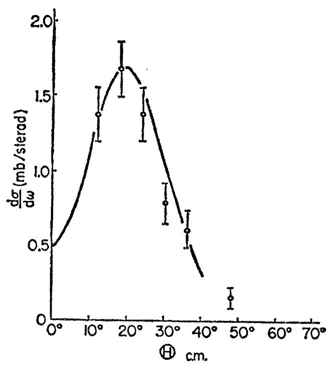

FIG. 8. Angular distribution of deuterons from $B^{11}(p,d)B^{10*}$ 2.15-MeV state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=6.0$ F. Proton energy was 19.0 MeV.

[7] R. B. Galloway and R. M. Sillitto, Proc. Roy Soc. (London 65, 247 (1961).

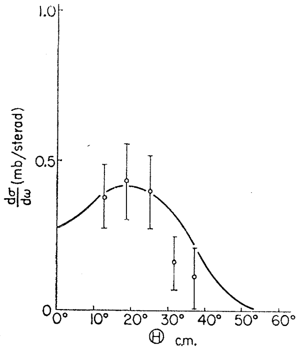

FIG. 9. Angular distribution of deuterons from $B^{11}(p,d)B^{10*}$ 3.58-MeV state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=6.0$ F. Proton energy was 19.0 MeV.

0.1 MeV because of the energy spread included in a single channel and uncertainties in the standard energies and calibration curve. These energies correspond to Q values of -9.14, -9.91, -10.94, -11.30, and -12.70 MeV. Comparison of these Q values to those of the known states of B^{10} (-9.22, -9.94, -10.96, -11.37, and -12.80 MeV) indicates that these deuteron groups may be assigned to the known B^{10} ground state, 0.72-MeV state, 1.74-MeV state, 2.15-MeV state, and 3.58-MeV state.

The angular distributions for deuterons leaving B^{10} in these states are shown in Figs. 5–9. Deuterons leaving

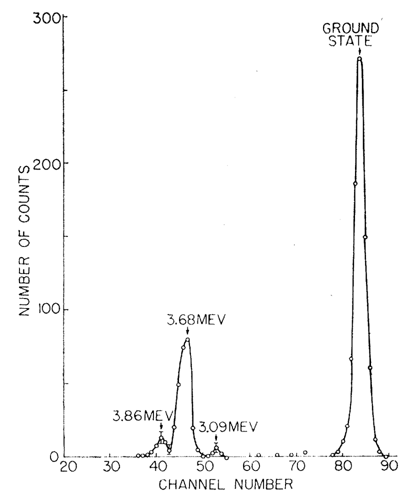

FIG. 10. Energy spectrum of deuterons from $C^{14}(p,d)C^{13}$. This spectrum was obtained using a NaI(Tl) detector placed at 15° lab. Proton energy was 18.5 MeV.

B^{10} in higher excited states were not observed. All five distributions could be fitted with $l=1$ curves in agreement with previous information on these levels.

The absolute cross section was determined by using a B_4C target. Deuterons were observed at 20° lab and their cross section was determined by comparison with protons scattered at 160° lab from the ground state and 4.43-MeV state of C^{12} at a proton bombarding energy of 16.7 MeV. Peelle's values for the proton cross sections were taken as standards, with the carbon elastic proton cross section used as the primary standard.[8] This method gave a center-of-mass cross section for deuterons leaving B^{10} in the ground state of 7.03 mb/sr$\pm$15% at $\theta_{lab}=20°$.

Since the deuterons leaving B^{10} in the various states were well resolved the probable error in θ^2 shown in Table I in all cases is due to statistics in the counting of deuterons, probable errors in the carbon elastic proton

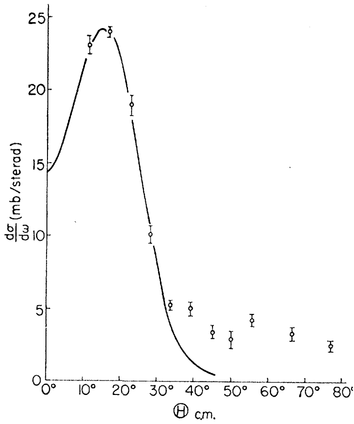

FIG. 11. Angular distribution of deuterons from $C^{14}(p,d)C^{13}$ ground state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=5.2$ F. Proton energy was 18.5 MeV.

cross section, and difficulties in resolving the C^{12} elastic protons from the B^{11} elastic protons.

$C^{14}(p,d)C^{13}$ and $C^{14}(p,t)C^{12}$

An energy spectrum of deuterons observed in this experiment is shown in Fig. 10. This spectrum was obtained using a NaI(Tl) E counter with thick absorber in front of it. This absorber (ΔE counter, mica windows, etc.) was equivalent to approximately 48 mg/cm^2 of aluminum. The presence of this thick absorber made possible the separation of the two deuteron groups labelled in Fig. 10 as the 3.68-MeV and 3.86-MeV groups.

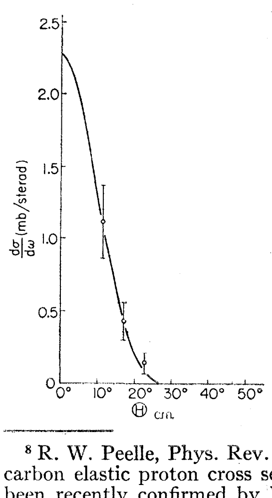

FIG. 12. Angular distribution of deuterons from $C^{14}(p,d)C^{13*}$ 3.09-MeV state. The solid line is a plane-wave Butler curve with $l=0$, $r_0=5.5$ F. Proton energy was 18.5 MeV.

[8] R. W. Peelle, Phys. Rev. **105**, 1311 (1957). The value of the carbon elastic proton cross section at this angle and energy has been recently confirmed by W. Daehnick (Private communication).

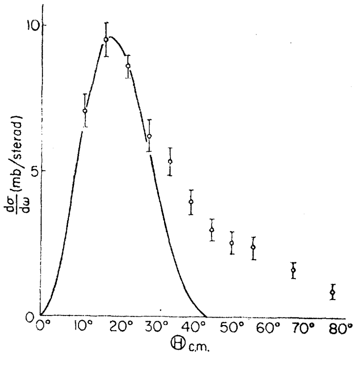

FIG. 13. Angular distribution of deuterons from $C^{14}(p,d)C^{13*}$ 3.68-MeV state. The solid line is a plane-wave Butler curve with $l=0$, $r_0=5.9$ F. Proton energy was 18.5 MeV.

Energy calibration was done using the $F^{19}(p,d)F^{18}$ reaction at 15° lab and the $O^{18}(p,d)O^{17}$ reaction at 16° lab as standards. Using these calibration points, Q values for the deuterons leaving B^{13} were calculated to be −5.97, −8.96, −9.50, and −9.79 MeV. These values were then compared to the Q values of the known levels (−5.94, −9.03, −9.62, and −9.80 MeV) and the deuteron groups observed were assigned to the known levels. It should be noted that the agreement of Q values is not quite as good as was obtained in the $B^{11}(p,d)B^{10}$ experiment. An explanation for this may be that the use of absorbers, whose uniformity is certainly questionable, was not a completely satisfactory experimental method. To check the possibility that one or more of the deuteron groups might be due to a contaminant reaction, the energies of the deuterons were compared from 10° to 60° lab. This comparison

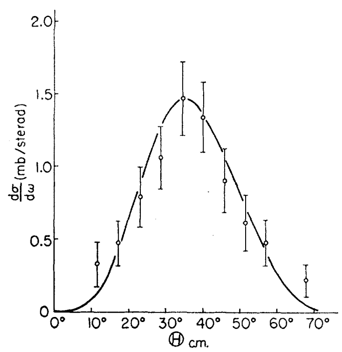

FIG. 14. Angular distribution of deuterons from $C^{14}(p,d)C^{13*}$ 3.86-MeV state. The solid line is a plane-wave Butler curve with $l=2$, $r_0=4.6$ F. Proton energy was 18.5 MeV.

indicated that any such contaminant must have approximately the same atomic weight as C^{14}. No nucleus with a mass between C^{12} and O^{16} could have contributed such a group of deuterons without contributing another easily recognizable group of deuterons which was not seen. Therefore, it was concluded that all deuteron groups observed were indeed a result of the $C^{14}(p,d)C^{13}$ reaction.

Angular distributions for deuterons leaving C^{13} in the ground state, 3.09-MeV state, 3.68-MeV state, and 3.86-MeV state are shown in Figs. 11–14. Deuterons leading to higher excited states were not observed. The angular distribution for tritons leaving C^{12} in the ground state is shown in Fig. 15. Tritons leaving C^{12} in the 4.43-MeV state were not observed and it is estimated that the cross section for this reaction is less than 1 mb/sr at any angle at which observations were made during this experiment.

Since the target thickness (0.27 ± 0.1 mg/cm^2) was not well known, the cross sections were normalized to the corrected values of McGruer *et al.*[9] for $C^{13}(d,p)C^{14}$,

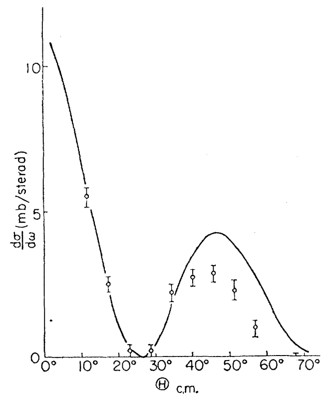

FIG. 15. Angular distribution of tritons from $C^{14}(p,t)C^{12}$ ground state. The solid line is a $Cj_0^2(qr_0)$ curve with C chosen to give fit on side of first maximum; $l=0$, $r_0=5.1$ F. Proton energy was 18.5 MeV.

which are given by MacFarlane and French (i.e., a value of $\theta^2=0.063$ was assumed for the ground state reduced width).[4] This procedure gave a $C^{14}(p,d)C^{13}$ cross section corresponding to a target thickness of 0.30 mg/cm^2, in good agreement with the value above.

Values used in fitting the $C^{14}(p,d)C^{13}$ data are listed in Table I. The probable errors in $\theta^2/[\theta^2(\text{ground state})]$ for the 3.09- and 3.68-MeV states are due to statistics. However, it was felt that, because of the thick absorber, small numbers of deuterons could be straggling down from the 3.68-MeV group to the 3.86-MeV group. This straggling would produce negligible changes in $\theta^2/[\theta^2(\text{ground state})]$ for the 3.68-MeV level but could increase the $\theta^2/[\theta^2(\text{ground state})]$ of the 3.86-MeV level by a large amount.

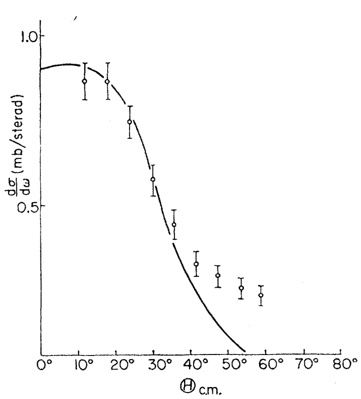

FIG. 16. Angular distribution of deuterons from $O^{16}(p,d)O^{15}$ ground state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=5.2$ F. Proton energy was 18.5 MeV.

[9] J. N. McGruer, E. K. Warburton, and R. S. Bender, Phys. Rev. **100**, 235 (1956).

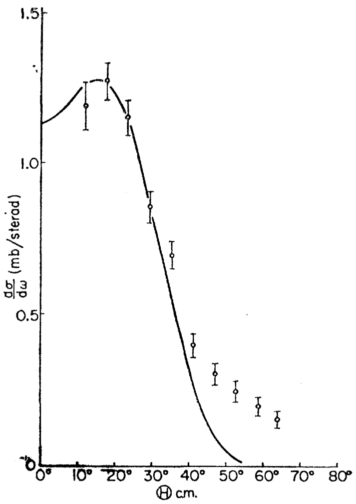

FIG. 17. Angular distribution of deuterons from $O^{16}(p,d)O^{15}$ ground state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=5.2$ F. Proton energy was 19.0 MeV.

This sort of straggling was observed in the $F^{19}(p,d)F^{18}$ calibration spectrum. Therefore, it was estimated that $\theta^2/[\theta^2(\text{ground state})]$ for the 3.86-MeV level could be too high by as much as 25% because of this. This was added to the probable error due to statistics to yield the probable error quoted below.

Again, it should be emphasized that the experimental method used was not completely satisfactory so that it is felt that this experiment could be profitably repeated with equipment capable of resolving the 3.86-MeV state from the 3.68-MeV state without resorting to the technique of thick absorbers.

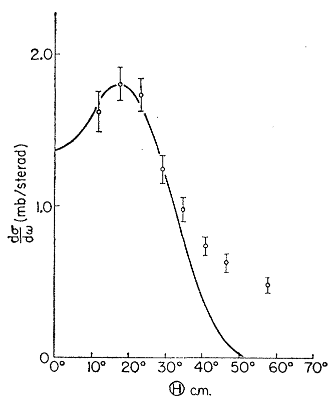

FIG. 18. Angular distribution of deuterons from $O^{16}(p,d)O^{15}$ ground state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=5.2$ F. Proton energy was 20.0 MeV.

$O^{16}(p,d)O^{15}$

Deuterons leaving the ground state of O^{15} were observed at bombarding energies of 18.0, 19.0, and 20.0 MeV. The angular distributions of the deuterons at these three energies are shown in Figs. 16–18.

Some experimental difficulty was experienced in observing these low-energy, low-cross-section deuterons. However, by operating the proportional counter at a pressure of 20 in. Hg, observation of these deuterons became experimentally feasible. The experimentally determined Q value at 19 MeV was -13.38 MeV, in good agreement with the listed Q of -13.43 MeV.

The cross sections were normalized to Hornyak and Sherr's data[10] on inelastic scattering of protons by O^{16} using their groups A and B. This normalization was done at 30° lab, since it was felt that the angle could be reproduced more accurately than the energy, and since it has been found that the cross sections for inelastic

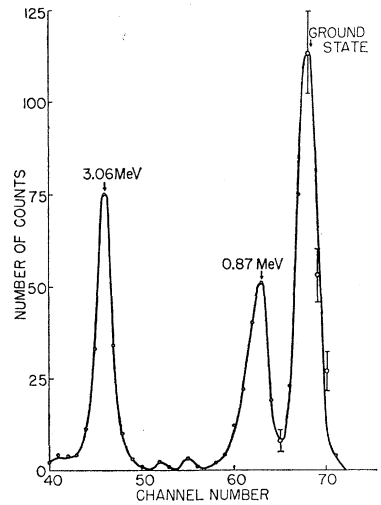

FIG. 19. Energy spectrum of deuterons from $O^{18}(p,d)O^{17}$. This spectrum was obtained using a solid-state detector placed at 16° lab. Proton energy was 17.6 MeV.

proton scattering by light nuclei often vary rapidly with energy at large angles. However, data taken on these groups and on protons elastically scattered from O^{16} at 135° lab confirmed the cross sections within 15%.

Parameters used to fit the experimental data are shown in Table I. The probable errors in θ^2 are due to statistics, estimated errors in proton cross section due to errors in reproducing the conditions of the standard measurements, and quoted errors in the standard measurement of the $O^{16}(p,p')O^{16*}$ proton cross section.

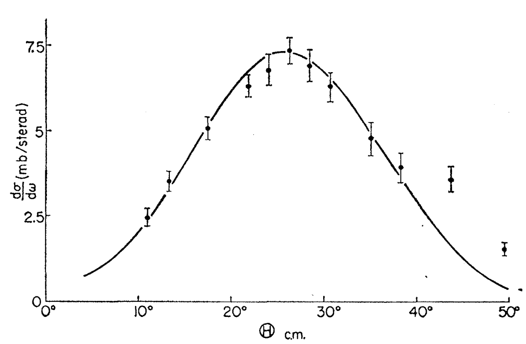

FIG. 20. Angular distribution of deuterons from $O^{18}(p,d)O^{17}$ ground state. The solid line is a plane-wave Butler curve with $l=2$, $r_0=5.7$ F. Proton energy was 17.6 MeV.

[10] W. F. Hornyak and R. Sherr, Phys. Rev. 100, 1409 (1955).

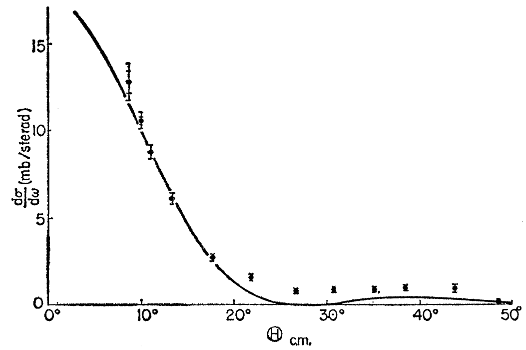

FIG. 21. Angular distribution of deuterons from $O^{18}(p,d)O^{17*}$ 0.87-MeV state. The solid line is a plane-wave Butler curve with $l=0$, $r_0=6.0$ F. Proton energy was 17.6 MeV.

$O^{18}(p,d)O^{17}$ and $O^{18}(p,t)O^{16}$

An energy spectrum of deuterons observed in this experiment is shown in Fig. 19. The E counter used was a solid-state detector. The $F^{19}(p,d)F^{18}$ reaction was used as a standard in energy calibration. Analysis yielded Q values within 0.05 MeV of the listed Q values for reactions to the ground state, 0.87-MeV state, and 3.06-MeV state.

Angular distributions of the deuterons leaving O^{17} in the ground state, 0.87-MeV state, and 3.06-MeV state are shown in Figs. 20–22. The angular distribution of tritons leaving O^{16} in the ground state is shown in Fig. 23.

The cross sections were determined by two semi-independent methods. The first determination was by direct comparison to the elastic protons scattered by the aluminum in the target at 160° lab at $E_p=17$ MeV (c.m.). The cross section for elastic scattering of protons by Al at this angles and energy as determined by Dayton and Schrank was used as a standard value.[11] Then the elastic protons scattered by the Al in the Al_2O_3 target were compared to the elastic protons scattered by a weighed Al target. This method gave the amount of Al in the O^{18} target which, it was assumed, determined the amount of O^{18} in the Al_2O_3 target. Then a cross section was obtained using the solid angle and integrated beam current. These methods yielded cross sections which agreed to within 5%. The center-of-mass cross section for the ground-state deutrons at 24° lab was $7.27\pm10\%$ mb/sr.

Values used in fitting the deutron data are shown in Table I. The probable errors in θ^2 for all states are due to statistics, quoted errors in the standard proton cross section, and estimated error due to errors in reproducing the conditions of the standard measurement.

V. DISCUSSION OF RESULTS

The results of these experiments will be discussed in terms of the plane-wave Butler stripping theory. In this theory, one extracts a reduced width, θ^2, from the

[11] I. E. Dayton and G. Schrank, Phys. Rev. **101**, 1358 (1956).

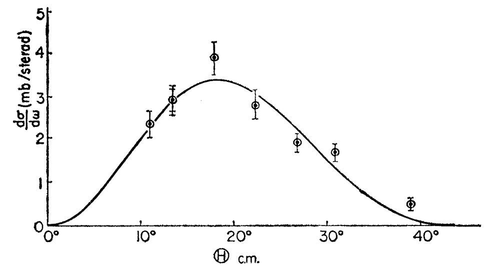

FIG. 22. Angular distribution of deuterons from $O^{18}(p,d)O^{17*}$ 3.06-MeV state. The solid line is a plane-wave Butler curve with $l=1$, $r_0=6.0$ F. Proton energy was 17.6 MeV.

experimental results and then expresses θ^2 as the product of S, the spectroscopic factor, and θ_0^2, the single-particle reduced width. The work of MacFarlane and French in determining probable values of θ_0^2 is drawn upon in the present discussion.

$B^{11}(p,d)B^{10}$

Intermediate coupling calculations in this mass region are extremely difficult and are only moderately successful, so for simplicity one can calculate what one expects in this reaction on the basis of a pure jj-coupling shell model. On this basis, there should be four strong $l=1$ transitions leading to the four states of B^{10} which may be performed by the $p_{3/2}{}^6$ configurations. Values can then be calculated for S for these transitions, a reasonable value for $\theta_0^2(1p)$ can be picked, and values of θ^2 for these transitions may be produced. Predicted values for θ^2 obtained by this procedure, using $\theta_0^2(1p)=0.045$, are compared to experimental values in Table II.

The experimental values of θ^2 for the ground state ($T_0=0$, $J_0\pi=3+$) and the 1.74-MeV state ($T_0=1$, $J_0\pi=0+$) are in reasonable agreement with the predictions of jj-coupling. Also the value of θ^2 for the 3.58-MeV state ($T_0=0$, $J_0\pi=2+$) is small, as predicted by jj-coupling. It is interesting also to note that the sum of the experimental values of θ^2 for the $T_0=0$,

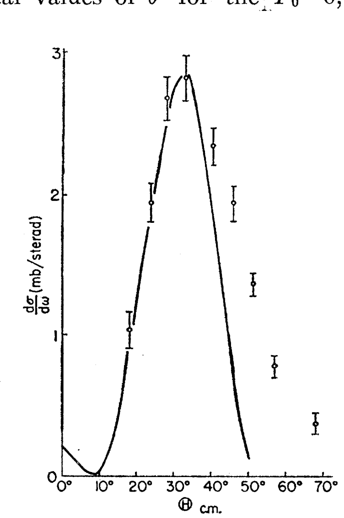

FIG. 23. Angular distribution of tritons from $O^{18}(p,t)O^{16}$ ground state. The solid line is $Cj_0{}^2(qr_0)$ curve with C chosen to give fit on small-angle data; $l=0$, $r_0=6.2$ F. Proton energy was 17.6 MeV.

TABLE II. Comparison of values obtained in this experiment with values predicted by a jj-coupling shell model for the θ^2 of low-lying states (letting $\theta_0^2=0.045$) in the $B^{11}(p,d)B^{10}$ reaction.

State	$\theta^2(jj)$	$\theta^2/\theta_{g.s.}^2(jj)$	θ^2(exp.)	$\theta^2/\theta_{g.s.}^2$(exp.)
Ground state	0.079	1	0.074	1
0.72 MeV	0.034	0.43	0.011	0.15
1.74 MeV	0.034	0.43	0.029	0.39
2.15 MeV	small	small	0.011	0.15
3.58 MeV	small	small	0.003	0.042
5.16(?) MeV	0.17	2.14	unobserved	unobserved

$J_0\pi=1+$ levels (0.72 MeV and 2.15 MeV) is fairly close to the value predicted by jj-coupling for its single $T_0=0, J_0=1$ state. This might be taken as evidence that these two states equally share the $p_{3/2}{}^6$ configuration predicted by jj-coupling.

The results of this experiment show poor agreement with the results obtained by Vlasov *et al.* from the $B^{11}(d,t)B^{10}$ experiment.[12] It is felt, however, that the (d,t) results may be somewhat questionable because of the poor energy resolution and experimental difficulties of the method used to observe tritons in that experiment.

$C^{14}(p,d)C^{13}$

MacFarlane and French have shown that the value of $S_{3.61\text{ MeV}}/S_{\text{ground state}}$ is relatively insensitive to the variation of $\zeta=a/K$, except near the LS limit. Letting $\zeta(C^{13})$ equal $\zeta(C^{14})$, they display values of this ratio of reduced widths for typical values of ζ and comment that these values do not change appreciably with separate variations of $\zeta(C^{13})$ and $\zeta(C^{14})$.[4] These values are all greater than 1.1, in contrast to the experimental values for $\theta^{*2}/[\theta^2(\text{ground state})]$ of 0.81 in the present work, and 0.7 in the (d,t) work of Moore *et al.*[13] This discrepancy may be due to a variation of $\theta_0^2(1p)$ with nucleon binding energy; that is, $\theta_0^2(1p)$ decreases with an increase in the binding energy of the transferred nucleon. This decrease with increasing binding energy seems fairly reasonable, especially in light of the $O^{16}(p,d)O^{15}$ results which will be discussed later.

Since the $\frac{1}{2}+$ 3.09-MeV state and $\frac{5}{2}+$ 3.86-MeV state are well separated from other $\frac{1}{2}+$ and $\frac{5}{2}+$ levels, they may be considered to be single particle $2s_{1/2}$ and $1d_{5/2}$ states. Therefore, measurements of these states reduced widths determine the admixtures of $(2s_{1/2})^2$ and $(1d_{5/2})^2$ in the C^{14} ground-state wave function.

If one assumes that the C^{14} ground-state wave function may be expressed as

$$\Psi(C^{14}\text{ g.s.})=\alpha\psi_\alpha[1p^{10}]+\beta\psi_\beta[(1p^8)_0(2s_{1/2}{}^2)_0]+\gamma\psi_\gamma[(1p^8)_0(1d_{5/2}{}^2)_0],$$

then $S_{3.09}=2\beta^2$ and $S_{3.86}=2\gamma^2$. Then the following values for β^2 and γ^2 may be extracted from the measured reduced widths, taking $\theta_0^2(2s)$ to be 0.17 and $\theta_0^2(1d)$ to be 0.07.

$$\beta^2=0.005\pm50\%,\quad \gamma^2=0.21\pm\begin{smallmatrix}50\%\\25\%\end{smallmatrix}.$$

These values may be compared to the values extracted, in a treatment similar to the one shown above, by MacFarlane and French from the $C^{14}(d,t)C^{13}$ data of Moore *et al.*[4]:

$$\beta^2=0.005,\quad \gamma^2=0.07.$$

The disparity in γ^2 indicates that it might be of value to repeat the $C^{14}(p,d)C^{13}$ experiment with much better resolution so that absorbers would not be required to separate the 3.86-MeV state from the 3.68-MeV state.

These results are applicable to the question of the long half-life of the C^{14} β decay. Baranger and Meshkov have published a qualitative argument that configuration mixing in the N^{14} and C^{14} ground states is the cause of this long half-life.[14] However, more detailed calculations are needed to establish whether this configuration mixing gives the right sign for accidental cancellation in the β-decay matrix element.

$C^{14}(p,t)C^{12}$

For the tritons leaving C^{12} in the ground state, it is interesting to compare the results with those of the $C^{12}(t,p)C^{14}$ experiment of Jaffe.[15]

Assuming that a simple plane-wave theory of double stripping is a good description of the process

$$A+2(J,T)+p\rightleftharpoons A(J_0,T_0)+t,$$

one expects

$$\left(\frac{d\sigma}{d\omega}\right)_{(p,t)}=CM_pM_t\left[\frac{(A+2)A}{(A+3)^2}\right]\frac{k_t}{k_p}F(q,r_0)$$

and

$$\left(\frac{d\sigma}{d\omega}\right)_{(t,p)}=C\frac{2J+1}{2J_0+1}M_pM_t\left[\frac{(A+2)A}{(A+3)^2}\right]\frac{k_p}{k_t}F(q,r_0),$$

where C contains, among other things, information about the overlap between final- and initial-state wave functions. Energy dependence enters only through the triton and proton momenta and the momentum transfer. For a given momentum transfer, the cross sections are related by the following expression:

$$\left(\frac{d\sigma}{d\omega}\right)_{(p,t)}=\frac{2J_0+1}{2J+1}\left(\frac{k_t}{k_p}\right)_{(p,t)}\left(\frac{k_t}{k_p}\right)_{(t,p)}\left(\frac{d\sigma}{d\omega}\right)_{(t,p)},$$

where

$$k_p{}^2=2EM_p[(A+2)^2/(A+3)^2],$$
$$k_t{}^2=2E_0M_t[A^2/(A+3)^2].$$

[12] N. A. Vlasov, S. P. Kalinin, A. A. Oglobin, and V. I. Chuev, Soviet Phys.—JETP 12, 1129 (1961).

[13] W. E. Moore, J. N. McGruer, and A. I. Hamburger, Phys. Rev. Letters 1, 29 (1958).

[14] E. Baranger and S. Meshkov, Phys. Rev. Letters 1, 30 (1958).

[15] A. A. Jaffe, in *Proceedings of the International Conference on Nuclear Structure, Kingston, Canada, 1960*, edited by D. A. Bromley and E. Vogt (University of Toronto Press, Toronto, 1960), p. 814.

Therefore, if a simple double stripping theory is adequate, one expects for the same momentum transfer

$$\left(\frac{d\sigma}{d\omega}\right)_{(p,t)}=\frac{2J_0+1}{2J+1}\times\left[3\frac{A^2}{(A+2)^2}\left(\frac{E_0}{E}\right)^{1/2}_{(t,p)}\left(\frac{E_0}{E}\right)^{1/2}_{(p,t)}\right]\left(\frac{d\sigma}{d\omega}\right)_{(t,p)}.$$

On this basis, the $C^{12}(t,p)C^{14}$ ($E_0=5.5$ MeV) and $C^{14}(p,t)C^{12}$ ($E_0=19.9$ MeV) cross sections for the same momentum transfer would be related to each other by

$$\left(\frac{d\sigma}{d\omega}\right)_{(p,t)}=1.53\left(\frac{d\sigma}{d\omega}\right)_{(t,p)}.$$

From Jaffe's results, a (p,t) cross section at 10° c.m. of 6 mb/sr is expected. In the present experiment a cross section of 6.5 mb/sr is obtained, which is considered to be very acceptable agreement.

This result seems to indicate that the cross section depends upon the energy and momentum transfer in the manner that the simple theory predicts. If this is the case, then in the future these experiments may furnish valuable information about wave functions through the "double neutron reduced width" concept.

$O^{16}(p,d)O^{15}$

The result of this experiment is of special interest because it should determine $\theta_0^2(1p)$ for the case where binding energy of the transferred nucleon is very large. The Q of this reaction is -13.43 MeV which corresponds to a neutron binding energy of 15.66 MeV.

Since one is fairly certain that O^{15} has a pure single-hole wave function [i.e., $\psi(O^{15}\text{ g.s.})=\psi(O^{16}\text{ g.s.})\times 1p_{1/2}^{-1}$], one may say that $S=4$ for this reaction and that any variation in θ^2 is due to a variation in $\theta_0^2(1p)$.

The analogous $O^{16}(d,t)O^{15}$ experiment was done at $E_d=14.9$ MeV by Keller.[4] Keller obtained a value for $\theta_0^2(1p)$ of 0.0031 which is considerably smaller than the range of 0.04–0.06 which has been determined experimentally for nuclei at the middle of the $1p$ shell. The $O^{16}(p,d)O^{15}$ experiment was done at $E_p=18$, 19, and 20 MeV in an attempt to give an independent check of this surprising result.

From the present experiment the following values of $\theta_0^2(1p)$ may be extracted: $0.0024\pm25\%$ at $E_p=18.0$ MeV, $0.0032\pm20\%$ at $E_p=19.0$ MeV, and $0.0042\pm15\%$ at 20 MeV. These values seem to indicate that at $E_p=20$ MeV, the deuteron energy ($E_0=6.1$ MeV) is not large enough for the experiment to yield a reduced width which is reasonably independent of energy—i.e., distortion of the low-energy deuterons is a sizable effect at these energies. Therefore, one may expect an experiment done at $E_p=25$–30 MeV to yield a slightly larger value for $\theta_0^2(1p)$. However, all of the values obtained are less than 1/10 of the values of $\theta_0^2(1p)$ obtained in experiments on nuclei in the middle of the $1p$ shell, where binding energies are on the order of 8–10 MeV. One would not expect a change in $\theta_0^2(1p)$ of a factor of 10 when one raises E_p from 20 to 30 MeV, so one would expect that $\theta_0^2(1p)$ is less than 0.01 for this transition.

These results seem to indicate that $\theta_0^2(1p)$ depends strongly on the binding energy of the transferred nucleon. A sharper decrease of $\theta_0^2(1p)$ with increasing binding energy than had previously been expected is suggested by the present results. Since this result is so unexpected, it would be interesting to have the $O^{16}(p,d)O^{15}$ experiments extended into the range 20 MeV$<E_p<$30 MeV so that a direct determination of the E_p-independent $\theta_0^2(1p)$ may be made. Certainly it would be of value to learn more specifically upon what factors θ_0^2 depends and how it varies as these factors change. Also these results may be an indication that even in light nuclei, distortions are important for transitions involving nucleons which have a large binding energy and that the plane-wave Butler formalism is not an adequate description for such transitions.

$O^{18}(p,d)O^{17}$

The ground state and the 0.87-MeV state have been identified as single-particle $1d_{5/2}$ and $2s_{1/2}$ levels, respectively, since there are no other $\frac{5}{2}+$ and $\frac{1}{2}+$ levels of O^{17} with an excitation of less than 5 MeV. Therefore, one can determine the admixtures of $(1d_{5/2}{}^2)_0$ and $(2s_{1/2}{}^2)_0$ in the O^{18} ground-state wave function.

Neglecting core excitation and including all shell-model levels which are present in levels below 5-MeV excitation in O^{17} we may have

$$\psi(O^{18}\text{ g.s.})=\alpha(1d_{5/2}{}^2)_0+\beta(2s_{1/2}{}^2)_0+\gamma(1d_{3/2}{}^2)_0+\delta(1f_{7/2}{}^2)_0+\epsilon(2p_{3/2}{}^2)_0.$$

Then, as in the case of $C^{14}(p,d)C^{13}$, one may say that $S_{\text{g.s.}}=2\alpha^2$ and $S_{0.87}=2\beta^2$. One then still has the problem of determining what single-particle widths to use in determining the admixtures. It is assumed in the present treatment that $\theta_0^2(1d)=0.025$ and $\theta_0^2(2s)=0.05$. These values are consistent with those of experiments on nuclei with A up to 28, although they are considerably smaller than the values found in $O^{16}(d,p)O^{17}$ experiments.[4] However, this discrepancy may be explained by assuming that the single-particle reduced widths, $\theta_0^2(1d)$ and $\theta_0^2(2s)$, vary inversely with the binding energy of the transferred nucleon. These values of $\theta_0^2(1d)$ and $\theta_0^2(2s)$ are also consistent with the values used in the analysis of the $O^{18}(d,t)O^{17}$ experiment of Armstrong and Quisenberry.[16]

Under these assumptions, we obtain values of admixtures, which are shown compared to the (d,t) results and to the shell-model predictions of Elliot and Flowers[17] and Redlich[18] in Table III.

[16] J. C. Armstrong and K. S. Quisenberry, Phys. Rev. **122**, 150 (1961).
[17] J. P. Elliot and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).
[18] M. G. Redlich, Phys. Rev. **110**, 468 (1958).

TABLE III. Shell-model configuration admixtures in the O^{18} ground-state wave function. The results of the present experiment are shown compared to those of the $O^{18}(d,t)O^{17}$ experiment and the theoretical predictions of Elliot and Flowers and Redlich.

Source	$(1d_{5/2}^2)_0$	$(2s_{1/2}^2)_0$	$(1d_{3/2}^2)_0$	$(1f_{7/2}^2)_0$	$(2p_{3/2}^2)_0$
Present experiment	80%	11%			
(d,t) results	75.2%	14.5%	3.3%	2.9%	0.7%
Elliot and Flowers[a]	79%	15.2%	5.8%		
Redlich[b]	74%	16%	9.6%		

[a] See reference 15.
[b] See reference 16.

In attempting to decide upon a wave function for the O^{17*} 3.06-MeV state, we are faced with a problem. Since the state was not observed in $O^{16}(d,p)O^{17}$ experiments, this state may be considered to be almost completely core excitation from the O^{16} core. If one accepts the value obtained from the $O^{16}(d,p)O^{15}$ experiment for $\theta_0^2(1p)$ of about 0.003, one would say that this state's wave function is almost completely $[O^{18}(\text{g.s.})\dot{\times}1p_{1/2}^{-1}]_{1/2}$. However, if one accepts the value obtained from the middle of the $1p$ shell at the appropriate binding energy for $\theta_0^2(1p)$ of 0.04, one would say that this state's wave function contains less than 10% $[O^{18}(\text{g.s.})\dot{\times}1p_{1/2}^{-1}]_{1/2}$. Unfortunately, there is no evidence in these experiments indicating which of these assumptions is correct. Therefore, no definite statement can be made about the percentage admixture of $(O^{18}\dot{\times}1p_{1/2}^{-1})_{1/2}$ in the wave function of the 3.06-MeV state of O^{17}.

$O^{18}(p,t)O^{16}$

As in the $C^{14}(p,t)C^{12}$ discussion, the $O^{18}(p,t)O^{16}$ results may be compared with the $O^{16}(t,p)O^{18}$ results of Jaffe.[15] Again assuming that this reaction may be described by a simple stripping theory, the (p,t) cross section should be 1.78 times the (t,p) cross section for the same momentum transfer. On this basis, a (p,t) cross section of 2.6 mb/sr at the peak of the second maximum is predicted from the $O^{16}(t,p)O^{18}$ results. In the present experiment a peak cross section of 2.8 mb/sr$\pm$20% is obtained which is in very good agreement with the prediction above.

Again, as in the case of $C^{14}(p,t)C^{12}$, this seems to be evidence that a simple theory is qualitatively correct, and that one may hope to extract a meaningful "double particle reduced width" from (p,t) and (t,p) experiments.

VI. CONCLUSION

Energy spectra and angular distributions have been measured for the reactions $B^{11}(p,d)B^{10}$, $C^{14}(p,d)C^{13}$, $C^{14}(p,t)C^{12}$, $O^{16}(p,d)O^{15}$, $O^{18}(p,d)O^{17}$, and $O^{18}(p,t)O^{16}$. No evidence for back angle peaking was observed in any of these reactions.

The $B^{11}(p,d)B^{10}$ results show fair qualitative agreement with the predictions of a jj-coupling model. However, there seems to be some disparity between these results and those of the $B^{11}(d,t)B^{10}$ experiment reported by Vlasov *et al.*

The $C^{14}(p,d)C^{13}$ results show fair agreement with those of the $C^{14}(d,t)C^{13}$ experiment reported by Moore *et al.* These results indicate that the configuration mixing in the C^{14} ground-state wave function is in agreement with the assumptions of Baranger and Meshkov. Baranger and Meshkov conclude that configuration mixing in the C^{14} and N^{14} ground-state wave functions is the cause of the long half-life of the C^{14} β decay. However, more detailed calculations are necessary to firmly establish that configuration mixing will lead to a cancellation in the β decay matrix element.

The $C^{14}(p,t)C^{12}$ and $O^{18}(p,t)O^{16}$ ground-state results are in good agreement with Jaffe's $C^{12}(t,p)C^{14}$ and $O^{16}(t,p)O^{18}$ results, under the assumption that a simple stripping theory will adequately describe these reactions. This agreement is very striking since the energies of the (t,p) experiments and the energies of the (p,t) experiments are quite different. In the $C^{14}(p,t)C^{12}$ experiment E_0 was 15.8 MeV, while in the $C^{12}(t,p)C^{14}$ experiment E_0 was 5.5 MeV. In the $O^{18}(p,t)O^{16}$ experiment E_0 was 15.3 MeV, while in the $O^{16}(t,p)O^{18}$ experiment E_0 was 5.5 MeV. This agreement gives some reason to hope that a simple stripping theory gives an adequate description of (p,t) and (t,p) reactions.

The $O^{18}(p,d)O^{17}$ results show very good agreement with Armstrong and Quisenberry's $O^{18}(d,t)O^{17}$ results. These results are also in good agreement with the O^{18} ground-state wave functions calculated by Elliot and Flowers and by Redlich. No definite statement can be made about the wave function of the 3.06-MeV state of O^{17} because of uncertainty about the proper value of $\theta_0^2(1p)$ to use in analyzing this reaction.

The most interesting results of these experiments are the indications that the stripping single-particle reduced widths used in plane-wave Butler analysis are more strongly dependent on the binding energy of the transferred nucleon than had been previously suspected. This seems to indicate that distorted-wave analysis of stripping experiments may be necessary to obtain trustworthy information about the excited states of light nuclei.

The usefulness of stripping and pickup reactions in determining the configurations of nuclei is well known and it appears that the double stripping and double pickup experiments will become very useful also as more experimental information on these reactions accumulates.

ACKNOWLEDGMENTS

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