

## Interband Faraday Effect in Aluminum Antimonide, Germanium, and Gallium Antimonide

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The Faraday rotation at the absorption edge corresponds to interband transitions. Data for both Ge and GaSb exhibit a reversal in sign of the rotation at photon energies approaching that required for direct transitions at the zone center. The rotation for AlSb shows no reversal which indicates indirect transitions. Measurements were made at 77 and 296°K. In the case of GaSb and Ge, the Faraday rotations show large shifts with temperature as the edge is approached associated with direct transitions, while this shift is small in AlSb where the rotation is associated with indirect transitions. The spectral dependence of the Faraday rotation at the direct energy gap is proportional to  $(\omega_g - \omega)^{-1/2}$ . For frequencies small compared to the gap frequency  $\omega_g$ , it is of the form  $\text{const} \cdot \omega^2$ . The resultant direct energy gaps calculated from our measurements are: Ge,  $(0.80 \pm 0.01)$  eV at 296°K,  $(0.89 \pm 0.01)$  eV at 77°K; GaSb,  $(0.74 \pm 0.02)$  eV at 296°K,  $(0.82 \pm 0.02)$  eV at 77°K. The temperature dependence of the indirect gap in AlSb is found to be  $-3.5 \times 10^{-4}$  eV (°K)<sup>-1</sup>. The temperature dependence of the rotation in GaSb suggests that the effective indirect gap increases with temperature.

### INTRODUCTION

THE rotation of the plane of polarization in a material in the presence of a magnetic field, where the field direction is parallel to the propagation of the incident radiation is called Faraday rotation. The Faraday rotation is very useful in the investigation of semiconductors because their Faraday rotation can be very large.<sup>1</sup> Further, it is possible to investigate this semiconductor property in a given energy region with only negligible influence from other contributions. In the free carrier region—that is, for energies well below the gap energy—the rotation is proportional to the square of the wavelength of the incident radiation. However, for the intrinsic semiconductor, and especially near the absorption edge, the rotation is related to the band structure.

Lax and Nishina have proposed a quantum mechanical approach to interband Faraday rotation associated with direct, indirect, and direct forbidden transitions. They applied the Kramers-Kronig method<sup>2</sup> and the dispersion theory<sup>3</sup> to derive approximate expressions for the Faraday rotation due to magneto-optical interband transitions. The Faraday effect due to interband magneto-optical transitions has also been calculated by Suffczynski.<sup>4</sup> He considers the direct interband transitions from both the uppermost Landau levels in the two light-hole ladders and the uppermost levels in the two heavy-hole ladders, to the lowest Landau levels in the conduction band in Ge. In a magnetic field, the first direct transitions are from the light-hole levels for the left circularly polarized radiation and from the heavy-hole levels for the right circularly polarized radiation.<sup>5</sup> In the absorption experiments the light-hole transitions

are observed more strongly<sup>6</sup>; therefore, the direct transition rotation in Ge, which is proportional to the difference between the two refractive indices corresponding to the right and left polarized light, is negative.

In the case of direct transitions at small magnetic fields for frequencies approaching the band gap frequency, the Faraday rotation is given approximately by<sup>2</sup>

$$\theta_d \approx - (C\hbar\Delta\omega_d/2\omega)(\omega_g - \omega)^{-1/2}, \quad (1)$$

where  $\omega_g$  is the frequency corresponding to the direct gap and  $C$  is a constant. Here  $\Delta\omega_d = \gamma_d H$  is the differential frequency shift for the left and right circularly polarized waves which propagate along the magnetic field  $H$ , and  $\gamma_d$  is a phenomenological parameter. In the case of indirect transitions for  $\omega < \omega_g$ , the Faraday rotation is given by the expression<sup>7</sup>

$$\theta_i = K(\omega/\omega_g)^k [1 + a(\omega/\omega_g)^m], \quad (2)$$

where  $K$  is a constant. With  $k=2$ , this equation describes the indirect rotation in silicon. One can see that, in general, the equation exhibits a long-wavelength  $\omega^2$  dependence.

In the present theories, besides other simplifications, the Coulomb interaction between the electron and the hole is neglected. It is difficult to take the Coulomb interaction into account in the presence of a magnetic field. However, the exciton effects for the lowest levels in the conduction band are most pronounced. In the case of absorption due to indirect transitions, it was shown that the temperature dependence and relative magnitude of the absorption components are not altered by exciton effects, but their detailed shapes and the interpretation of their threshold energies are considerably modified.<sup>8</sup> In any case, the direct rotation offers in

<sup>1</sup> H. Kimmel, *Z. Naturforsch.* **12a**, 1016 (1957).

<sup>2</sup> B. Lax and Y. Nishina, *Phys. Rev. Letters* **6**, 464 (1961).

<sup>3</sup> B. Lax and Y. Nishina, *J. Appl. Phys.* **32**, 2128S (1961).

<sup>4</sup> M. Suffczynski, *Proc. Phys. Soc. (London)* **77**, 1042 (1961).

<sup>5</sup> R. J. Elliott, T. P. McLean, and G. G. Macfarlane, *Proc. Phys. Soc. (London)* **72**, 553 (1958).

<sup>6</sup> E. Burstein, G. S. Picus, R. F. Wallis, and F. Blatt, *Phys. Rev.* **113**, 15 (1959); L. M. Roth, B. Lax, and S. Zwerdling, *ibid.* **114**, 90 (1959).

<sup>7</sup> H. Piller and R. F. Potter, *Phys. Rev. Letters* **9**, 203 (1962).

<sup>8</sup> T. P. McLean, in *Progress in Semiconductors*, edited by A. F. Gibson (Heywood and Company, Ltd., London, 1960), Chap. V.

some materials a sensitive experimental method for investigating the direct gap  $E_g$ , as well as the optical direct gap  $E_g + E_m$  in degenerate samples.

### EXPERIMENTAL METHOD

The experimental setup at NOLC is similar to that used by Austin<sup>9</sup> for measuring the Faraday rotation. Instead of a longitudinal optical arrangement, a transversal type with respect to the magnet is used. By means of two 45° mirrors (Fig. 1), the polarized radiant energy passes through the sample parallel to the field  $H$ . Monochromatic energy is provided by a Leiss double monochromator equipped with calcium fluoride prisms, and a Nernst glower or a tungsten lamp is used as a source. A pair of Polaroid polarizers and a set of silver chloride sheets stacked at the Brewster angle were employed—the Polaroid polarizers for the wavelength region between 0.8 and 2.3  $\mu$ , and the silver chloride sheets for the long-wavelength region. Different detectors were used for special wavelength regions: silicon below 1  $\mu$ , lead sulfide between 1 and 3  $\mu$ , and gold-doped germanium cooled by liquid nitrogen for the longer wavelengths. Field strengths up to 20 kG were obtained with a 4-in. magnet in a gap of 16 mm. With a metal Dewar having magnesium oxide windows, it was possible to investigate in the temperature region between 77 and 297°K. The sample surfaces were mechanically finished with a Buehler No. 3 polish. The measurements were made without any mechanical rotation by either the analyzer or the polarizer. The rotations were calculated from the measured intensities corresponding to opposite field directions and were corrected for the nonpolarization amount of the polarizers. The accuracy of the measured rotations for different wavelengths is dependent on the equipment used—especially the light source and detectors in that

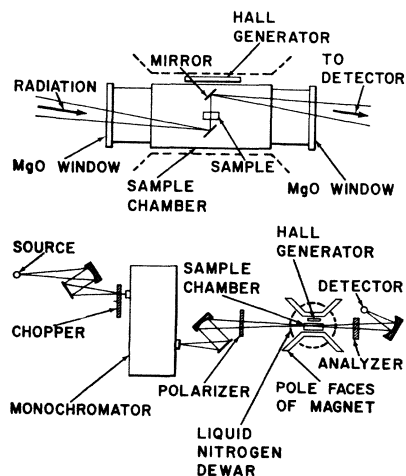


Fig. 1. Schematic of the optical arrangement. The Hall generator was used to provide an accurate magnetic field measurement.

<sup>9</sup> I. G. Austin, Proc. Phys. Soc. (London) **76**, 169 (1960).

region. The error, which shows up in noise in the intensity measurements, was calculated from the following approximate expression:

$$m_\theta/\theta \approx 1.4m/(I_2 - I_1),$$

where  $m_\theta$  is the error in the rotation measurement,  $I_1$  and  $I_2$  are the measured intensities for the opposing field directions, and  $m$  is the error in the intensity measurement. Accuracy was increased by repeating the measurement at a given wavelength and taking the average value. Thickness of the samples varied between  $10^{-2}$  and  $10^{-1}$  cm.

### RESULTS

#### Aluminum Antimonide

Faraday rotation in AlSb is shown in Fig. 2, at 77 and 296°K. (The data in Figs. 2, 3, 4, and 5, are normalized to the Verdet coefficient.) The shift of the Faraday rotation to shorter wavelengths with decreasing temperature is similar to the shift in Si.<sup>7</sup> Further-

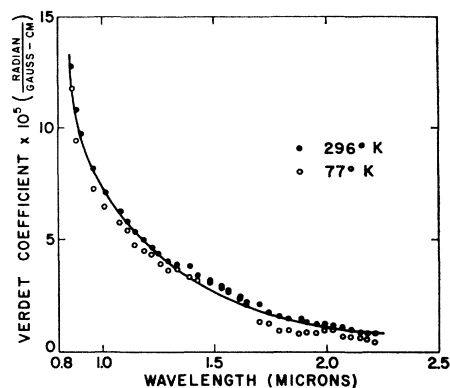


Fig. 2. Verdet coefficient for aluminum antimonide.

more, as in Si, the Faraday rotation does not change the sign. The temperature dependence of the Faraday rotation is

$$\theta^{-1}(d\theta/dT) \approx 2.2 \times 10^{-4} (\text{°K})^{-1}.$$

Equation (2) describes the rotation in AlSb with  $k \approx 2$  ( $a \leq 1, m \approx 2$ ). Figure 2 shows an increase in rotation around 1.6  $\mu$ , and a transmission measurement taken at room temperature shows an increase in absorption in the same region. The observed increase in rotation around 1.6  $\mu$  is apparently related to that absorption.

#### Germanium

Germanium was chosen for our investigation of the interband Faraday rotation because the temperature dependence of the energy gap in Ge is well known. Figure 3 shows the Faraday rotation in Ge (*n* type) as a function of wavelength, at 77 and 296°K. The rotation at 77°K is similar to the rotation at room temperature; however, the difference between the two rotations shows

a strong temperature dependence as the direct edge is approached and a weak temperature dependence of the rotation at long wavelengths. The increase of the positive peak of the rotation at 77°K with respect to 296°K shows that at 77°K the positive rotation due to other transitions gives a larger contribution to the total rotation in that region than at 296°K. The direct rotation at the edge for frequencies  $\omega$  approaching the gap frequency  $\omega_0$  is given by the approximate expression (1). Using that expression, we get for the direct energy gap in Ge the values  $(0.80 \pm 0.01)$  eV at 296°K and  $(0.89 \pm 0.01)$  eV at 77°K. The Faraday rotation beyond a wavelength of  $2.2 \mu$  shows an  $\omega^2$  dependence.

**Gallium Antimonide**

Figure 4 shows the Verdet coefficient of *p*-type GaSb as a function of wavelength. It can be seen that it

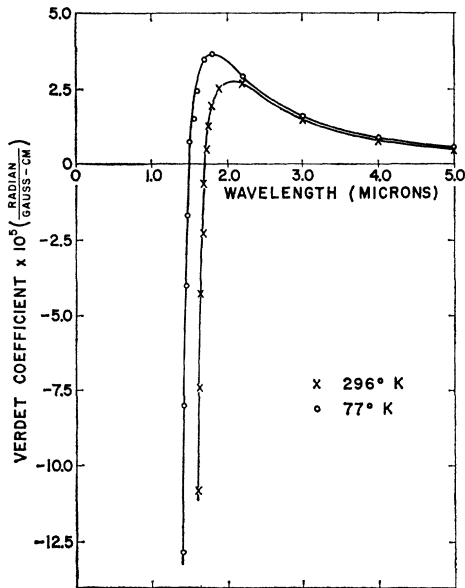


FIG. 3. Verdet coefficient for germanium.

behaves like intrinsic Ge, having large negative values at frequencies near the edge. Using the same approximation as we used on Ge, from these experimental values of the Verdet coefficient we calculated the direct gap energies for GaSb as  $(0.74 \pm 0.02)$  eV at 296°K and  $(0.82 \pm 0.02)$  eV at 77°K. Assuming a linear change of the gap with temperature, we obtain from these values the temperature coefficient of the direct gap,  $-3.7 \times 10^{-4}$  eV (°K)<sup>-1</sup>.

The term  $\hbar\omega_c/2$  contributes to the gap energy  $E_0$  (where  $\omega_c = eH/m^*c$  is the cyclotron frequency for electrons in the conduction band. The term involving the cyclotron frequency is, in general, larger than the term involving the *g* factors, and the effect of the magnetic field will be to increase the band gap).<sup>6</sup> For

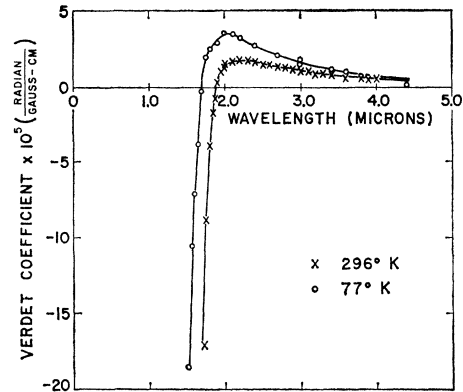


FIG. 4. Verdet coefficient for gallium antimonide.

$H = 19.1$  kG and  $m^* = 0.05m_0$ , its value is

$$\hbar\omega_c/2 \approx 2 \times 10^{-3} \text{ eV.}$$

The values for the energy gap given above should be correspondingly reduced for  $H = 0$ . The temperature dependence of the rotation shows, as in Ge, that the positive rotation gives a larger contribution at 77°K than at 296°K, but also a larger temperature dependence of the total rotation in that region. For the temperature coefficient of the direct rotation at  $1.7 \mu$ , we get

$$\theta^{-1}(d\theta/dT) = 4.5 \times 10^{-3} \text{ (°K)}^{-1}.$$

Figure 5 shows the Faraday rotation as a function of temperature in the range between 25 and  $-196^\circ\text{C}$  at two wavelengths. The Faraday rotation at  $1.75 \mu$  is mainly due to direct transitions at  $k = 0$ , whereas the rotation at  $2 \mu$  is mainly due to transitions at other points of the Brillouin zone and due to indirect transitions.

**DISCUSSION**

From Eq. (1), the following approximate expression is obtained for the temperature coefficient of the

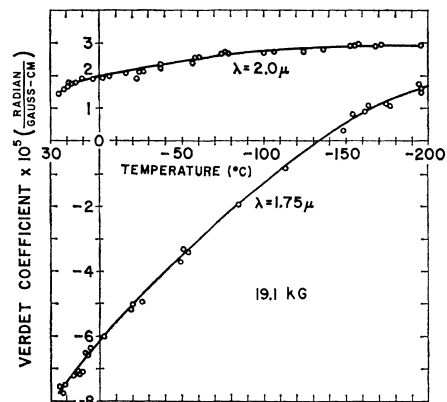


FIG. 5. Verdet coefficient as a function of temperature in gallium antimonide.

Faraday rotation due to direct transitions:

$$\theta_a^{-1}(d\theta_a/dT) \approx -[\omega_g/2(\omega_g - \omega)]\omega_g^{-1}(d\omega_g/dT). \quad (3)$$

Equation (3) shows that for frequencies approaching the gap frequency, the temperature coefficient of the Faraday rotation due to direct transitions becomes very large. The temperature coefficient for the rotation due to indirect transitions can be derived from Eq. (2), and is given by

$$\theta_i^{-1}(d\theta_i/dT) \approx -[1 + 2a(\omega/\omega_g)^m]\omega_g^{-1}(d\omega_g/dT). \quad (4)$$

Equation (4) shows that in the case of indirect transitions at frequencies well below the energy gap, the temperature coefficient of Faraday rotation is about equal to the temperature coefficient of the energy gap. By using the approximate equation (1) for the direct transition, the gap frequency  $\omega_g$  can be evaluated. With further assumptions, the temperature dependence of the Faraday rotation can be used to give the temperature dependence of the energy gap.

### Aluminum Antimonide

The red shift of the absorption edge of AlSb with pressure indicates that the transitions may be in the [100] direction, as is generally regarded to be the case in Si.<sup>10</sup> It has been proposed that the absorption around 1.6  $\mu$  is due to localized levels split off from the valence band.<sup>11</sup> The conclusion was drawn from alloy investigations that the [000] minimum lies about 0.3 eV above the [100] valleys.<sup>12</sup> From the indirect rotation that is seen in the experiment, we conclude that direct transitions are forbidden. [Valence and conduction band at  $k=0$  ( $\Gamma_{15}$ ) consist of the same wave functions.] Direct forbidden transitions require such a high photon energy that they should have little or no effect on the Faraday rotation at wavelengths beyond the indirect gap. From the temperature dependence of the indirect rotation [Eq. (4)], one can calculate the temperature dependence of the indirect gap. We obtained  $dE_g/dT \approx -3.5 \times 10^{-4}$  eV ( $^\circ\text{K}$ )<sup>-1</sup>.

### Germanium

In contrast with the indirect rotation, a direct test of the frequency dependence of the direct rotation is not possible because of the contribution of other transitions in the wavelength region  $\lambda > \lambda_g$ . Figure 6 shows a schematic representation of the Faraday rotation due to interband transitions. The difference in the temperature dependence between the negative and positive rotation gives, in the presence of both contributions, an expected temperature dependence of the total rotation shown for 77 and 300°K, because the total

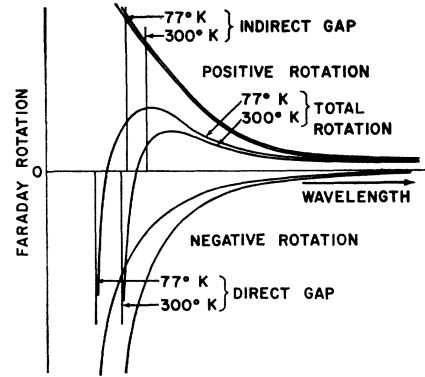


FIG. 6. Representation of the Faraday rotation due to interband transitions.

rotation is merely the sum of these rotations for a given wavelength.

For wavelengths greater than 2.2  $\mu$ , our measurements show an  $\omega^2$  dependence of the Faraday rotation. If we assume that the positive rotation also has an  $\omega^2$  dependence for  $\omega < \omega_g$ , as in Si, then our result leads to the conclusion that the negative rotation has the same  $\omega^2$  dependence in that wavelength region. Hartmann and Kleman<sup>13</sup> have investigated the Faraday rotation in Ge at room temperature, and our results at that temperature are in excellent agreement with their measurements.

Walton and Moss<sup>14</sup> have measured the interband Faraday effect on *p*-type Ge. Their results indicate a very small shift to longer wavelengths at both temperatures.

The direct energy gaps calculated from our measurements are in good agreement with the values given by McLean.<sup>8</sup>

Recent experiments by Nishina, Kolodziejczak, and Lax<sup>15</sup> on the oscillatory interband Faraday rotation at room temperature in very thin Ge samples (4.10<sup>-4</sup> cm) have shown that the important term of the direct Faraday rotation is of the form of Eq. (1), including losses due to scattering. In the high-field case the frequency corresponding to the energy gap  $\omega_g$  in (1) is replaced by

$$\omega_n = \omega_g + (n + \frac{1}{2})\omega_c,$$

where  $\omega_c$  is the cyclotron frequency for the reduced effective mass for electrons and holes. The first few peaks of the Faraday rotation correlate exactly with the magneto-absorption data of Zwerdling, Lax, and Roth.<sup>16</sup> But there is an unexplained difference in the amplitudes of the peaks of the direct rotation in the results of Nishina *et al.*<sup>15</sup> and those of Hartmann and

<sup>13</sup> B. Hartmann and B. Kleman, *Arkiv Fysik* **18**, 75 (1960).

<sup>14</sup> A. K. Walton and T. S. Moss, *Proc. Roy. Soc. (London)* **78**, 1393 (1961).

<sup>15</sup> Y. Nishina, J. Kolodziejczak, and B. Lax, *Phys. Rev. Letters* **9**, 55 (1962).

<sup>16</sup> S. Zwerdling, B. Lax, and L. M. Roth, *Phys. Rev.* **108**, 1402 (1957).

<sup>10</sup> A. L. Edwards and H. G. Drickamer, *Phys. Rev.* **122**, 1149 (1961).

<sup>11</sup> R. F. Blunt, H. P. R. Frederikse, J. H. Becker, and W. R. Hosler, *Phys. Rev.* **96**, 578 (1959).

<sup>12</sup> H. Ehrenreich, *J. Appl. Phys.* **32**, 2155S (1961).

Kleman,<sup>13</sup> as well as ours. The large difference in the thickness of the samples ( $4 \times 10^{-4}$  cm as opposed to  $\approx 10^{-2}$  cm) might have some influence, along with contributions due to internal reflections, which increases the measured rotation for very parallel samples. Our calculation shows that this increase due to internal reflections is relatively small, having a maximum value of 3.7 deg for a rotation of 22.5 deg in Ge.

### Gallium Antimonide

Cardona<sup>17</sup> pointed out that the band structure of GaSb has marked similarity to that of Ge, with the exception that the [000] minima are lowest in energy. The similarity in the band structure of Ge and GaSb should show up in Faraday rotation experiments. Therefore, it should be possible to evaluate the temperature dependence of the direct gap and reach conclusions with respect to the contributions due to transitions at other points of the Brillouin zone.

The fact that the lowest conduction band minimum is likely to occur at<sup>18</sup>  $k=0$  is confirmed by the measured direct rotation (Fig. 5), which has the same sign as in Ge and which is related to the different contributions of direct transitions from light- and heavy-hole levels. Equation (3) gives the expected approximate agreement between the temperature coefficient of the rotation and the temperature coefficient of the direct gap.

Sagar<sup>19</sup> concluded from elastoresistance measurements that subsidiary conduction band minima lie in the  $\langle 111 \rangle$  directions about 0.08 eV above the [000] minima at room temperature. The existence of subsidiary minima has been confirmed by measurements of magneto-resistance, Hall effect, pressure, free carrier reflectivity, thermoelectric power,<sup>12</sup> and free-carrier Faraday rotation.<sup>20</sup> Ramdas and Fan<sup>21</sup> attributed the

absorption at high levels to direct transitions but found a temperature-dependent absorption tail as indication of indirect transitions.

To explain the absorption data on  $n$ -type and  $p$ -type samples,<sup>22</sup> indirect transitions were assumed from a valence band having two off-center maxima. The possibility of having the minimum of the conduction band and the maximum of the valence band at different points in  $k$  space was also considered. Our interband Faraday rotation results show that the positive Faraday rotation is larger than the negative direct rotation for frequencies  $\omega < \omega_g$ , where  $\omega_g$  corresponds to the direct gap. Boswarva and Lidiard have recently shown that direct transitions at the  $\langle 111 \rangle$  zone edge can give a strong contribution to the Faraday rotation at low frequencies in germanium.<sup>23</sup> The band structure of GaSb as given by Ehrenreich<sup>12</sup> indicates that these transitions also contribute to the observed positive rotation in GaSb. The negative rotation at the zone center, because of its much larger amplitude, determines the total rotation. On that basis, the observed temperature dependence of the positive rotation in GaSb seems to indicate that the effective indirect gap increases with temperature.

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<sup>17</sup> M. Cardona, Z. Physik **161**, 99 (1961).

<sup>18</sup> S. Zwerdling, B. Lax, K. Button, and L. M. Roth, J. Phys. Chem. Solids **9**, 320 (1959).

<sup>19</sup> A. Sagar, Phys. Rev. **117**, 93 (1960).

<sup>20</sup> H. Piller, J. Phys. Chem. Solids (to be published).

<sup>21</sup> A. K. Ramdas and H. Y. Fan, Bull. Am. Phys. Soc. **3**, 121 (1958).

<sup>22</sup> W. M. Becker, A. K. Ramdas, and H. Y. Fan, J. Appl. Phys. **32**, 2094 (1961).

<sup>23</sup> I. M. Boswarva and A. B. Lidiard, Atomic Energy Research Establishment Report HL62/3539, Harwell, Berkshire, England (unpublished).