$J=13/2$

	(2, 9/2)	(2, 11/2)	(2, 15/2)
(2, 9/2)	$-18F_1-4215/286F_3$	$375/143(39)^{1/2}F_3$	$-120/143(935)^{1/2}F_3$
(2, 11/2)	$375/143(39)^{1/2}F_3$	$-700/121F_1-255/77F_3$	$160/1001(14586)^{1/2}F_3$
(2, 15/2)	$-120/143(935)^{1/}F_3$	$160/1001(14586)^{1/2}F_3$	$105/11F_1-3135/91F_3$

$J=15/2$

	(0, 15/2)	(2, 11/2)	(2, 15/2)
(0, 15/2)	0	$-20/7(210)^{1/2}F_2$	$5/7(1785)^{1/2}F_2$
(2, 11/2)	$-20/7(210)^{1/2}F_2$	$-200/11F_1-5/14F_3$	$-60/7(34)^{1/2}F_3$
(2, 15/2)	$5/7(1785)^{1/2}F_2$	$-60/7(34)^{1/2}F_3$	$30/11F_1-95/7F_3$

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Quasi-Deuteron Model and the Internal Momentum Distribution of Nucleons in C^{12}

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The quasi-deuteron model was assumed in computing the internal momentum distribution of nucleons in C^{12} using the monoergic photodata of Cence and Moyer and the bremsstrahlung photodata of the Purdue group. The computed quasi-deuteron momentum distribution has the form $A \exp(-p^2/4ME_1)/(4\pi ME_1)^{1.5} + Bp^2 \exp(-p^2/4ME_2)/1.5\pi^{1.5}(4ME_2)^{2.5}$, where E_1 is 1.5 MeV and E_2 is 5 MeV. The ratio B/A is approximately 2. These two components resemble the (1*s*) and (1*p*) wave functions of the shell model. The (1*s*) component is associated with a binding energy 40 MeV while the (1*p*) component is associated with a binding energy 10 MeV. The internal momentum distribution of nucleons in C^{12} was calculated from this quasi-deuteron momentum distribution assuming that the nucleons exist in proton-neutron pairs in the nucleus.

INTRODUCTION

SEVERAL independent sources of information are available on the internal momentum distribution of nucleons in light nuclei.[1–8] Two of these are the quasi-elastic proton-proton scattering data[6–8] and the nucleon photoproduction data.[1–5] In a quasi-elastic *p-p* scattering experiment (see, for example, reference 6) a target is bombarded by a fairly monochromatic proton beam and some suitable angular and energy correlations of the scattered and scattering protons are measured and analyzed to yield the proton binding energy and momentum distribution in the nucleus. These experiments indicate that there are several groups of protons in the nucleus with some characteristic binding energy and momentum distribution. The transparent nucleus model (impulse approximation) is usually employed in these analyses. The binding energy and momentum distribution thus computed are reasonably consistant with the predictions of the shell theory.[6]

In a nucleon photoproduction experiment, a target is bombarded by a photon beam (either bremsstrahlung[1–3] or monochromatized bremsstrahlung[4,5]) and a proton with a given momentum is detected with[2,3] or without[1,4,5] a neutron in coincidence. The *n-p* coincidence rate as a function of the detection geometry has been seen to peak where the free deuteron photodissociation *n-p* coincidence rate peaks.[2,3] The width of the *n-p* coincidence rate as a function of the neutron emission angle is considerably wider than the resolution of the detector system and can be related to the intranucleus momentum distribution of nucleons in nuclei. In addition to the momentum distribution, the energy spectrum of the singly detected protons yields information on the binding energy of nucleons in nuclei. Of course, the width of the angular correlation of *n-p*

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is related to the distribution function for the total momentum of two nucleon subunits inside the nucleus, and some assumptions concerning the binding of the two nucleons in the subunit are necessary in order to obtain the nucleon momentum distribution function from the quasi-deuteron momentum distribution.[1]

In general, the quasi-elastic p-p scattering data yield more precise and conclusive information than the photodata. This is primarily due to the relatively large experimental uncertainties which are very nearly unavoidable in a high-energy photonuclear experiment. The transparent nucleus model (independent particle model) assumed in the p-p data analysis does not give any satisfactory result in the photodata analysis. This is presumably due to the correlation between nucleons which plays a crucial role in photon absorption.[1–5]

The analysis of the photodata can either be made to study the quasi-deuteron model assuming some known momentum distribution, or vice versa. In this paper the latter type of analysis is given. A somewhat modified version of the quasi-deuteron model[9] is assumed and the bremsstrahlung photodata of the Purdue group[1] and the monoergic photodata of the Berkeley group[5] are analyzed to give the momentum distribution of the quasi-deuterons in C^{12}. The intranucleus momentum distribution of nucleons in C^{12} is then related to the quasi-deuteron momentum distribution.

ANALYSIS

Introduction

It has been observed (see, for example, reference 1) that the photonucleon production cross section for high-energy gamma rays has several unique characteristics: (1) angular dependence of the form $a+b\sin 2\theta \times(1+c\cos\theta)$, characteristic of the $E1$ and $E2$ transitions with a large isotropic term a, (2) E^{-n} energy dependence, (3) linear dependence on Z, and (4) emission of correlated nucleons somewhat consistent with the free deuteron photodisintegration kinematics.

Several authors[1–5] have studied these characteristics in terms of the quasi-deuteron model originally suggested by Levinger.[9] The quasi-deuteron model assumes that the pairing effect of nucleons in nuclei is large and that photons are absorbed by pairs of nucleons in the photoemission of nucleons in complex nuclei. In the Born approximation such pairing is necessary to account for the large number of high-energy nucleons observed. The pairing effect is presumably due to the short-range two-nucleon forces.

An approach, somewhat more fundamental, is made in terms of the isobar model (see, for example, reference 3). For photons with energy above the pion production threshold the photopion production cross section from hydrogen is by several orders of magnitude larger than the free deuteron photodissociation cross section.[10–14] In the light of the impulse approximation[15] and the available experimental data on photopion production cross section (see, for example, reference 13), we can assume that the high-energy photonucleon production process goes via the pion-production-reabsorption channel, as is well known in the case of the free deuteron photodissociation (see, for example, reference 14). If the reabsorption process were random, one would expect an isotropic distribution for the emitted nucleons.[13]

If two nucleons were in a sphere of radius $\hbar/\mu c$ when a pion is produced from one or the other nucleon in the sphere, the phase-space factor strongly favors the two nucleon emission over pion emission. Therefore, if the probability that two nucleons are confined to a sphere of radius $\hbar/\mu c$ were appreciable, then we could assume that the photoemission of high-energy nucleons from complex nuclei proceeds via the two nucleon photodissociation process. There is much evidence for pairing of a proton and a neutron in light nuclei (see, for example, Sachs[16]). One classic example is the fact that for most of the stable light nuclei the mass number A is twice the charge number Z, and adding one proton or neutron results in instability of the nucleus.

If indeed the nucleons in C^{12} existed in n-p pairs, then instead of the conventional random coupling of nucleons[12] which results in the familiar ZN factor, one would have a factor Z multiplying the free deuteron photodisintegration cross section. Furthermore, if the shell theory prediction of the nucleon grouping is correct as it is in the case of the quasi-elastic scattering of protons,[6] then one can assume that two $(1s)$ pairs and four $(1p)$ pairs contribute to the nucleon photoemission. The "pairing" of a $(1s)$ nucleon with a $(1p)$ nucleon [i.e., the probability of finding a $(1s)$ nucleon and a $(1p)$ nucleon in a sphere of radius $\hbar/\mu c$] is much less probable than the pairing of two nucleons belonging to the same shell.[17] And in the spirit of the shell theory[17] one can assume that these two types of pairs—$(1s)$ and $(1p)$ pairs—have different binding energy and momentum distribution.

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[16] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company Inc., Reading, Massachusetts, 1955), Chap. II, p. 10.

[17] E. Feenberg, *Shell Theory of the Nucleus* (Princeton University Press, Princeton, New Jersey, 1955), Chap. X.

Monochromatic Photodata

The differential cross section $d^2\sigma/d\Omega dT_p$ of Cence and Moyer[5] gives the number of protons detected per proton MeV per photon per sr per C^{12} nucleus per cm^2. According to the quasi-deuteron model described previously, this second-order differential cross section is related to the momentum distribution of the quasi-deuterons and the free deuteron photodisintegration cross section $d\sigma/d\Omega$ through Eq. (1)

$$\frac{d^2\sigma}{d\Omega dT_p}=\int d\mathbf{P}_d XF(\mathbf{P}_d)(1-\beta_d\cos\theta_d)Jd\sigma/d\Omega, \quad (1)$$

where the subscript d refers to the quasi-deuteron variables in the laboratory system, $(1-\beta_d\cos\theta_d)$ is the Doppler shift correction factor for the photon flux, J is the laboratory-quasi-deuteron rest system transformation Jacobian as given in the Appendix, X is the energy resolution function of the electron counter as given in reference 5, and F is the quasi-deuteron momentum distribution function. The free deuteron photodissociation cross section is evaluated in the rest system of the quasi-deuteron, and the triple integral is evaluated under the four momentum conservation constraint as given in (A6) and (A7) in the Appendix. The conventional constant L (see reference 9) which multiplies $d\sigma/d\Omega$ is contained in F for reasons explained later in this paper. In Fig. 1 it is seen that the cross-section peaks near 120 MeV and its shape is very suggestive of a superposition of two independent curves, one with a broad width peaking near 140 MeV and the other with a narrow width peaking near 110 MeV. It is interesting to note that for the free deuteron photodisintegration by 240 MeV quanta 133-MeV protons will be emitted at 60° in the laboratory system.

In evaluating the integral the distribution function F was broken into two parts as shown in Eq. (2) and Fig. 2.

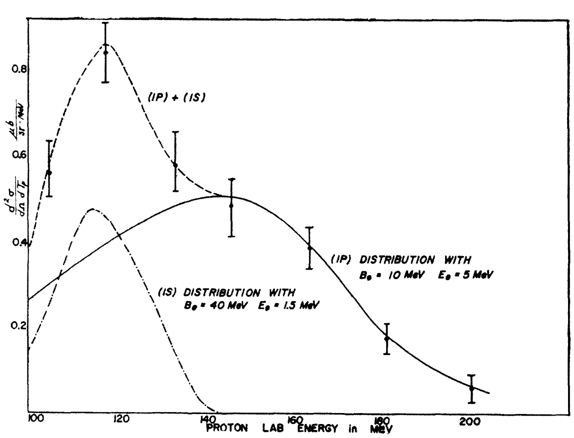

Fig. 1. The calculated differential cross section is compared with the experimental data of Cence and Moyer. The best fit is achieved with $E_1=1.5$ MeV, $E_2=5$ MeV, $B_{01}=40$ MeV, $B_{02}=10$ MeV, $A=0.37$, and $B=0.83$.

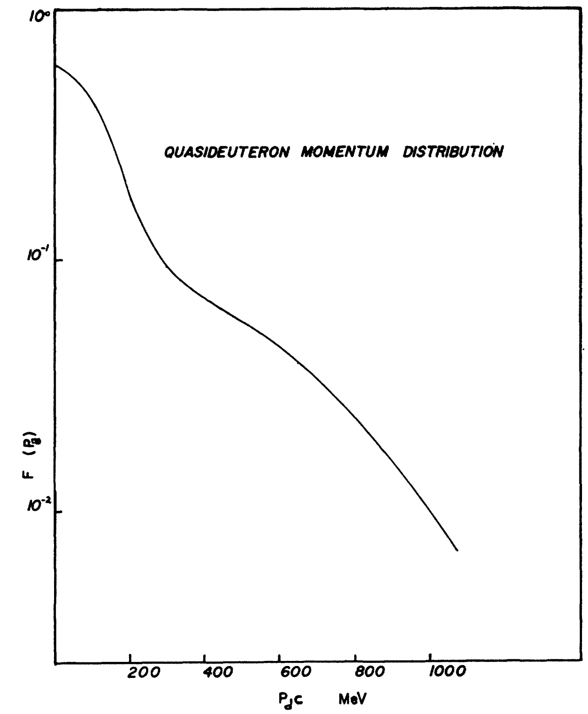

Fig. 2. $F(P_d)$ is plotted as a function of P_d for illustration only. The scales used are arbitrary.

$$F(\mathbf{P}_d)=A\frac{\exp(-P_d^2/4ME_1)}{(4\pi ME_1)^{1/2}}+B\frac{P_d^2\exp(-P_d^2/4ME_2)}{1.5\pi^{1/2}(4ME_2)^{2.5}}. \quad (2)$$

The parameters E_1 and E_2 determine the width of the curves while the parameters B_{01} and B_{02} determine the position of the peak of the curves in Fig. 1.[18] As shown in the figure the computed differential cross section agrees very well with the experimental data for $E_1=1.5$ MeV, $B_{01}=40$ MeV, $E_2=5$ MeV, $B_{02}=10$ MeV, $A=0.37$, and $B=0.82$. It should be noted that the absolute value of the cross section both experimental and theoretical should not be taken too seriously and only the shape of the cross section should be dealt with. Thus the absolute value of A and B are not important but the ratio B/A is important and is related to the relative weight of the (1s) and (1p) contributions.[19]

Bremsstrahlung Photodata

The bremsstrahlung photodata of the Purdue group[1] give the number of protons detected in a given energy

[18] B_{01} and B_{02} are the energy balance terms contained in the four-momentum conservation constraint.

[19] The integrals in (1) and (3) were numerically evaluated with an error of less than 1%.

interval per effective quantum per MeV per steradian per C^{12} nucleus per cm^2. In analogy to (1) this differential cross section is related to the quasi-deuteron momentum distribution and the free deuteron photodissociation cross section through Eq. (3).

$$\frac{d^2\sigma}{d\Omega dT_pQ}=\int d\mathbf{P}_d(1-\beta_d\cos\theta_d)\times F(\mathbf{P}_d)d\sigma/d\Omega JdE'/dT_p'B(E_m,E)/E. \quad (3)$$

The notations used in (3) are explained in the Appendix. $B(E_m,E)$ is the Schiff bremsstrahlung distribution function for peak energy E_m and photon energy E. The free deuteron photodissociation cross section as in (1) is given in the rest system of the quasi-deuteron. The momentum distribution function F has the same form as in (2).

In Fig. 3 the calculated cross section is compared with the experimental data of the Purdue group.[1] Here instead of an energy distribution as in Fig. 1 the angular distribution of the 156-MeV protons is studied. The fit is quite reasonable. It should be noted that the same parameters except A and B are used for F as in the monoergic case. The absolute values of A and B are larger than in the monoergic case but the ratio B/A is approximately the same.

Discussions and Conclusions

The ratio B/A is seen to be a little over 2 (2.2), but as mentioned earlier the constants A and B contain the conventional L factor which multiplies the free deuteron photodissociation cross section. It is reasonable to assume that the pairing strength[20] of $(1s)$ is stronger than that of $(1p)$ pairs and since L depends on the pairing strength, $(1s)$ pairs and $(1p)$ pairs would have different L values.[9] Nevertheless, in view of the large experimental errors contained in the photodata analyzed here, a 10% fractional deviation from 2 may be insignificant and all pairs may contribute with equal weight to the emission of photonucleons.[21]

Now since the nucleons were assumed to be paired[22] in the nucleus in our analysis, the intranucleus nucleon momentum distribution is equivalent to the quasi-deuteron momentum distribution except for the mass involved. Thus, the $(1s)$ nucleons have a momentum distribution $(3\pi M)^{-3/2}\exp(-p^2/3M)$ and the $(1p)$ nucleons have a momentum distribution $\frac{2}{3}\pi^{-1/2}(10M)^{-5/2}\times\exp(-p^2/10M)$. The binding energy of the $(1s)$ nucleons is about 20 MeV, and that of the $(1p)$ nucleons is about 5 MeV. Of course, this is only a rough estimate and neglects the Coulomb effects and the intranucleus collision loss which may be quite appreciable at the intermediate energy.[3] The Harvard group[6] gives a range of binding energy 13–20 MeV for $(1p)$ protons and 27–47 MeV for $(1s)$ protons. Their analysis of the proton momentum distribution is not quite complete,[6] but their E_2 is roughly 4 MeV which compares very well with our 5-MeV value.

The excellent qualitative agreement between this analysis and that of the Harvard group is very significant and encouraging. With the advent of faster logic circuitry and solid-state detectors, it is now possible to obtain sharper monochromatization of bremsstrahlung. And with the advent of spark chambers, one can accumulate sufficient amount of data in a reasonably short time interval. It is hoped that with these means better and more precise photodata will be made available for our analysis in the very near future. Measurements of the coincident p-n emission using sharply monochromatized photon beam will be immensely valuable in testing the photonuclear interaction models and studying the intranucleus momentum distribution of nucleons.

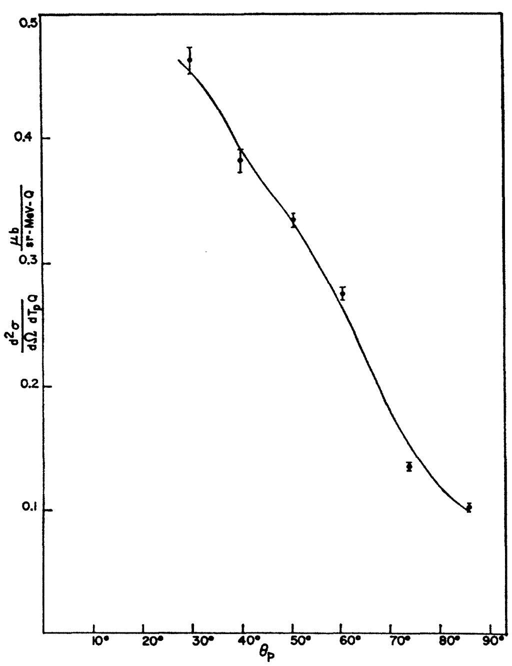

FIG. 3. Comparison of the calculated differential cross section with the Purdue data for 156-MeV protons produced by 340-MeV bremsstrahlung in C^{12}. The fit is made with $E_1=1.5$ MeV, $E_2=5$ MeV, $B_{01}=40$ MeV, $B_{02}=10$ MeV, $A=0.63$, and $A=1.4$.

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[20] The "pairing strength" would be large if the probability of finding two nucleons (the pair) in a sphere with radius $\hbar/\mu c$ is large.

[21] The effects of scattering and absorption of the outgoing nucleons by the residual nucleus are expected to be small at proton energies above 100 MeV and hence are neglected in our analysis. (See, for example, reference 2 for these effects for proton energies below 100 MeV.)

[22] Here we are assuming that the pairing strength is large so that the pair can be treated as one particle.

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APPENDIX

Some of the kinematic relations used in the text are derived here. The general form of the Lorentz transformation of the four-momentum is

$$\mathbf{P}_p' = \mathbf{P}_p - \mathbf{v}_d[\gamma E_p\beta_d^2 - \mathbf{v}_d\cdot\mathbf{P}_p(\gamma-1)]/\mathbf{v}_d^2, \quad \text{(A1)}$$

$$E_p' = \gamma(E_p - \mathbf{v}_d\cdot\mathbf{P}_p), \quad \text{(A2)}$$

where the primed system is moving with a velocity $\mathbf{v}_d$ in the unprimed system. Let the rest system of the quasi-deuteron be primed and the laboratory system unprimed. Assume that the photon is incident along the positive Z axis. The proton makes an angle, θ_p' and θ_p, respectively, in the primed and the unprimed system, with the polar axis. These two angles are related by

$$\cos\theta_p' = (P_p/P_p')[\cos\theta_p + \cos\theta_d\alpha(\gamma-1)] - \gamma E_p\beta\cos\theta_d/P_p', \quad \text{(A3)}$$

where α is the cosine of the angle between $\mathbf{P}_p$ and $\mathbf{v}_d$, and θ_d is the angle between $\mathbf{v}_d$ and the polar axis. α is given by

$$\alpha = \cos\theta_d\cos\theta_p + \sin\theta_p\sin\theta_d\cos(\phi_d - \phi_p). \quad \text{(A4)}$$

The partial derivative of α with respect to $\cos\theta_p$ is

$$\alpha' = \cos\theta_d - \cot\theta_p\sin\theta_d\cos(\phi_d - \phi_p). \quad \text{(A5)}$$

For small detectors ϕ_p is very nearly 90° and $\cos(\phi_d - \phi_p)$ in (A5) can be replaced by $\sin\phi_d$.

The four-momentum is conserved in the following way:

$$E_\gamma + T_d = B_0 + T_p + T_n, \quad \text{(A6)}$$

and

$$\mathbf{P}_\gamma + \mathbf{P}_d = \mathbf{P}_n + \mathbf{P}_p, \quad \text{(A7)}$$

where the subscript d stands for the quasi-deuteron and the rest are the standard notations. B_0 is the energy balancing term and includes the binding energy of the two nucleons. Solving (A6) and (A7) for E_γ and eliminating the neutron coordinates, we obtain

$$E_\gamma = D/(M_n - B - T_p - P_d\cos\theta_d + P_p\cos\theta_p), \quad \text{(A8)}$$

and

$$dE/dT_p = (A - P_pE_p\alpha P_d^{-1})D^{-1}E - (E_p\cos\theta_pP^{-1} - 1)D^{-1}E^2, \quad \text{(A9)}$$

where $B = B_0 - T_d$, $A = M_p + M_n - B$, $2C = P_d^2 - B^2 + 2M_nB$, and $D = T_pA + C - P_pP_d\alpha$.

In the rest system of the quasi-deuteron

$$E' = D'/(M_n - B_0 - T_p' + P_p'\cos\theta_p'), \quad \text{(A10)}$$

$$dE'/dT_p' = (M_n + M_p - B_0)E'D'^{-1} - (E_p'\cos\theta_p'P_p'^{-1} - 1)E'^2D'^{-1}, \quad \text{(A11)}$$

and

$$dT_p'/dT_p = (1 - \beta_d\alpha E_pP_p^{-1}), \quad \text{(A12)}$$

where

$$D' = T_p'(Mn + M_p - B_0) - B_0^2/2 + M_nB_0.$$

Now the solid angles $d\Omega'$ and $d\Omega$ are related by

$$d\Omega' = Jd\Omega,$$

where

$$J = \begin{vmatrix} \partial\cos\theta_p'/\partial\cos\theta_p & \partial\cos\theta_p'/\partial\phi_p \\ \partial\phi_p'/\partial\cos\theta_p & \partial\phi_p'/\partial\phi_p \end{vmatrix}.$$

The elements of J are given by

$$\partial\cos\theta_p'/\partial\cos\theta_p = (P_p/P_p')[1 + \cos\theta_D(\gamma-1)\alpha'] + \cos\theta_p'\gamma^2\beta P_p\alpha'(E_p - \alpha\beta P_p)P_p'^{-2}, \quad \text{(A13)}$$

$$\partial\phi_p'/\partial\phi_p = [P_p\sin\theta_p - \sin\theta_d\cos\phi_d \times(\gamma-1)P_p(d\alpha/d\phi_p)]/(P_p'\sin\theta_p'\sin\phi_p') - \cos\phi_p'L/\sin\phi_p', \quad \text{(A14)}$$

$$\partial\cos\theta_p'/\partial\phi_p = (P_p/P_p')[\cos\theta_d(\gamma-1)d\alpha/d\phi_p] + \cos\theta_p'\gamma^2(E_p - v_dP_p\alpha)v_DP_p \times d\alpha/d\phi_pP_p'^{-2}, \quad \text{(A15)}$$

$$\partial\phi'/\partial\cos\theta_p = P_p\sin\theta_d\cos\theta_d \times(1-\gamma)\alpha'/(P_p'\sin\theta_p'\sin\phi_p' - \cos\phi_p'K/\sin\phi_p', \quad \text{(A16)}$$

where

$$d\alpha/d\phi_p = -\sin\theta_p\sin\theta_d\cos\phi_d,$$

$$L = \gamma^2(E_p - v_dP_p\alpha)v_dP_pd\alpha/d\phi_pP_p'^{-2} + \cos\theta_p'(\sin\theta')^{-2},$$

and

$$K = \gamma^2(E_p - v_dP_p\alpha)v_dP_p\alpha'/P_p'^2 + \cos\theta_p'(\partial\cos\theta_p'/\partial\cos\theta_p)\sin^{-2}\theta_p'.$$

For $\theta_D = 0$ these reduce to the usual solid angle transformation factor.