Analysis of the Energy Dependence of the Isomer Ratio in the $Sn^{120}(p,\alpha)In^{117,117m}$ Reaction

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The method of Huizenga and Vandenbosch for calculating isomer ratios has been extended to reactions that constitute a small fraction of the total cross section. In particular, the method determines the effective distribution in J for emission of particles a brought about by competition between the emission of neutrons and particles a. The method is then applied to the reaction $Sn^{120}(p,\alpha)In^{117,117m}$, and values of σ, the parameter that characterizes the spin distribution, are extracted by comparison with experiment. The values of σ obtained for proton bombarding energies of 12 and 18 MeV are 2.7 and 4.2, respectively, and do not depend strongly upon the value of σ chosen for the neutron evaporation, but do depend strongly upon the fractional amount of direct interaction assumed. Similar values for σ are obtained when competition is not considered. The energy dependence of σ is discussed in the light of present theory.

INTRODUCTION

A METHOD for calculating the isomer ratio in (x,n) reactions applicable to situations where the statistical model of the nucleus is a good approximation has been developed by Huizenga and Vandenbosch.[1] In their formulation the calculated values depend upon a parameter of the theory called σ, and they obtain values of σ by fitting experimental isomer ratios. They have dealt with the case where the final state of interest is produced by neutron emission (or a sequence of neutron emissions) in reactions where neutron emission is overwhelmingly the most probable decay mode of the compound system. They, therefore, do not consider the dependence of the decay probability upon the spin of the compound system, since these probabilities are all so very close to one.

It is the object of this paper to develop a similar formalism applicable to reactions which constitute a small fraction of the total cross section and in particular to investigate the question of the spin dependence of the partial widths. The formation and gamma-ray decay processes are treated in the manner of Huizenga and Vandenbosch but are included here for completeness.

The formalism is then applied to a calculation of the isomer ratio for the $Sn^{120}(p,\alpha)In^{117,117m}$ reaction. These results are modified by considering the effect of direct reactions and finally σ is obtained by comparison with the experimental isomer ratios of Need and Linder.[2]

GENERAL FORMULATION

The cross section for the formation of a residual nucleus in a state i by the $(b,a\gamma)$ reaction can be written as

$$\sigma_i(b,a)=\sum_{J_C}\sigma_b(J_C,E_b)\times\int\sum_{J_F}\frac{\Gamma_a(J_C,E_a,J_F)dE_a}{\Gamma(J_C)}P_i(N_\gamma J_F),\quad(1)$$

where $\sigma_b(J_C,E_b)$ is the cross section for formation of a compound nucleus with spin J_C by a particle b of energy E_b; $\Gamma_a(J_C,E_a,J_F)$ is the width for decay of a state with spin J_C by emission of a particle a of energy E_a to a state in the residual nucleus with spin J_F; $\Gamma(J_C)$ is the total width for decay of the state with spin J_C; and $P_i(N_\gamma,J_F)$ is the probability that a state in the residual nucleus with spin J_F will decay by emission of N_γ photons to the ith state. The number N_γ depends upon the excitation energy of the state formed by the decay particle a and thus upon E_b, Q, and E_a. For a fixed bombarding energy N_γ is a function of E_a, and $P_i(N_\gamma,J_F)$ can be written $P_i(E_a,J_F)$.

Now $\sigma_b(J_C,E_b)$ is given by[1,3,4]

$$\sigma_b(J_C,E_b)=\pi\lambda_b^2\frac{2J_C+1}{(2I+1)(2s+1)}\sum_{S=|I-s|}^{I+s}\sum_{l=|J_C-S|}^{J_C+S}T_{lb}(E_b),\quad(2)$$

where λ_b is the de Broglie wavelength of the projectile, S is the channel spin, s is the spin of the projectile, and $T_{lb}(E_b)$ is the barrier transmission coefficient of particle b with angular momentum l and energy E_b.

The values of $P_i(E_a,J_F)$ are calculated in the following manner. It is assumed that only dipole radiation is emitted in each step of the cascade except for the last gamma ray. For each such step the relative probabilities for a state of spin J_F to decay to states of J_F-1, J_F, and J_F+1 are determined by the spin density of the final state. Thus

$$P(J,J_F)=\frac{\rho(J)}{\rho(J_F-1)+\rho(J_F)+\rho(J_F+1)},\quad(J_F-1\leq J\leq J_F+1)$$
$$=0,\qquad\text{(otherwise)}\quad(3)$$

where $P(J,J_F)$ is the probability of populating a final state with spin J starting from an initial state with spin J_F. The form of the spin dependence of the energy

[1] J. R. Huizenga and R. Vandenbosch, Phys. Rev., **120**, 1305, 1313 (1960).
[2] J. L. Need, and B. Linder, preceding paper [Phys. Rev. **129**, 1298 (1963)].
[3] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[4] L. Wolfenstein, Phys. Rev. **82**, 690 (1951).

level density is taken to be[5–7]

$$\rho(J)=\rho(0)(2J+1)\exp[-(J+\tfrac{1}{2})^2/2\sigma^2], \quad (4)$$

where $\rho(0)$ is the density of states with spin zero. For the last gamma ray it is assumed that the excited nucleus chooses to feed the isomeric or ground state depending on which spin change is the smaller (if the spin changes are equal, it is assumed that both states are fed equally). A discussion of the assumptions involved here is given by Huizenga and Vandenbosch.[1]

The expression for $\Gamma_a(J_C,E_a,J_F)dE_a$ is obtained by integrating Eq. (2) of Wolfenstein[4] over all emission angles and summing over all final states. The $2J_F+1$ degenerate states corresponding to different m_{J_F} values are counted as one state. It is assumed that the sticking probabilities are unity and that there are states of both parities available. Thus,

$$\Gamma_a(J_C,E_a,J_F)dE_a = \frac{D^JP}{2\pi}\sum_{S=|J_F-s|}^{J_F+s}\sum_{l=|J_C-S|}^{J_C+S} T_{al}(E_a)\rho_a(E_a,J_F)dE_a, \quad (5)$$

where $\rho_a(E_a,J_F)$ is the density of final states with spin J_F in the residual nucleus formed by the emission of particle a of energy E_a.

Equation (1) calls for the integration of $\Gamma_a(J_C,E_a,J_F)$ over all energies E_a. Unfortunately, the transmission coefficients are tabulated only at a few discrete energies so that the calculation can be performed only at these discrete energies and the results summed in a manner to be described.

Because the quantity of interest is a ratio of cross sections, any constant factors in Eq. (1) can be ignored, as can absolute magnitudes. Thus, for instance, $D^JP/2\pi$ in Eq. (5) can be dropped, and $\sigma_b(J_C,E_b)$ in Eq. (1) can be replaced by

$$P(J_C,E_b)\equiv\sigma_b(J_C,E_b)/\sum_{J_C}\sigma_b(J_C,E_b), \quad (6)$$

which is the normalized distribution of J_C in the compound nucleus.

If particle a is a neutron, it is possible to take the above spin distribution as the distribution which serves as a source of neutrons because the neutron width is the overwhelming fraction of the total width. When particle a is not a neutron, then the effect of the competition between emission of a neutron and particle a must be considered. In this regard it is illuminating to rewrite the expression $\Gamma_a(J_C,E_a,J_F)/\Gamma(J_C)$ as

$$[\Gamma_a(J_C,E_a)/\Gamma(J_C)][\Gamma_a(J_C,E_a,J_F)/\Gamma_a(J_C,E_a)], \quad (7)$$

where the first factor describes how particles a at energy E_a compete with the total emission from a compound state with spin J_C and the second describes how these particles a are distributed over final spin states J_F. Combining the first factor of (7) with $P(J_C,E_b)$ and normalizing on J_C leads to the normalized distribution of J_C that serves as the source for the emission of particles a at energy E_a and can be written as $P(J_C,E_b,E_a)$. In this way the effect of the competition between the emission of particles a and the total emission is made graphic.

For the calculation the total decay width $\Gamma(J_C)$ is approximated by $\Gamma_n(J_C)$, the width for neutron decay:

$$\Gamma(J_C)\approx\Gamma_n = \int\sum_{J_F=0}^{\infty}\sum_{S=|J_F-\frac{1}{2}|}^{J_F+\frac{1}{2}}\sum_{l=|J_C-S|}^{J_C+S} T_{nl}(E_n)\rho_n(E_n,J_F)dE_n. \quad (8)$$

At this point it is assumed that the dependence of final states on spin and energy can be factored as $\rho(E_a,J_F)=\rho(E_a)\rho(J_F)$ [where $\rho(J_F)$ is given by (4)] and further that

$$\int T_{nl}(E_n)\rho_n(E_n)dE_n\approx T_{nl}(\bar{E}_n)\int\rho_n(E_n)dE_n,$$

where $\bar{E}_n$ is some appropriate average energy. This last point was checked by Huizenga and Vandenbosch[1] and found to be satisfactory. With these assumptions

$$\Gamma(J_C)\approx\sum_{J_F=0}^{\infty}\sum_{S=|J_F\frac{1}{2}|}^{J_F+\frac{1}{2}}\sum_{l=|J_C-S|}^{J_C+S} T_{nl}(\bar{E}_n)\rho_n(J_F). \quad (9)$$

Here the factor

$$\int\rho_n(E_n)dE_n$$

has been dropped because it cancels in the calculation of the isomer ratio.

APPLICATION TO $Sn^{120}(p,\alpha)In^{117,117m}$

The above formulation will now be used to estimate the isomer ratio resulting from the reaction $Sn^{120}(p,\alpha)In^{117,117m}$ for certain values of the parameters involved, and values of σ will be extracted by comparison with the experimental results of Need and Linder.[2]

The entrance channel transmission coefficients were taken from Feshbach, Shapiro, and Weisskopf[8] for a value of $r_0=1.5$ F. The two proton bombarding energies for which the calculation was done are 12 and 18 MeV. These energies are, respectively, the energy of the minimum in the experimental isomer ratio and the maximum energy for which the T_{pl} are tabulated. The values of $T_{nl}(\bar{E}_n)$ were taken from Feld *et al*[9] for a

[5] T. Ericson, Phil. Mag. Suppl. **9**, No. 36, 425 (1960).
[6] C. Bloch, Phys. Rev. **93**, 1094 (1953).
[7] H. Bethe, Rev. Mod. Phys. **9**, 84 (1937).

[8] H. Feshbach, M. M. Shapiro, and V. F. Weisskopf, Atomic Energy Commission Report NYO-3077, 1953 (unpublished).
[9] B. T. Feld, H. Feshbach, M. L. Goldberger, and V. F. Weisskopf, Atomic Energy Commission Report 636, 1951 (unpublished).

square well with $r_0=1.5$ F. $\bar{E}_n$ was taken to be 2 MeV.

Calculations were made with three values (3, 5, and ∞) for σ_n, the spin cutoff parameter for Sb^{120}, and three values (2, 3, and 5) for σ_α, the corresponding parameter for In^{117}. The notation σ_n and σ_α is a convenient way to indicate the spin-cutoff parameters characterizing final nuclei produced by neutron and α-particle emission, respectively, and should not be construed to have any other connection with neutrons or α particles.

Theoretically σ^2 is proportional to the product of the nuclear temperature and a moment of inertia. The energy dependence of σ is therefore model dependent. However, for this calculation the energy dependence is ignored.

There are no transmission coefficients available for α particles on In^{117}. There are some calculated by Huizenga and Igo[10] for α particles on Sn^{119} and these were used in the numerical calculation. Because the integration over E_α is going to be replaced by a weighted sum over those E_α for which the calculation is performed, it is useful to know the spectrum of the α particles emitted from Sn^{120} bombarded by 12- and 18-MeV protons. No experimental data exist for these

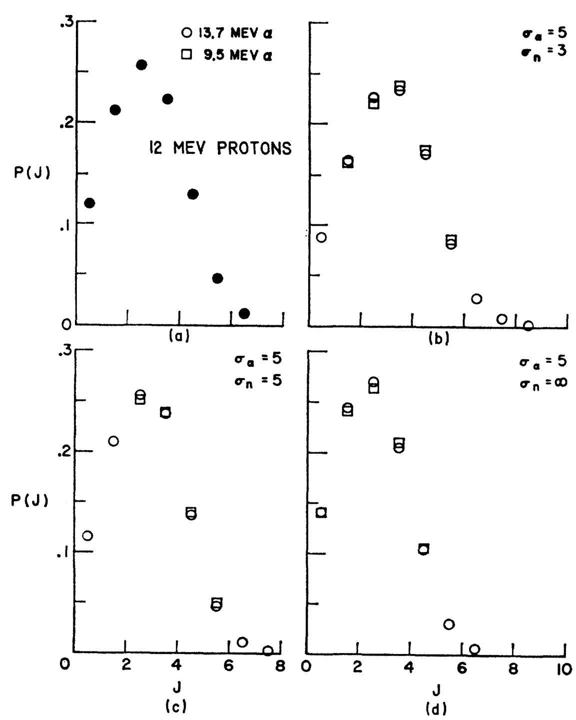

FIG. 1. The spin distributions in the Sb^{121} compound nucleus as directly produced by 12-MeV protons and as modified by competition between α particles and neutrons for a value of $\sigma_\alpha=5$ and $\sigma_n=3$, 5, and ∞ for two values of outgoing channel energy.

[10] J. R. Huizenga, and G. J. Igo, Argonne National Laboratory Report ANL-6373, 1961 (unpublished).

spectra. The best information for extrapolation is the data of Fulmer *et al.*[11,12] and Sherr *et al.*[13,14] on Rh^{103}.

The energy of the peak in the α-particle spectrum of $Rh^{103}(p,\alpha)$ at 17-MeV proton bombarding energy is between 12.5 and 13.5 MeV (total energy in c.m. system). For the α particles from $Co^{59}(p,\alpha)$ the peak lies between 8.5 and 9.5 MeV. When the Q values and the shift in peak energy due to the Z dependence of the Coulomb barrier are considered, the peak of the α particle spectrum from the $Sn^{120}(p,\alpha)$ reaction at 17-MeV bombarding energy is estimated to be at 10 MeV.

The 90° α-particle spectra from protons on Rh^{103} of Fulmer and Goodman for 13- and 17-MeV bombarding energy show little difference when normalized to the same peak value. This is also true of the 135° α-particle spectra from protons on Co and Ni. Thus, it is not unreasonable to assume that the α-particle spectrum from $Sn^{120}+p$ will not change shape drastically in going from 12- to 18-MeV bombarding energy. The available $T_l(E_\alpha)$ are tabulated[9] for α-particle laboratory energies of 10, 12, 14, 16, $\cdots$ MeV, which correspond to c.m. energies of 9.7, 11.6, 13.5, 15.5, $\cdots$ MeV. Thus, there are T_l values available for energies near the peak energy and above but none for lower energies. Calculations were done at 9.7 and 13.5 MeV, the latter being at about the upper 1/4 point of the evaporation spectrum.

The excitation energy in In^{117} is determined uniquely by E_p and E_α. Two choices were made for N_γ the number of γ rays in the cascade. In one calculation N_γ was taken to be the two integers closest to $E^*/2$ MeV and the results averaged (except for the case where $E^*=1$ MeV for which N_γ was taken to be 1). For the other N_γ was taken to be the integer nearest $(E^*a)^{1/2}/2$ where a was taken as 10 MeV^{-1}.[15] The values for the isomer ratio resulting from these calculations are presented as functions of E_α in Table I.

Figures 1 and 2 show some values of $P(E_p,J_C,E_\alpha)$ for selected values of the parameters. The other distributions are similar. Each figure also contains the appropriate unmodified spin distribution, $P(E_p,J_C)$, for comparison purposes. As might be expected, the distributions as modified by competition differ the least from the unmodified distribution in those cases where $\sigma_\alpha=\sigma_n$. The second interesting point is that these distributions do not depend strongly on the energy chosen for the emitted α particle. This fact may well depend upon the particular energies and the particular nucleus for which this calculation was performed. In particular it would be interesting to extend the calculation to α-particle energies below the peak.

Strictly speaking, the values in Table I are not the

[11] C. B. Fulmer and B. L. Cohen, Phys. Rev. **112**, 1672 (1958).
[12] C. B. Fulmer and C. D. Goodman, Phys. Rev. **117**, 1339 (1960).
[13] R. Sherr and F. P. Brady, Phys. Rev. **124**, 1928 (1961).
[14] H. A. Hill and R. Sherr, Bull. Am. Phys. Soc. **5**, 249 (1961).
[15] C. T. Bishop, Argonne National Laboratory Report ANL-6405, 1961 (unpublished).

TABLE I. Calculated values for the isomer ratio σ_g/σ_m at various α-particle energies.

		$E_p=12$ MeV				$E_p=18$ MeV			
		$E_\alpha=9.7$ MeV		$E_\alpha=13.5$ MeV		$E_\alpha=9.7$ MeV		$E_\alpha=13.5$ MeV	
σ_α	σ_n \ N_γ	$E^*/2$	$(2.5E^*)^{1/2}$	$E^*/2$	$(2.5E^*)^{1/2}$	$E^*/2$	$(2.5E^*)^{1/2}$	$E^*/2$	$(2.5E^*)^{1/2}$
2	...	0.55	...	0.69	...	0.49	...	0.59	...
	3	0.51	0.49	0.64	0.56	0.46	0.48	0.56	0.51
	∞	0.45	0.44	0.55	0.49	0.42	0.43	0.48	0.46
3	...	1.44	...	1.52	...	1.81	...	1.94	...
	3	1.49	1.49	1.58	1.56	1.90	1.88	2.05	2.00
	5	1.33	1.34	1.39	1.40	1.63	1.64	1.73	1.71
	∞	1.23	1.25	1.29	1.31	1.50	1.50	1.57	1.56
5	...	2.71	...	2.69	...	4.41	...	4.60	...
	3	3.71	3.79	3.24	3.40	5.99	5.98	6.05	6.05
	5	3.29	3.38	2.73	2.92	4.52	4.47	4.46	4.52
	∞	3.07	3.16	2.47	2.67	3.87	3.80	3.80	3.88

"true" isomer ratios since the integration over E_α has not yet been performed. However, it is seen that there is generally only a slight decrease in σ_g/σ_m in going from $E_\alpha=13.5$ MeV to $E_\alpha=9.7$ MeV. It is assumed that this decrease continues for lower values of E_α. The energy distribution is generally symmetric around the peak, and since there are no drastic changes in σ_g/σ_m with energy it is assumed that

$$\frac{\sigma_g}{\sigma_m}=\int N(E_\alpha)\sigma_g(E_\alpha)dE_\alpha \Big/ \int N(E_\alpha)\sigma_m(E_\alpha)dE_\alpha \approx\frac{\sigma_g(9.7)}{\sigma_m(9.7)}.$$

The isomer ratios calculated above should not yet be compared with the experimental values because they represent only the isomer ratio brought about by processes that are described as compound nucleus; direct reactions have not yet been considered.

The angular distribution of α particles from Rh^{103} bombarded by 17-MeV protons[13] is forward peaked. This forward peaking is produced by high-energy α particles that leave the residual nucleus at excitations below about 7 MeV, whereas the angular distribution of the lower energy α particles is symmetric about 90°. The minimum amount of direct reactions in the total cross section is 15%, this being the nonsymmetric part of the total α-particle cross section. It is expected that the direct contribution is larger than this since direct interactions can contribute a symmetric part to the angular distribution. The direct reaction contribution is expected to decrease at lower bombarding energy although this may not necessarily be the case. For the present calculation the proportion of direct reaction α particles is taken to be 25% at 18-MeV proton bombarding energy and both 5 and 25% at 12 MeV.

In a direct interaction it is expected that large spin changes do not occur—the incident particle does not react with the target nucleus as a whole and thus does not transfer all its angular momentum to the residual nucleus. These direct reactions populate the low-lying ($E^*<4$ MeV) levels of the residual nucleus so that the number of cascade gamma rays is small. Thus, direct reaction favors the formation of the residual nucleus in a low spin state. A value of $\sigma_g/\sigma_m\approx0.32$ for the direct reaction portion is obtained with reasonable assumptions about the spin changes involved. The dependence of this number on σ_α is small, being from 0.26 for $\sigma_\alpha=2$ to 0.37 for $\sigma_\alpha=\infty$ for a particular choice that was calculated.

With these assumptions the final values for σ_g/σ_m were calculated. The results are given in Table II. To illuminate the role of the decay competition, calculations were also performed with $\Gamma_\alpha(J_C,E_\alpha)/\Gamma_n(J_C)$ taken to be unity and these appear in the tables with a dash under σ_n.

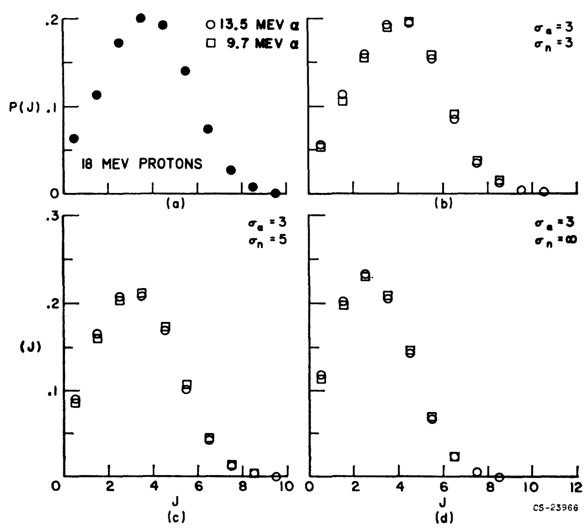

FIG. 2. The spin distributions in the Sb^{121} compound nucleus as directly produced by 18-MeV protons and as modified by competition between α particles and neutrons for a value of $\sigma_\alpha=3$ and $\sigma_n=3$, 5, and ∞ for two values of outgoing channel energy.

TABLE II. Calculated values for the isomer ratio σ_g/σ_m with direct interactions included.

σ_α	σ_n \ N_γ	$E_p=12$ MeV $E^*/2$	$E_p=12$ MeV $(2.5\times E^*)^{1/2}$	$E_p=18$ MeV $E^*/2$	$E_p=18$ MeV $(2.5\times E^*)^{1/2}$
2	...	0.54	...	0.45	...
	3	0.50	0.48	0.42	0.44
	∞	0.44	0.43	0.39	0.40
3	...	1.34	...	1.19	...
	3	1.39	1.38	1.24	1.26
	5	1.24	1.25	1.11	1.13
	∞	1.16	1.18	1.05	1.06
5	...	2.41	...	2.06	...
	3	3.18	3.23	2.38	2.40
	5	2.86	2.93	2.08	2.09
	∞	2.69	2.76	1.92	1.92

Figure 3 shows these results for $N_\gamma=E^*/2$ and Fig. 4 shows the results when no direct reactions are considered. The values of σ appropriate to the spin distribution in In^{117} are determined by the intercepts of the curves with the experimental values[2] and are given in Table III. The derived values are seen to depend upon proton bombarding energy, percentage of direct reaction assumed, and to a lesser degree upon the value of σ_n. It is also seen that the calculation that ignores competition gives similar results so that the role of competition is not made completely clear by this particular example.

DISCUSSION

In other determinations of σ Wolfe and Hummel[16] looked at the $Sb^{121}(\gamma,\alpha)In^{117,117m}$ reaction and obtained a value of 4 for σ. Huizenga and Vandenbosch[1] obtained $\sigma=4\pm1$ in their study of the isomeric pair $Hg^{197,197m}$ and $\sigma\leq5$ from investigations of (n,γ) and (γ,n) reactions. Bishop[15] obtained $3\leq\sigma\leq5$ for (n,γ) reactions and an energy-dependent σ (from 2.5 at low excitation to 7 at high excitation) in the $Ag^{107}(\alpha,n)In^{110,110m}$ reaction. Ericson[17] obtained values of $\sigma\approx4$ for S^{33}, Mn^{56}, and Fe^{57} by counting low-lying nuclear levels and projecting to the neutron binding energy, while Douglas and MacDonald[18] analyzed the angular distributions obtained in (n,p) and (n,α) reactions and obtained $\sigma>2.2$ for the $Cu^{63}(n,p)$ reaction and $\sigma<1.6$ for various nuclei from Si^{27} to Cu^{59} for the (n,α) reaction.

The results of Wolfe and Hummel[16] are of particular interest because they produced the same final states, $In^{117,117m}$, via the same compound nucleus, Sb^{121}, as did Need and Linder.[2] In the (γ,α) reaction the isomer ratio was 2.60 ± 0.40 and was constant to 7% over the range of gamma-ray energies between 15.5 and 24 MeV. The same range of excitation energies in Sb^{121} is produced by protons between 9.6 and 18.0 MeV, and Need and Linder[2] observed a strong variation in the isomer ratio in this region. Wolfe and Hummel did not consider the spin dependence of the total α-particle decay width in their calculation but since the present calculations show little dependence on σ_n (cf. Table III) it seems safe to conclude that their result of $\sigma=4$ with no energy dependence will not be changed greatly by the inclusion of this consideration in their calculations.

The values of σ obtained in the present calculation with no direct interaction are all low and show a mild dependence on bombarding energy. The values obtained for a constant 25% direct component are in general agreement with previous results and show a larger energy dependence. Finally, when the comparison is made using the most reasonable assumptions as to the fraction of direct reactions, i.e., 5% at 12 MeV and 25% at 18 MeV, the energy dependence of σ is substantial, and the values obtained at 12 MeV are low. The discrepancy between these results and those of Wolfe and Hummel is not understood.

The energy dependence of σ is dependent upon the model used to describe the nucleus. By introducing the the concepts of nuclear temperature τ and a moment of inertia $\mathscr{I}$ it is possible to show[4,6] that

$$\sigma^2=\mathscr{I}\tau/\hbar^2.$$

For a Fermi gas of protons and neutrons the energy dependence of τ is given by[6]

$$1/\tau=(A/fE)^{1/2}-2/E, \tag{9}$$

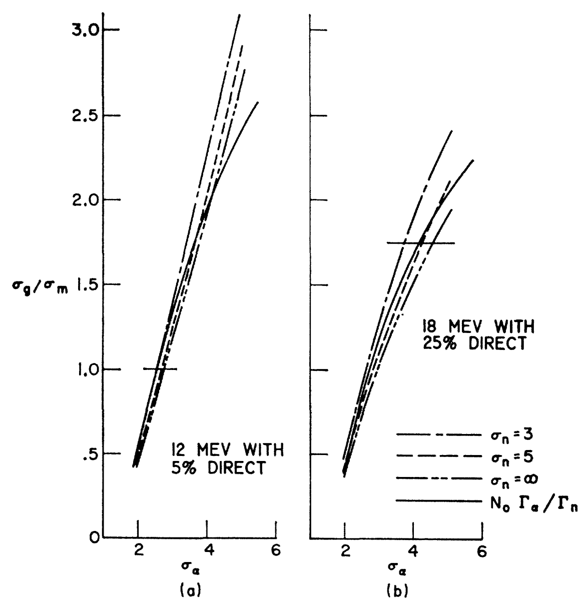

FIG. 3. Values of the isomer ratio vs σ_α with σ_n as a parameter under the conditions of 5% direct reaction at 12 MeV and 25% at 18 MeV.

[16] J. H. Wolfe and J. P. Hummel, Phys. Rev. **123**, 898 (1961).
[17] T. Ericson, Nucl. Phys. **11**, 481 (1961).
[18] A. C. Douglas and N. MacDonald, Nucl. Phys. **13**, 382 (1959).

so that σ varies roughly as $E^{1/4}$. A numerical calculation (with $f=10\,\text{MeV}^{-1}$) for residual nucleus excitations corresponding to the most probable α-particle energy at proton energies of 12 and 18 MeV gives $\sigma(18)/\sigma(12)=1.2$ with the assumption that $\mathscr{I}$ does not change. The experimental value of this ratio is 1.5.

It is customary to compare the value of $\mathscr{I}$ derived from (8) with the rigid-body moment of inertia, which is given by[5]

$$\mathscr{I}_R=\tfrac{2}{5}MAR^2.$$

At 12 MeV the experimental value of $\mathscr{I}/\mathscr{I}_R$ is 0.27 and at 18 MeV it is 0.48. These results are in general agreement with other values.[17,18]

The experimental results of Sherr and Brady[13] indicate that their α-particle spectra from proton-induced reactions can be fitted best with a constant value of nuclear temperature. Equation (8) implies, then, that the value of σ should not change with proton energy if $\mathscr{I}$ is a constant. However, Strutinski[19] has calculated both the energy level density and σ^2 at low values of excitation energy of nuclei. He finds that for 3 MeV $\lesssim E^*\leq 10$ MeV the energy level density is well represented by the constant-temperature form (which agrees with the conclusion of Sherr and Brady) and that σ is roughly a linear function of E. The ratio of σ at 10 and 5 MeV excitation as obtained from his results is 1.8. This is larger than the experimentally determined value of 1.5 which is in turn larger than the value of 1.2 calculated for the Fermi gas model.

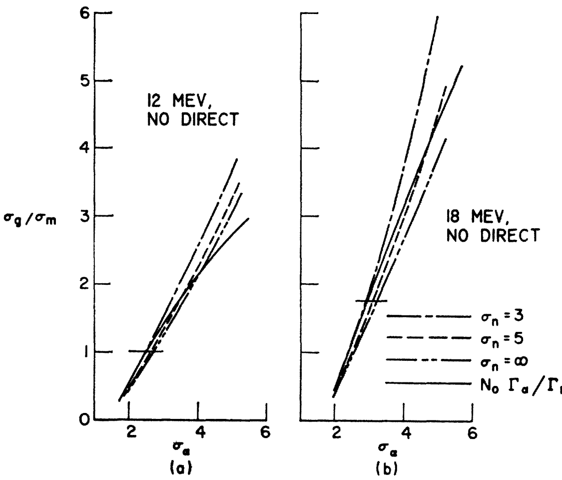

FIG. 4. Values of the isomer ratio vs σ_α with σ_n as a parameter under the conditions of no direct reactions.

TABLE III. Derived values of σ.

σ_n \ E_p	With direct interaction 12.1 MeV (5%)	12.1 MeV (25%)	18.1 MeV (25%)	No direct interaction 12.1 MeV	18.1 MeV
...	2.6	2.9	4.2	2.5	2.9
3	2.6	2.9	3.7	2.5	2.9
5	2.7	3.1	4.2	2.6	3.1
∞	2.8	3.2	4.6	2.7	3.2

It is unfortunate that the presence of so many places in this type of calculation where approximations are necessary makes uncertain the actual numerical values of the parameters obtained by comparison with experiment. However, the area of agreement that has been found gives support to the belief that the general framework of the statistical theory is valid and that meaningful values can be obtained by proper calculations. The present results are not inconsistent with a model that predicts an energy dependence of σ, one for which the moment of inertia is below the rigid-body value at low excitation energy and tends toward it at high excitation energy.

Several discussions of the calculation with J. R. Huizenga, B. Linder, and W. F. Ford are gratefully acknowledged. I wish to thank R. Sherr and J. R. Huizenga for sending me their data before publication.

[19] V. Strutinski, *Comptes Rendus Congrès International de Physique Nucléaire, Paris 1958* (Dunod, Paris, 1959), p. 617.