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Effect of Angular Localization on Direct Nuclear Reactions*

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Some interesting effects of the angular localization of reaction sites caused by the damping of the incident and exit particle waves by the nuclear optical potential are described. Using the reactions $Li^7(p,2p)He^6$ and $Be^9(p,2p)Li^8$, as examples, we show that, due to the angular localization, the total reaction cross section is different for aligned than for unaligned target nuclei. For the Li^7 target in j-j or L-S coupling the fractional change, f, in the cross section for an incident energy of 6 BeV was given by $f=0.165(2x-1)$, where x is the fractional population of Li^7 in the $M=|\frac{3}{2}|$ substates. At 140 MeV the value of the constant in the above expression was slightly but not significantly smaller. For the reaction $Be^9(p,2p)Li^8$ the fractional change in the cross section is one half of and of the opposite sign of the Li^7 value for the same amount of target alignment. Another result of angular localization of reaction sites in the nucleus is that $J\neq 0$ product nuclear states will be oriented with respect to the incident beam. Using (p,pn) and $(p,2p)$ reactions on O^{16} and Ni^{58} as examples, the amount of product nuclear state alignment was computed for the $1p_{3/2}$ and $1f_{7/2}$ excited hole states of O^{15}, N^{15}, and Ni^{57}. The population ratios obtained for the substates in O^{15} or N^{15} were $|\frac{3}{2}|:|\frac{1}{2}|=0.58:0.42$ and for Ni^{57}, $|\frac{7}{2}|:|\frac{5}{2}|:|\frac{3}{2}|:|\frac{1}{2}|=0.34:0.25:0.21:0.20$. The angular anisotropies, $[W(\pi/2)-W(0)]/W(\pi/2)]$, expected from gamma decay of these states were computed to be -0.125 assuming an $M1$ transition for the O^{15} or N^{15} $\frac{3}{2}$-states and 0.162 assuming an $E2$ transition for the Ni^{57} $\frac{7}{2}$-state.

I. INTRODUCTION

RECENTLY, nuclear reaction cross section calculations using distorted-wave approximations have been enjoying success in fitting various features of nuclear reactions, such as angular distributions, polarization, etc. One high-energy approximation often used in these calculations is to evaluate the distortion effects as line integrals through the optical potential along the incident and exit particle momenta.[1,2] The particle waves are damped along their trajectories by absorption represented by the imaginary part of the nuclear optical potential seen by the passing particle. Because of this absorption, the amplitude for the requisite nucleon-nucleon encounters to produce the required final states is greatest in nuclear regions which give the least damping. As has been discussed in other work[2] this localization to preferred reaction regions in the nucleus can be divided into angular and radial localization. One often used example of pure radial localization is the Butler stripping theory where the radial integrals are cut off for r less than an appropriate R. The surface equatorial localization (the incident beam is the z axis) found for $(p,2p)$ and (p,pn) reactions[3,4] contains both angular and radial components.

In this work, we would like to discuss some interesting effects of the angular localization on direct nuclear reactions. Recently a general theoretical framework has appeared[5] with which the results of this paper could be derived. However, we prefer to use a less elegant approach in order to clarify the relationship of the angular localization to the effects to be discussed. Most of our discussion is limited to single nucleon ejection reactions, such as the (p,pn) or $(p,2p)$ reactions as they give clear-cut results.

If there is an angular region of the nucleus which is a preferred reaction site, then the target nucleon wave function which maximizes the amplitude for the nucleon being at this site will contribute most to the reaction. Now the angular variation of a single nucleon wave function is contained in the spherical harmonic part, Y_{lm}, and, for a given l, is dependent on the value of m. As a result there is an m variation in the relative contribution to the reaction cross section with some values of m contributing more than others. In the Born approximation all values of m contribute equally to the cross section as the effect of the optical potential on the incident and exit waves is neglected.

We shall discuss two possible laboratory tests of this effect of the angular localization. One experiment consists of measuring the total (p,pn) or $(p,2p)$ reaction cross section for a $J\neq 0$ target nucleus aligned along the incident beam and comparing the result with the unaligned value. By aligning the target nucleus, we tend to align the target nucleons, or holes, and change the cross section by altering the relative population of the varous m states.

Another result of the m dependence of the contributions to the total reaction cross section is that the $J\neq 0$ product nuclear states from unpolarized targets are oriented with respect to the incident beam. The orientation can be observed as an angular anisotropy in the emission of the de-excitation gamma rays from excited $J\neq 0$ product states. Since several excited product states are always strongly populated in (p,pn) and $(p,2p)$ reactions, there is no lack of test cases for this type of experiment.

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Before discussing these effects in more detail, we need to derive the total reaction cross section from the distorted-wave impulse approximation matrix element.

II. REACTION CROSS SECTION

Since the cross section for the $(p,2p)$, (p,pn), and closely similar reactions in the distorted-wave impulse approximation has been derived several times before,[4,6–9] we briefly sketch the derivation here.

The factored distorted-wave impulse approximation matrix element for the $(p,2p)$ or (p,pn) reactions is[2,6,8,9]

$$M_{if}=\langle e^{i\mathbf{q}\cdot\mathbf{r}_1}D_1^-(\mathbf{r}_1)D_0^-(\mathbf{r}_1)|D_0^+(\mathbf{r}_1)\theta_{if}(\mathbf{r}_1)\rangle \times\langle e^{i\mathbf{k}_p\cdot\mathbf{r}}|t(\mathbf{r})|e^{i\mathbf{k}\cdot\mathbf{r}}\rangle. \quad (1)$$

To obtain this expression we have defined the distortion factors, $D_1^\pm(\mathbf{r})$ by[1]

$$\psi_i^\pm(\mathbf{r}_i)=D_i^\pm(\mathbf{r}_i)e^{i\mathbf{k}_i\cdot\mathbf{r}_i}, \quad (2)$$

where $\psi^\pm$ is the distorted wave of the incident (+) or exit (−) particle and we have used the approximation[2,6] that the distortion factors do not change much over the range for which $t(\mathbf{r}_0-\mathbf{r}_1)\neq 0$. In the interests of simplicity we have neglected including explicitly the spin-dependent parts of t as their inclusion does not remove the angular effects. (Actually we are neglecting only the target-nucleon spin-flip amplitudes of t as our results are in a form which includes much of the spin dependence of t.) This is discussed more fully later on. In Eq. (1) $\mathbf{q}$ is defined by

$$\mathbf{q}=\mathbf{k}-\mathbf{k}_p-\mathbf{k}', \quad (3)$$

where $\mathbf{k}$, $\mathbf{k}_p$, and $\mathbf{k}'$ are the c.m. incident proton, outgoing proton, and outgoing nucleon [proton or neutron for $(p,2p)$ or (p,pn) reactions] momenta. We have neglected any $1/A$ recoil effects. The nuclear overlap wave function, $\theta_{if}(\mathbf{r}_1)$, includes Clebsch-Gordan and fractional parentage coefficients.

The c.m. differential cross section can be written as[8–10]

$$d\sigma/d\mathbf{k}_pd\mathbf{k}'=[1/(2\pi)^3](d\sigma_{\mathrm{fr}}/d\mathbf{k}_p) \times\sum|\langle e^{i\mathbf{q}\cdot\mathbf{r}_1}|D(\mathbf{r}_1)\theta_{if}(\mathbf{r}_1)\rangle|^2, \quad (4)$$

where the sum represents a sum and average over the final and initial states, respectively. The three distortion factors are collected into $D(\mathbf{r}_1)$. To obtain this result which contains the free two-nucleon differential scattering cross section, one neglects the $1/A$ terms and the Q value in the energy-conserving delta function. The neglect of the Q value is justified for bombarding energies, T_L, such that $|Q|\ll T_L$.

To get the total cross section we change variables from $\mathbf{k}'$ to $\mathbf{q}$ and integrate first over $\mathbf{q}$. The energy delta function limits the $\mathbf{q}$ integration by requiring $q\leq 2k$. If the bombarding energy is 100 MeV or more, the $\mathbf{q}$-containing matrix element is quite small[6] at the upper limit of q and we can ignore this limitation. We get for the total (p,pn) or $(p,2p)$ reaction cross section

$$\sigma=\sigma_{\mathrm{fr}}\sum\langle\theta_{if}(\mathbf{r}_1)D(\mathbf{r}_1)|D(\mathbf{r}_1)\theta_{if}(\mathbf{r}_1)\rangle, \quad (5)$$

where σ_{fr} is the appropriate total, free proton-nucleon cross section. If we consider reaction cross sections determined by the radioactive decay of the product, the final-state sum must be limited to nucleon stable states. This is taken care of by including in Eq. (5) a factor

$$(\Gamma_\gamma/\Gamma)^f,$$

where $(\Gamma_\gamma)^f$ and $(\Gamma)^f$ are the respective gamma and total widths of the final state f. We take this factor to be either 0 or 1.

Each of the three distortion factors included in $D(\mathbf{r}_1)$ is given by the exponential of a line integral along the particle trajectory[1,2,6–9] through the optical potential. We replace the product of these three factors by[6]

$$D(\mathbf{r}_1)=\exp\left[-i\eta\int_{-\infty}^{\infty}\rho(r_1)dz_1\right], \quad (6)$$

where

$$\eta=\frac{1}{2}\left[\frac{E_LV_p}{k_L\hbar^2c^2}+\frac{E_{pL}V_p'}{k_{pL}\hbar^2c^2}+\frac{E_L'V'}{k'_L\hbar^2c^2}\right]. \quad (7)$$

The total lab system particle energy, including the rest mass for the incident and two exit particles, is given by E_L, E_{pL} and E_L', respectively, V_p, V_p', and V' are the central values of the optical potential seen by each nucleon and $\rho(r_1)$ is the nuclear density distribution.

To obtain these equations we have assumed that the exit nucleon momenta are parallel to that of the incident proton. Then the product of the three individual distortion factors is replaced by an average factor[6,7] which depends only on the impact parameter. This averaging approximation, Eq. (7), is good as has been shown earlier[4] in cases where the coefficients in the exponents of the distortion factors are much larger for all the exit waves than for the incident wave. There the localization was found to be only slightly asymmetric about the $Z=0$ plane. (The Z axis is along the incident particle momentum.) The forward scattering approximation will contribute some errors which become small at high bombarding energies.[4] In any case these approximations would only affect the numerical results but will not destroy the effect of the angular localization.

The final and initial $j-j$ coupled nuclear state wave functions are taken to be $|J_1M_1T_1M_{T1}\rangle$ and $|JMTM_T\rangle$, respectively, and θ_{if} is obtained by projecting the final state out of the initial state. Using this result for θ_{if} and

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substituting Eq. (6) into Eq. (5), we get

$$\sigma=\sigma_{tr}\frac{1}{2J+1}\sum_{J_1T_1n'lj}N_{n'lj}\left(\frac{\Gamma_\gamma}{\Gamma}\right)^{n'ljJ_1}\times\sum_{M,M_1m}(B_{mMM_1M_TM_{T1}}{}^{ljJJ_1TT})^2I_{n'ljm},\quad(8)$$

where $N_{n'lj}$ is the number of nucleons in the $n'lj$ shell and

$$B_{mMM_1M_TM_{T1}}{}^{ljJ_1TT_1}=\langle j^{N-1}T^{N-1}(J_1T_1)j\tau]\!]j^N\tau^NJT\rangle\times C(T_1\tau T;M_{T1}M_T-M_{T1})C(J_1jJ;M_1,M-M_1)\,C(lsj;m,M-M_1-m),\quad(9)$$

and

$$I_{n'ljm}=\int R_{n'lj}{}^2(r_1)\,|Y_{lm}(\theta_1\varphi_1)|^2\times\exp\left[2\,\mathrm{Im}\eta\int_{-\infty}^{\infty}\rho(r_1)dz_1\right]d\mathbf{r}_1.\quad(10)$$

The factor $\langle]\!]\rangle$ in Eq. (9) is a total spin-orbit isospin fractional parentage coefficient and the three other factors are Clebsch-Gordan coefficients. If the target nucleon is a neutron (proton) one must take $M_T-M_{T1}=\frac{1}{2}[-\frac{1}{2}]$ in the isospin C coefficient. We have also kept only the imaginary part of η in Eq. (10) because the real part of η which gives a purely imaginary exponential, disappears in Eq. (5). This is as expected, since the real part of the optical potential changes only the angular distribution of the emitted particles but not the total cross section.

A further point to note about Eqs. (9) and (10) is that we have kept no cross terms in the quantum number m. These cross terms are associated with the target nucleon spin-flip amplitude of the t matrix which we have neglected as being small.[10] However, even if we had included the spin-flip part of t, the cross terms in m would disappear. This is due to our forward scattering approximation which makes the distortion factor of Eq. (10) independent of the angle φ. As a consequence, we would have $\int Y_{lm'}{}^*Y_{lm}d\varphi\propto 2\pi\delta(m',m)$. Furthermore, some of the spin dependence of t is included as σ_{tr} is the value of the two nucleon cross section including spin effects. These comments would also be modified if we included the small spin-orbit part of the optical potential in the distortion effect.

III. EFFECT OF THE ANGULAR LOCALIZATION

The main point we wish to make is that $I_{n'lm}$ in Eq. (10) depends on m. Since the distortion factor is largest in the nuclear equatorial region $I_{n'lm}$ will be largest for $m=\pm l$ as $Y_{l\pm l}$ peaks in the equatorial region. Conversely, $I_{n'l0}$ will be small for $m=0$ as Y_{l0} peaks in the nuclear polar regions where the distortion factor is small. This illustrates the previously mentioned general feature of the distortion effect: $I_{n'lm}$ will be largest for that value of m for which Y_{lm} and the distortion-effects factor are large in the same nuclear regions. Also $I_{n'lm}$ will be small if Y_{lm} is small wherever the distortion factor is large. We also note that this effect depends only on the distortion: if $\eta=0$ then $I_{n'lm}$ is independent of m.

A. Changes in σ

Suppose we consider a $J\neq0$ target nucleus with only one open shell aligned along the direction of the incident beam. If the value of $J\neq0$ comes from a less than half-filled shell, the alignment, by increasing the population of the $m\sim|l|$ states, preferentially places particles in the nuclear equatorial regions. Since $I_{n'lm}$ is largest for $m\sim|l|$ for (p,pn) or $(p,2p)$ reactions, we would expect the contribution to the total reaction cross section from this shell to increase and consequently the $(p,2p)$ or (p,pn) reaction cross section should be larger for aligned target nuclei than for unaligned nuclei. On the other hand, if $J\neq0$ comes from a more than half-filled shell in the target nucleus, the alignment tends to place holes in the equatorial regions and particles in the polar regions. We would then expect that the observed cross section for the aligned target nuclei would be *smaller* than the value for unaligned nuclei.

This change in the cross section is also dependent on the coupling scheme assumed for the target nucleus. For example, a nucleus with two neutrons and a proton in an open shell would show no change in the (p,pn) cross section if, in the target nuclear wave function, the two neutrons were coupled to zero spin only. All the effect would reside with the $(p,2p)$ reaction as only the proton could be aligned. However, if the three nucleons were coupled to a definite value of J and isospin T, then both the neutrons and the proton would be partially aligned and we would expect a change in both the $(p,2p)$ and (p,pn) reaction cross section.

In searching for particular reactions which would best exhibit this effect one is faced with some conflicting requirements. In order to maximize the equatorial localization one would choose as large a value of the target atomic weight as is possible. However, most of the cross section for large A targets comes from closed shells[4] which would give no effect as they cannot be aligned. On this basis we would want a low Z target where often only the one open shell gives the observed cross section.[9,11] Besides these considerations one has to consider the ease and accuracy of determining the cross section for the product nucleus as well as the alignment and spin relaxation times of the target.

Two possible candidates in the light of the above considerations are the reactions $Li^7(p,2p)He^6$ and $Be^9(p,2p)Li^8$. Both product nuclei have short half-lives and good decay characteristics, He^6 and Li^8 have only one and two bound levels, respectively,[12] and at liquid-

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helium temperatures Li^7 in Li metal has a nuclear spin relaxation time measured in seconds.[13] The alignment of Be^9 does not seem to have been studied much, but we have chosen it as an example of the effect of the more-than-half-filled shell.

1. *Reaction* $Li^7(p,2p)He^6$

The only state[12] of He^6 we need consider is the $J=0+$ ground state which we take to be pure $T=1$. The ground state[12] of Li^7 has $J=\frac{3}{2}-$ and we assume it to be pure $T=\frac{1}{2}$. The sum over J_1, n', l, j in Eq. (8) reduces to one term, $J_1=0$, $n'lj=1p_{3/2}$ as $(\Gamma_\gamma/\Gamma)=0$ for all other terms. From tables of fractional parentage and Clebsch-Gordan coefficients,[14] the factor, B, in Eq. (8) can be determined. We define R by

$$R=I_{1p0}/I_{1p1}, \tag{11}$$

(note that $I_{n'lm}=I_{n'l-m}$) and the fractional ppoulation of the target $M=J=|\frac{3}{2}|$ and $M=|\frac{1}{2}|$ states by x and $1-x$, respectively. This normalization requres Eq. (8) to be multiplied by 2 as it contains part of the $1/(2J+1)$ factor. With these definitions the cross section, σ_A, for aligned target nuclei is found to be

$$\sigma_A=(5/18)I_{1p1}\sigma_{\rm fr}[1+2x+2(1-x)R]. \tag{12}$$

The cross section, σ_U, for unaligned target nuclei is obtained from Eq. (12) by setting $x=\frac{1}{2}$. The fractional change, f ,of σ_A over σ_U, given by

$$f=(\sigma_A-\sigma_U)/\sigma_U, \tag{13}$$

is found to be

$$f=\tfrac{1}{2}(2x-1)(1-R)/(1+\tfrac{1}{2}R). \tag{14}$$

From this equation we see that there is no effect, $f=0$, if $x=\frac{1}{2}$, no alignment; or $R=1$, no distortion effect.

It remains to estimate from Eqs. (7) and (10), the values of I_{1p1} and R. We take $\rho(r_1)$ to be a Gaussian,

$$\rho(r_1)=e^{-\beta'^2 r_1^2}.$$

The constant, β' is found by fitting the Gaussian to the Woods-Saxon distribution with[15] $r_0=1.25$F and $\alpha=0.60$F. For Li^7 $\beta'=0.39$. If $R_{n'lj}$ is taken to be the harmonic oscillator radial wave function and the double exponential expanded in a power series, the resulting expression can be integrated easily in cylindrical coordinates. The result is

$$I_{1pm}=\sum_{n=0}^{\infty}[\epsilon^n/n!(1+n^2/b^2)^{|m|+1}], \tag{15}$$

where

$$\epsilon=(2\pi^{1/2}/\beta')\,\mathrm{Im}\eta,$$

and

$$b=\beta/\beta'.$$

The spring constant in the radial wave function is denoted by β, which we estimate from electron scattering results[16] to be $\simeq 0.66$ for Li^7.

We calculate I_{1p1} and R at two widely spaced bombarding energies, 140 MeV and 6 BeV, to see if there is any energy dependence in R and f. At 140-MeV incident energy the imaginary part of the optical potential is $-18i$ MeV.[10,15] We take $-12i$ MeV for the imaginary part of the optical potential of each outgoing proton which we take to have 70-MeV kinetic energy. Using these figures we find that $b=1.69$ and $\epsilon=-2.36$. Equation (15) can be easily evaluated by hand as the series converges rapidly. For an upper limit $n\leq 9$ the results, which are accurate to better than 0.2%, are $I_{1p1}=0.310$ and from Eq. (11) $R=0.634$. With these numbers Eq. (14) gives

$$f=0.139(2x-1). \tag{16}$$

At 6 BeV η contains optical potential terms summed over all particles (mesons, etc.) emitted in the nucleon-nucleon event weighted by the probability of occurrence of a certain type of event. For these high energies it is easier to use the meson-nucleon and nucleon-cross sections to determine Imη. Making use of the optical theorem and the relation between the nucleon-nucleon scattering amplitude and the optical potential we have[1,10,17]

$$E_i\,\mathrm{Im}V_i/k_i\hbar^2c^2=-(\sigma_i/2)(A-1)\beta'^3/\pi^{3/2},$$

where the factor multiplying $-\sigma_i/2$ normalizes the radial distribution $\exp(-\beta'^2 r^2)$ to $A-1$ nucleons.[10] We can easily obtain the value of Imη and ϵ as the summing and weighting of σ_i over the various meson multiplicities and energy spectra has already been done.[4] The value obtained is $\sigma_t=180$ mb. Combining this with the value of[18] 34 mb for the 6-BeV p-n cross section, we get $\mathrm{Im}\eta=-0.322$ and $\epsilon=-2.925$. The evaluation of Eq. (15) with $n\leq 10$ gives results accurate to $\leq 0.5\%$, $I_{1p1}=0.243$, $R=0.576$, and from Eq. (14)

$$f=0.165(2x-1). \tag{17}$$

We see from a comparison of Eqs. (16) and (17) that the effect is somewhat larger at 6 BeV than at 140 MeV This difference is due to the mesons emitted in the p-n interactions at 6 BeV. The average exit imaginary optical potential due to the mesons + two nucleons is larger

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than that for just two nucleons at the lower energy. This increase of the effect with increasing optical potential indicates that it might be better to consider using deuterons or alphas as bombarding particles as they see a larger optical potential than do protons.

The above calculations have been done assuming that the Li^7 ground state is a j-j coupled state. However, the level scheme of Li^7 indicates that it is better described by intermediate coupling which is closer to the L-S limit than to the j-j limit.[19] We have consequently determined σ_A and f in the L-S coupling limit by taking the Li^7 and He^6 ground states as belonging to the ^{22}P and ^{31}S supermultiplets, respectively. It turns out that

$$\sigma_A=(5/27)I_{1p1}\sigma_{\mathrm{fr}}[1+2x+2(1-x)R], \quad (18)$$

and that f is the same as for the j-j limit and is given by Eq. (14).

2. *Reaction* $Be^9(p,2p)Li^8$

The evaluation of f is more difficult for the reaction $Be^9(p,2p)Li^8$ than for $Li^7(p,2p)He^6$ due to the increased number of particles in the open p shell and the fact that Li^8 has two levels stable to nucleon emission. The ground and first excited states of Li^8 have spins of $2+$ and $\leq 3+$, respectively[12]; we take the spin of the first excited state to be $1+$.[19,20] Using j-j coupling and evaluating the necessary coefficients,[14] we find that

$$\sigma_A=(\sigma_{\mathrm{fr}}I_{1p1}/225)[287-114x+(58+114x)R], \quad (19)$$

and

$$f=-(57/230)(2x-1)(1-R)/(1+\tfrac{1}{2}R). \quad (20)$$

In comparing this expression for f with that for the $Li^7(p,2p)He^6$ reaction, Eq. (14), we note the appearance of the minus sign which means that $\sigma_A<\sigma_U$. As has been discussed this is due to the aligning of the holes in a more-than-half-filled shell of the target nucleus. The $1p_{3/2}$ shell of Be^9 has 5 nucleons and 3 holes. From the fact that the numerical coefficient for this reaction is about one-half that for the $Li^7(p,2p)He^6$ reaction, we see that the latter reaction is a more sensitive indicator of the effect of the angular localization. In general, one would expect nuclei with a few nucleons in a shell (subject to $J\neq 0$) to be more sensitive than nuclei with many particles in a shell. The reason for this is that for a given projection of the target nucleus, the population of the available particle m states is more evenly distributed for many than for few nucleons in a shell. In this respect the effect of holes is not the same as for particles as can be seen by calculating f for the reaction $B^{11}(p,2p)Be^{10}$. (B^{11} has one $1p_{3/2}$ proton hole.) For this reaction we get

$$f=-\tfrac{1}{6}(2x-1)(1-R)/(1+\tfrac{1}{2}R),$$

which shows that the fractional cross-section change for this reaction is $\frac{1}{3}$ that for the $Li^7(p,2p)He^6$ reaction. It should be noted that a minor part of this difference is due to the fact that both parent states in Be^{10} are available whereas only one in He^6 is available.

The methods we have used to determine R, I_{1p1}, and σ_U can be checked against experiment for the reaction $Be^9(p,2p)Li^8$. The experimental value of σ_U for this reaction for 150-MeV protons is 9 mb.[20] To estimate the value of σ_U we set $x=\frac{1}{2}$ in Eq. (19) and take the values of I_{1p1} and R equal to those determined for the $Li^7(p,2p)He^6$ reaction at 140 MeV. With a value of $\sigma_{\mathrm{fr}}\simeq 27$ mb[18] we find that $\sigma_U\simeq 11$ mb which is in good agreement with the experimental value.

B. Product Nuclear Orientation

An equivalent effect of the angular localization, which creates a dependence of the reaction cross section on the value of m of the removed nucleon, is that the product nuclei from bombardments of unoriented nuclei will be oriented.[5,21] This orientation is due to the unequal weighting of the contributions to the cross section of the nucleons with different m values. For example, with the equatorial localization in the (p,pn) and $(p,2p)$ reactions, the target nucleons with $m=|l|$ are most likely to be knocked out as $Y_{l|l|}$ is largest in the equatorial region. Since there is a strong correlation between the nucleon m and the final state nuclear M_1, the states with large values of $|M_1|$ will be preferentially populated. This alignment of (p,pn) or $(p,2p)$ product nuclei is of more general occurrence than the change in cross section as we only require that the product nuclear states have spin be $>\frac{1}{2}$.

It should be noted that, in general, we have product nuclear orientation (and the requirement that the spins of the product nuclear states be $\neq 0$) rather than nuclear alignment. However, the approximations used in obtaining $I_{n'lm}$, Eq. (10) cause the angular localization to be symmetric about 90° so that the contribution from $m=l$ equals that from $m=-l$. Even if the approximation of using an average optical potential for the incident and exit waves, is relaxed the loss of asymmetry of the localization about 90° is small even for extreme cases[4] so we shall consider the orientation to be alignment. A possible way[21] to detect the alignment is to measure the angular anisotropy of the gamma rays emitted from such oriented excited product nuclear states. As examples we estimate the expected angular anisotropies of the 6-MeV decay gammas from the $1p_{3/2}$ hole states in O^{15} or N^{15} and of the decay gammas from the $1f_{7/2}$ hole state in Ni^{57}. These states are produced in good abundance[4,9,11,22] from the (p,pn) or $(p,2p)$ reaction on Ni^{58}.

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[21] The author is indebted to Professor H. Lipkin and Professor P. Hillman for pointing out the existence of this equivalent effect and suggesting a possible method for its experimental verification.
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1. $O^{16}(p,pn)O^{15}$ *and* $O^{16}(p,2p)N^{15}$ *Reactions*

In order to estimate the angular anisotropy of the gamma decay of the $1p_{3/2}$ hole states[12,23] in O^{15} at 6.14 MeV and in N^{15} at 6.33 MeV, we must first compute the amount of alignment expected. This is obtained by computing separately in Eqs. (8)–(10) each term of the M_1 sum over the final nuclear state projections. We take O^{16} to be a j-j coupled $J=0+T=0$ nucleus. As was done with Li^7, the values of β and β' are obtained from electron scattering work[16] and from a fit of the Gaussian to the Woods Saxon distribution[15] and are found to be $0.57F^{-1}$ and $0.32F^{-1}$, respectively. Taking the same values of the optical potential parameters and incident energy, 140 MeV, as we used for Li^7, we find that $\epsilon=-2.91$. After some computation, using Eqs. (11), (15), (8), and (9) we find that $I_{1p1}=0.223$, $R=0.585$ and finally that the fractional populations, $P_{|M_1|}$ of the $M_1=|\frac{3}{2}|$ and $M_1=|\frac{1}{2}|$ states in O^{15} or N^{15} are 0.58 and 0.42, respectively. These values correspond to a 16% M_1 substate population change which is about the same magnitude as the percent change in the cross section for $Li^7(p,2p)He^6$ if Li^7 were completely aligned.

From these values of $P_{|M_1|}$ and the decay characteristics of the hole states the gamma-ray angular anisotropy can be found from theory already developed.[24] The $1p_{3/2}$ hole states in O^{15} and N^{15} decay appear to decay only to the ground states which have a total spin of $\frac{1}{2}-$ so the gamma transitions are either $E2$ or $M1$ or mixtures. Since the single-particle lifetime estimates give the $M1$ transition a width 100 times larger than the $E2$ width and the enhancement factors do not seem to be known,[25] we assume the gamma ray to be only $M1$. The theoretical angular distribution for a $\frac{3}{2}-\longrightarrow\frac{1}{2}-$ $M1$ gamma transition is given by[24]

$$W(\theta)=1+\tfrac{3}{2}N_2f_2P_2(\cos\theta), \qquad (21)$$

where N_2 is a nuclear factor, f_2 describes the nuclear orientation, and P_2 is a Legendre polynomial. Evaluating the literature expressions[24] for N_2, f_2, and P_2 with the values of P_{M1} already obtained, we find that

$$W(\theta)=1+0.04(3\cos^2\theta-1).$$

This function is plotted as the $1p_{3/2}$ curve in Fig. 1 against the emission angle and gives an anisotropy parameter, $[W(\pi/2)-W(0)]/W(\pi/2)$ equal to -0.125.

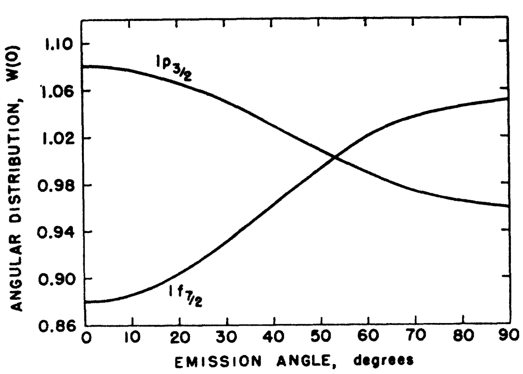

FIG. 1. The estimated relative intensity of decay gammas from the $1p_{3/2}$ hole states of N^{15} or O^{15} and the $1f_{7/2}$ hole state of Ni^{57} as a function of emission angle (0° is along the incident beam).

2. $Ni^{58}(p,pn)Ni^{57}$ *Reaction*

For the reaction $Ni^{58}(p,pn)Ni^{57}$ we are concerned with an excited $\frac{7}{2}-$ state in Ni^{57} corresponding in the shell model to the removal of a $1f_{7/2}$ neutron. This state will be strongly populated in the shell model and should be stable to nucleon emission.[4] For the purposes of estimating an angular anisotropy we assume the $\frac{7}{2}-$ state decays directly to the $\frac{3}{2}-$ Ni^{57} ground state[26] by $E2$ emission.

To compute the alignment of the Ni^{57} $\frac{7}{2}-$ state we need explicit expressions for I_{1fm} given by Eq. (10). Using radial oscillator wave functions and a Gaussian optical potential as was done for Li^7 we find that for $m=\pm3$ or ±2

$$I_{1fm}=\sum_{n=0}^{\infty}\frac{\epsilon^n}{n!(1+n/b^2)^{|m|+1}}, \qquad (22)$$

and for $m=\pm1$ or 0

$$I_{1fm}=\sum_{n=0}^{\infty}\frac{\epsilon^n}{n!(1+n/b^2)^{|m|+1}}\times\left[\frac{|m|+5}{5}-\frac{6-2|m|}{5(1+n/b^2)}+\frac{6-3|m|}{5(1+n/b^2)^2}\right], \qquad (23)$$

where b and ϵ have the same definition as in Eq. (15).

For 140-MeV bombarding energy we again take for the imaginary part of the optical potential $W_{\text{in}}=-18i$ MeV and $W_{\text{out}}=-12i$ MeV for each proton. A fit of the Gaussian to the Woods-Saxon potential gives $\beta'=0.22$. With a value of β equal to[4] 0.48 we obtain the results given in Table I. The first column gives the m

TABLE I. The distortion effect and nuclear alignment of the $1f_{7/2}$ hole state in Ni^{57}.

m	I_{1fm}	$\lvert M_1\rvert$ [a]	$P_{\lvert M_1\rvert}$
3	0.173	$\frac{7}{2}$	0.345
2	0.118	$\frac{5}{2}$	0.251
1	0.0987	$\frac{3}{2}$	0.208
0	0.0985	$\frac{1}{2}$	0.197

[a] We are not implying that $|M_1|=m+\frac{1}{2}$.

[23] E. K. Warburton and J. McGruer, Phys. Rev. **105**, 639 (1957).
[24] S. R. DeGroot and H. Tolhoek, in *Beta and Gamma Ray Spectroscopy*, edited by Kai Siegbahn (Interscience Publishers, Inc., New York, 1955), pp. 613–623.
[25] D. H. Wilkinson, Phil. Mag. **1**, 127 (1956).
[26] D. Strominger, J. M. Hollander, and G. T. Seaborg, Rev. Mod. Phys. **30**, 585 (1959).

values for which I_{1fm} was calculated. Columns three and four give the values of $|M_1|$ and the fractional population $P_{|M_1|}$ for each substate of the $1f_{7/2}$ hole state in Ni^{57}. The "apparent" relationship $|M_1|=m+\frac{1}{2}$ only occurs because we have put the values of I and P in the same table.

We see from these results that there is an appreciable alignment especially of the $M_1=\pm\frac{7}{2}$ substates. The populations of these states show a 38% increase over the unaligned values. All of this increase is at the expense of the $M_1=\pm\frac{3}{2}$, $\pm\frac{1}{2}$ substates whose populations show a respective decrease of 17 and 21%, respectively. The populations of the $M_1=\pm\frac{5}{2}$ substates are essentially unchanged from the unaligned values. A comparison of these results with those for the (p,pn) or $(p,2p)$ reaction on O^{16} shows that the alignment is somewhat larger for Ni than for O. This increase is probably due to the difference in target atomic weight and attendant greater angular localization in Ni than in O.

The expected angular distribution of the assumed $E2\ \frac{7}{2}- \rightarrow \frac{3}{2}-$ gamma decay of the $1f_{7/2}$ Ni^{57} hole state can be determined using the available theory and the population values of Table I. The angular distribution is given by[24]

$$W(\theta)=1-(15/7)N_2f_2P_2(\cos\theta)-5N_4f_4P_4(\cos\theta), \quad (24)$$

where the N_i's, f_i's, and P_i's have the same meaning as in Eq. (21). Evaluation of the literature expressions[25] for N_i, f_i, and P_i with the values of $P_{|M_1|}$ from Table I gives the result,

$$W(\theta)=1-0.0542(3\cos^2\theta-1) -0.00145(35\cos^4\theta-30\cos^2\theta+3). \quad (25)$$

The $1f_{7/2}$ curve in Fig. 1 gives $W(\theta)$ as a function of the gamma emission angle, θ. From Eq. (25) we find that the anisotropy parameter equals 0.162.

From the values of the anisotropy parameter and the curves in Fig. 1 it can be seen that the angular localization causes an appreciable angular anisotropy in the decay gammas from excited (p,pn) or $(p,2p)$ product states. The values of the anisotropy parameters estimated here are in the same range as those obtained in several nuclear orientation studies.[27] However, the observation of the anisotropy of the (p,pn) or $(p,2p)$ product decay gammas is complicated by the presence of many other decay gammas coming from other types of nuclear reactions. Each of these other gammas will exhibit some type of angular anisotropy as the angular localization effects are general and apply to many nuclear reactions.

Foley *et al.*[22] have already shown that in some cases these decay gammas can be easily detected. These authors bombarded O^{16} with 150-MeV protons and found an intense 6.2-MeV gamma peak which they attribute to the 6.15- and 6.33-MeV states in O^{15} and N^{15}, respectively. However, they did not report an angular distribution.

[27] L. Roberts and J. W. T. Dabbs, Ann. Rev. Nucl. Sci. **11**, 175 (1961).

IV. CONCLUSION

In closing it is worthwhile to caution against taking the numerical values estimated here of the cross section changes and gamma anisotropies too literally. The magnitude of both these effects is quite dependent on the amount of angular localization which does in fact occur. The forward scattering approximation, used to obtain Eq. (6), results in a strong equatorial or near equatorial localization. This approximation, while more justified at very high bombarding energies[4] than lower down, is by no means exact. Inclusion of a more realistic angular distribution of the exit particles in the computation of the distortion effect could change both the magnitudes and angular dependence of the regions of preferred reaction sites. This would, in turn, affect the magnitude of the estimates of the reaction cross-section changes and the gamma angular anisotropy.

It should also be kept in mind that we neglect some effects of spin such as the spin-flip amplitudes in the two-body t matrix and the spin-orbit part of the nuclear optical potential. We also assume a simple coupled shell model with harmonic oscillator radial wave functions to represent the nucleus. The removal of these approximations will definitely affect the magnitude of the estimations made here; however, one might expect the changes to be fairly small. In any case, a more exact treatment would not remove the experimental observables but would only change their values.

Our assumption of pure $M1$ and pure $E2$ for the $1p_{3/2}$ and $1f_{7/2}$ hole-state decays of O^{15}, N^{15}, and Ni^{57} was only used to demonstrate the alignment effect. Actually the O^{15} and N^{15} states should be taken as $M1$ and $E2$ mixtures and the excited $1f_{7/2}$ hole state in Ni^{57} may have several decay branches of different multipolarity.

Although the discussion of the effects of the angular localization has been limited here to (p,pn) or $(p,2p)$ reactions, it should be stressed that these effects are more general and apply to many direct and spallation nuclear reactions. The damping of particle waves along their trajectories during the course of a reaction will lead to some type of angular dependence however complex of the contributions to the cross sections by various nuclear regions.

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