dependence of the cross sections. However, an order-of-magnitude comparison is made here to check the consistency of the carbon data of this experiment with that of other experiments. Table III gives a comparison to some of the previous data. The units of the cross section are μb/sr-MeV per equivalent quantum.

The linear dependence of the cross sections on ZN/A as seen in Fig. 6 is in agreement with other experiments.[6,8,9]

Future experiments should look carefully into the effect, if any, of alpha-particle substructures in the nucleus on the angular distribution of photoprotons. Furthermore, as Cence and Moyer have suggested, future experiments should either be done with photon-beam monochromators, or with photon-difference techniques. Either of these techniques would permit a reasonable separation of the effect of scattering in the parent nucleus from other effects.

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Hyperons with and without Doublet Symmetry. II*

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Earlier work on hyperon isobars (resonances) based on a static strong-coupling approximation is continued. New results are derived, in particular for values near ± 1 and near 0 of the ratio of the two coupling constants ($\Sigma\Sigma\pi$ to $\Lambda\Sigma\pi$).

The orbital angular momentum l of a stationary (or resonant) state is unambiguously defined through the eigenfunctions of the bound-pion Hamiltonian. For instance a resonance with $l=2$ and isospin 0 is predicted if the direct $\Sigma\Sigma\pi$ coupling is very weak or absent.

1. INTRODUCTION

A STRONG-COUPLING treatment of hyperon-pion interactions, with emphasis on bound states or resonances, was carried through, with some limitations, in an earlier paper under the same title.[1] Additional results on the same topic are reported here.

The interaction Hamiltonian to be studied is

$$H_1=g(\Lambda^\dagger\sigma\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger\sigma\Lambda)\cdot\nabla\boldsymbol{\Psi}_{\mathrm{av}}+g'i(\boldsymbol{\Sigma}^\dagger\times\sigma\boldsymbol{\Sigma})\cdot\nabla\boldsymbol{\Psi}_{\mathrm{av}}. \quad (1)$$

In I, the corresponding scalar interaction ($\sigma\cdot\nabla\rightarrow 1$) was treated for an arbitrary ratio ($\alpha=g'/g$) of the two independent coupling constants. The more interesting, but more difficult, pseudoscalar interaction (1) was explored only for the special case $g'=0$ (I, Appendix), while the results are well known for $g'=\pm g$ by virtue of doublet (or global) symmetry. Generalizations of these special results are our concern here.

For reference, previous findings have been summarized in Table I. Excitation energies, in arbitrary units, are listed in the third and fourth columns for the two special cases. While these energy differences become continuous functions of the coupling constants g, g', the quantum numbers assigned to the various states, as listed in the second column, are adiabatic invariants (although not obvious, this turns out to be true even for l). Hence, these quantum numbers retain their significance if the coupling strengths are scaled down to more realistic values (intermediate coupling). This provides a kind of information which is not so easily accessible to the more popular methods of analysis,[2] concerned primarily with the S matrix. Instead, we study the Schrödinger equation of the problem; we can explicitly construct the eigenfunctions of "stationary" states and by inspection determine their quantum numbers. For instance, if the state "Y_0*" is called a d state, this has the precise meaning that its eigenfunctions are linear combinations (corresponding to $j=\frac{3}{2}$) of the five spherical harmonics $l=2$ (see I, p. 776).[3] The physical Σ appears as a $p_{1/2}$ state. Of course, all states have the same parity.

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[1] G. Wentzel, Phys. Rev. **125**, 771 (1962), to be quoted as I.

[2] D. Amati, A. Stanghellini, and B. Vitale, Nuovo Cimento **13**, 1143 (1959); Phys. Rev. Letters **5**, 524 (1960); M. Nauenberg, *ibid.* **2**, 351 (1959); J. Franklin, R. C. King, and S. F. Tuan, Phys. Rev. **124**, 1995 (1961); T. L. Trueman, *ibid.* **127**, 2240 (1962).

[3] Note, however, that these are the (one-component) eigenfunctions of the transformed Hamiltonian $U^\dagger HU$.

TABLE I. Energy levels as derived in reference 1.

	l, j, i	$g'=0$	$g'=\pm g$
Λ	$s, \frac{1}{2}, 0$	0	0
Σ	$p, \frac{1}{2}, 1$	6	0
Y_1^*	$p, \frac{3}{2}, 1$	9	3
Y_2^*	$d, \frac{3}{2}, 2$	15	3
Y_0^*	$d, \frac{3}{2}, 0$	15	high
	$p, \frac{1}{2}, 1$	16	high
	$s, \frac{1}{2}, 2$	18	high
	$p, \frac{3}{2}, 1$	19	high
	$d, \frac{5}{2}, 2$	20	8
	$d, \frac{5}{2}, 0$	20	high

We now proceed to examine the problem more generally than was done in I (Sec. 5 and Appendix).

2. EIGENVALUES OF H_1

As was explained in I, the first task is the diagonalization of H_1 which is an 8×8 matrix (8 being the number of bare baryon states Λ, Σ_1, Σ_2, Σ_3, with 2 spin orientations each), involving the 9 pion field variables $q_{i\rho}$ ($=\text{const}\times\partial\psi_\rho/\partial x_i$ projected into the baryon source function). With the notation

$$Q_\rho=\sum_i \sigma_i q_{i\rho}, \quad \text{or} \quad \mathbf{Q}=\sum_i \sigma_i \mathbf{q}_i, \tag{2}$$

the pertinent linear equations for the spinor amplitudes X_0, $\mathbf{X}$ are

$$\begin{aligned} \lambda X_0&=\mathbf{Q}\cdot\mathbf{X}, \\ \lambda\mathbf{X}&=\mathbf{Q}X_0-i\alpha\mathbf{Q}\times\mathbf{X}, \end{aligned} \tag{3}$$

($\alpha=g'/g$). From (3), we have deduced the secular equation

$$\lambda^4-\lambda^2(1+\alpha^2)Q^2-\lambda(3+\alpha^2)2\alpha\Delta +\alpha^2[2\textstyle\sum_{\rho\sigma} T_{\rho\sigma}^2-(Q^2)^2]=0, \tag{4}$$

where

$$T_{\rho\sigma}=\sum_i q_{i\rho}q_{i\sigma}, \quad Q^2=\sum_\rho T_{\rho\rho}, \tag{5}$$

$$\Delta=\det|q_{i\rho}|. \tag{6}$$

Each root of Eq. (4) counts twice (spin degeneracy). Writing out the roots as algebraic functions of the three field parameters Q^2, $\sum T_{\rho\sigma}^2$, and Δ, leads to expressions much too complicated to be of any use in solving the ensuing wave-mechanical problem. We, therefore, resort to expansions near $|\alpha|=1$ (Sec. 3) and near $\alpha=0$ (Sec. 4).

3. NEAR DOUBLET SYMMETRY

For $\alpha=\pm 1$, the solution is well known from the corresponding pion-nucleon problem,[4] and we try to follow essentially the same procedure here, for $|\alpha|$ near unity.

Define a real orthogonal matrix $S_{i\rho}$, depending on three Euler angles θ, ϕ, ψ, which rotates the x_i into the x_ρ' coordinate system, as written out in (I, 25);

$$\textstyle\sum_i S_{i\rho}S_{i\sigma}=\delta_{\rho\sigma}, \quad \sum_\rho S_{i\rho}S_{j\rho}=\delta_{ij}. \tag{7}$$

In addition, introduce a symmetric tensor in charge-space, $\xi_{\rho\sigma}$, and substitute for $q_{i\rho}$

$$q_{i\rho}=RS_{i\rho}+\textstyle\sum_\sigma \xi_{\rho\sigma}S_{i\sigma}, \tag{8}$$

where R is a constant to be chosen later. The Ansatz (8) expresses the 9 variables $q_{i\rho}$ in terms of 9 new ones, viz., θ, ϕ, ψ, ξ_{11}, $\xi_{12}\equiv\xi_{21}$, $\cdots$. Note that for $\xi_{\rho\sigma}=0$ the three isovectors $\mathbf{q}_i$ would rotate like a rigid orthogonal cross. The variables $\xi_{\rho\sigma}$ allow for deformations of this cross, but these (as we know already for $|\alpha|=1$) will amount to small oscillations only.

We express the coefficients of Eq. (4) as functions of the new variables and find the following simple results.

$$Q^2=3r^2+\eta^2, \quad \Delta=r^3-\tfrac{1}{2}R\eta^2+\cdots, \tag{9}$$

$$2\textstyle\sum_{\rho\sigma} T_{\rho\sigma}^2-(Q^2)^2=-3r^4+6R^2\eta^2+\cdots, \tag{10}$$

where

$$r=R+\tfrac{1}{3}\textstyle\sum_\rho \xi_{\rho\rho}, \tag{11}$$

$$\eta^2=\textstyle\sum_{\rho\sigma} \xi_{\rho\sigma}^2-\frac{1}{3}(\sum_\rho \xi_{\rho\rho})^2 \quad (\gtrless 0). \tag{12}$$

Terms of third and fourth orders in the $\xi_{\rho\sigma}$ have been omitted in (9) and (10).

For $\alpha=-1$, the roots of Eq. (4) are those known from the pion-nucleon problem:

$$\lambda_0=-3r, \quad \lambda_1\cong\lambda_2\cong\lambda_3\cong+r.$$

(For $\alpha=+1$, replace R by $-R$.) If the coupling is very strong, R will be very "large" (proportional to the coupling constant g), and only the lowest eigenvalue of H_1 is relevant, corresponding to $\lambda_0=-3r$, if $R>0$.

Next consider

$$\alpha=-1+\delta, \quad |\delta|\ll 1. \tag{13}$$

Then, expanding λ_0 (i.e., the smallest root of Eq. 4) in powers of δ, up to δ^2, we find

$$\lambda_0=-3r(1-\tfrac{1}{2}\delta+\tfrac{1}{16}\delta^2+\tfrac{3}{64}\delta^2\eta^2/R^2+\cdots). \tag{14}$$

We can now write down the total Hamiltonian, or rather that part of it which involves the bound-field variables $q_{i\rho}$ and their canonical conjugates $p_{i\rho}$. As in I, we do this only for the simpler case of low cutoff momentum, again emphasizing that the more general procedure is well known. The free-field Hamiltonian then contains the "kinetic energy" term (as for U, see below)

$$T=U^\dagger T_0 U, \quad T_0=\tfrac{1}{2}\textstyle\sum_{i\rho} p_{i\rho}^2, \tag{15}$$

and the "potential energy"

$$V_0=\tfrac{1}{2}\mu^2\textstyle\sum_{i\rho} q_{i\rho}^2=\tfrac{1}{2}\mu^2Q^2, \tag{16}$$

which combines with the interaction $H_1=\gamma\lambda_0$ ($\gamma=$ large positive constant $\sim g$) to give the "total potential energy"

[4] A. Houriet, Helv. Phys. Acta 18, 473 (1945) [Eqs. (2.37)–(2.52)]. See also G. Wentzel, *ibid.* 16, 551 (1943); W. Pauli and S. M. Dancoff, Phys. Rev. 62, 85 (1942).

$$V=\tfrac{1}{2}\mu^2Q^2+\gamma\lambda_0. \tag{17}$$

With (9) and (14), this is seen to be a "potential valley," with minimum at $\eta=0$ and $r=r_0$:

$$r_0=\mu^{-2}\gamma(1-\tfrac{1}{2}\delta+\tfrac{1}{16}\delta^2+\cdots). \tag{18}$$

It is then convenient to choose the constant R in (8) equal to this r_0, so that, according to (11), the equilibrium position of $\sum_\rho \xi_{\rho\rho}$ is zero, and $\eta=0$ implies that the whole tensor $\xi_{\rho\sigma}$ vanishes [see (12)]. Then V, Eq. (17), may be rewritten as

$$V=-\tfrac{3}{2}\mu^2r_0^2+\tfrac{1}{2}\mu^2(1-\tfrac{9}{32}\delta^2)\sum_{\rho\sigma}\xi_{\rho\sigma}^2 +\tfrac{3}{64}\mu^2\delta^2(\sum_\rho \xi_{\rho\rho})^2+\cdots. \tag{19}$$

Next, the "kinetic energy" T_0, Eq. (15), has to be expressed in terms of second derivatives with respect to the new variables θ, Φ, Ψ, $\xi_{\rho\sigma}$. This calculation is precisely the same as in the pion-nucleon problem, and we merely quote the result. In the "low cutoff" version, one finds

$$T_0=(2r_0^2)^{-1}\mathbf{P}^2-\tfrac{1}{2}\sum_\rho \partial^2/\partial\xi_{\rho\rho}^2-\tfrac{1}{4}\sum_{\rho<\sigma}\partial^2/\partial\xi_{\rho\sigma}^2, \tag{20}$$

where the isovector $\mathbf{P}$ denotes the angular momentum of the "spherical top" θ, ϕ, ψ [see I, Eqs. (31), (32)]. The rotational term, $\sim\mathbf{P}^2$, is the one of major interest, whereas the vibrational energy (remaining terms in T_0, plus V) may be replaced by its zero-point value. (This is so because the vibrational excitation energies, of order μ, are "large" compared with the rotational energies of order $r_0^{-2}\sim g^{-2}$. In the "high cutoff" version, the $\xi_{\rho\sigma}$ variables become intimately coupled with the "free" meson field.) Note that, so far, the quantity δ which measures the deviation from doublet symmetry [see (13)] affects the result in a very minor way [mainly through (18)].

However, one crucial step is still to be carried out, namely the unitary transformation of T_0, indicated in Eq. (15), by the same matrix U which diagonalizes H_1. Since only the lowest eigenvalue of H_1, viz., $\gamma\lambda_0$, is under consideration, only one column of the matrix U is needed, namely, the one given by the normalized eigenvector X_0, $\mathbf{X}$ of Eqs. (3) with $\lambda=\lambda_0$. More precisely, there are two such columns due to spin degeneracy. For λ_0, we use the solution (14), neglecting $\delta^2\eta^2/r_0^2$ as $\ll\delta^2$. As solution of Eq. (3), in corresponding approximation, we find

$$X_\rho=(\lambda_0/3r)\sum_i \sigma_i S_{i\rho}X_0, \quad (\rho=1,2,3). \tag{21}$$

Here, X_0 will be interpreted as a constant spinor, for "spin up or down" [const$\times u_s$, in the language of I, Appendix I], because allowing X_0 to depend on θ, ϕ, ψ, would give the "Λ state" redundant degrees of freedom. (In other words, we exclude "symmetry-breaking solutions," as has been tacitly done in earlier work.) The normalization condition is

$$X_0^\dagger X_0+\sum_\rho X_\rho^\dagger X_\rho=[1+3(\lambda_0/3r)^2]X_0^\dagger X_0=1. \tag{22}$$

Using the isovector commutation rules

$$[P_\rho,S_{i\rho}]=0, \quad [P_1,S_{i2}]=-[P_2,S_{i1}]=iS_{i3}, \cdots,$$

one obtains easily

$$U^\dagger\mathbf{P}^2U=\mathbf{P}^2+U^\dagger[\mathbf{P}^2,U] =\mathbf{P}^2+\kappa(-\sum_{i\rho}\sigma_iS_{i\rho}P_\rho+\tfrac{3}{2}), \tag{23}$$

where

$$\kappa=4(\lambda_0/3r)^2[1+3(\lambda_0/3r)^2]^{-1}. \tag{24}$$

Now it is well known[4] that the quantities

$$l_i=-\sum_\rho S_{i\rho}P_\rho, \quad (i=1,2,3) \tag{25}$$

are the angular momentum components (referring to fixed space coordinates x_i) of the "bound pion field." So, finally, we have for the rotational energy[5]

$$T_{\text{rot}}=(2r_0^2)^{-1}\{\sum_i l_i^2+\kappa[\sum_i \sigma_i l_i+\tfrac{3}{2}]\}, \tag{26}$$

with eigenvalues

$$T_{\text{rot}}=(2r_0^2)^{-1}\{l(l+1)+\kappa[j(j+1)-l(l+1)+\tfrac{3}{4}]\}, \tag{27}$$

($j=l\pm\frac{1}{2}$). Since $\mathbf{P}^2=\sum_i l_i^2$, the isotopic spin i equals the orbital angular momentum quantum number l. The charge m_i runs from $-l$ to $+l$, and m_j (independently) from $-j$ to $+j$.

The value of κ, (24), may be written as an expansion in δ, using (14),

$$\kappa=1-\tfrac{1}{4}\delta-\tfrac{3}{32}\delta^2+\cdots. \tag{28}$$

In the case of doublet symmetry, $\delta=0$, $\kappa=1$, we have, as expected, $T_{\text{rot}}=(2r_0^2)^{-1}j(j+1)+\text{const}$, each j level ($\frac{1}{2},\frac{3}{2},\cdots$) being degenerate ($i=l=j\pm\frac{1}{2}$). A small δ removes the degeneracy:

$$T_{\text{rot}}(l=j+\tfrac{1}{2})-T_{\text{rot}}(l=j-\tfrac{1}{2}) =(2r_0^2)^{-1}(1-\kappa)(2j+1). \tag{29}$$

If this splitting is to account for the Σ–Λ mass difference, one would have to assume $\kappa<1$, $\delta>0$, i.e., $|g'|<|g|$. The Y_2^*–Y_1^* mass difference ($j=\frac{3}{2}$) should then be twice the Σ–Λ mass difference ($j=\frac{1}{2}$), which is not incompatible with present information. However, with $|\delta|\ll1$, these splittings would be small compared with the Y_1^*–Λ mass difference, and this is not really so ($|g'/g|$ would have to be chosen much smaller than 1). Note also that there is no Y_0^* in this level scheme.

One remarkable fact is the rapid convergence of the δ expansion. We have intentionally written the Eqs. (21)–(27) in an unexpanded form, because they are accurate as long as the expectation value of η^2, Eq. (12), remains much smaller than r_0^2, and judging from (18), (19) (where δ^2 appears with rather small numerical factors, in the vibrational terms), and (20), this condition is still satisfied for $|\delta|\sim\frac{1}{2}$, and perhaps even for $|\delta|$ approaching 1. In this case, also, the root λ_0 of Eq. (4) remains well separated from the three other

[5] In the case of high cutoff, the factor $(2r_0^2)^{-1}$ is replaced by $3\pi a/g^2$, where a=source radius. See reference 4.

roots.[6] In short, we see no indication why the isobar formula (27) should become invalid as long as $|\delta| \lesssim 1$, with the sole exception of the case $\delta \approx 1$, or $|g'| \ll |g|$, which is discussed in Sec. 4.

The situation described here is surprisingly different from the scalar coupling case studied in I. There, we had to deal with two potential valleys which coincide for $|g'| = |g|$, and a resonance-like coupling between the two kinds of states caused a radical change of the level scheme within a narrow α interval ($\sim g^{-4}$). We anticipated then a similar "resonance" to occur in the pseudoscalar coupling case, but this conjecture was obviously wrong: There is only one potential valley (though 6-dimensional), and there is no critical value of $|\delta|$ ($\ll 1$) where a qualitative change of the spectrum could take place. The origin of the difference is, of course, the σ_i-dependence of the isovector $\mathbf{Q}$ (2).

4. CASE $|g'| \ll |g|$

At first sight, the question seems to arise whether the g' interaction in (1) is still "strong" or "weak." In the first case, one would solve Eq. (4) for $|\alpha| \ll 1$:

$$\lambda_0 = -|Q| + 3\alpha Q^{-2}\Delta + \cdots, \tag{30}$$

construct the corresponding eigenvector X, use it to compute $U^\dagger T_0 U$, and so on. Terms $\sim\alpha^2$ give the first energy correction. We have convinced ourselves, however, that, as long as $|g|$ is "large," the α^2 correction is practically the same as that calculated by taking the g' term in H_1 straightway as a weak perturbation. For the sake of simplicity, we adopt here this second approach.

As to the unperturbed problem ($g'=0$, while g is large) we can then refer to the Appendix in I, and we shall use the same notations as there. [In particular, r will be defined by (I, 45) (viz., $r^2 = \sum_{i\rho} q_{i\rho}^2$); this is different from the r as used in Sec. 3.] The unperturbed U matrix which diagonalizes the g term in H_1, is given by (I, 48), and it must be used also to transform the perturbation term ($\sim g' = \alpha g$), with the result

$$H' = 3\gamma\alpha r^{-2}\Delta. \tag{31}$$

[Note that this is equal to the term $\sim\alpha$ in $\gamma\lambda_0$, Eq. (30).]

The determinant Δ [see (6)] is invariant under rotations in ordinary, spin, and charge space. Hence, the mixing of the unperturbed eigenfunctions (I, Table I) caused by H' is subject to the selection rules

$$\delta l = \delta j = \delta i = 0. \tag{32}$$

Furthermore, δL is odd, because H' is odd under reflection, $q_{i\rho} \to -q_{i\rho}$. The leading energy correction, $\sim\alpha^2$, is

$$E''(L,l,j,i) = \sum_{L'} \frac{|(L',l,j,i|H'|L,l,j,i)|^2}{C[L(L+7) - L'(L'+7)]}. \tag{33}$$

For the denominator, see Eq. (I, 57). The factor $C[=(2r^2)^{-1}]$ is "small," viz., $\sim g^{-2}$. This fact restricts the validity of the perturbation theory very drastically, because $|E''| < C$ requires

$$|\alpha| = |g'/g| < (d/g)^4, \tag{34}$$

where d is a constant length (viz., the source radius in the "high cutoff" version) such that $|d/g| \ll 1$ is the "strong-coupling condition" for the unperturbed system.

The inequality (34) suggests that $(d/g)^4$ is a critical value of $|\alpha|$ above which the level spectrum changes rapidly into that studied in Sec. 3. In I, we did not foresee that this critical $|\alpha|$ value might be so small, but this conjecture is supported by the general argument presented at the end of Sec. 3. It should be noted that the wavefunctions in the two regimes are quite different. Whereas in Sec. 3 [see Eq. (8)] the three vectors $\mathbf{q}_i$ form a rigid (or rather, elastic) orthogonal cross, here ($\alpha \approx 0$) their relative orientations are free to vary, with $r^2 = \sum_i \mathbf{q}_i^2$ constrained to the potential valley (sphere in the 9-dimensional $q_{i\rho}$ space). The transition, resembling certain oscillation-rotation transitions in molecular physics, is a complicated problem, and we have found no way to deal with it.[7]

Returning to Eq. (33), we shall survey the lowest states in a qualitative fashion. As to the unperturbed eigenfunctions, examples ($L=0,1,2$) are listed in (I,51), and they are catalogued in I, Table I, according to their transformation character under rotations in ordinary and charge space. To calculate the corresponding matrix elements of H' (31), the following formula was used. If q_n ($n = 1 \cdots N$) are Cartesian coordinates in N-dimensional space (here $N=9$), and $r^2 = \sum_n q_n^2$, then the

$$\left.\begin{aligned} &\text{angular average of } \textstyle\prod_n (q_n/r)^{2m_n} \\ &\quad = \frac{\Gamma(N/2)}{\Gamma(N/2 + \sum_n m_n)} \prod_n \frac{\Gamma(m_n + \tfrac{1}{2})}{\Gamma(\tfrac{1}{2})} \\ &\quad [m_n = \text{integers} \geqslant 0; \quad \Gamma(\kappa+1) = \kappa\Gamma(\kappa)] \end{aligned}\right\}. \tag{35}$$

The ground state, called Λ, with $L=0$, $l=0$, $j=\frac{1}{2}$, $i=0$, and eigenfunction $F_0 = f(r)$ [we always omit the irrelevant spinor factors], mixes only with the corresponding state $L'=3$ whose eigenfunction is

$$F_3 = c_3 f(r) r^{-3}\Delta.$$

The normalization constant is determined by Eq. (35) with $m_1 = m_2 = m_3 = 1$, all other $m_n = 0$, viz., $c_3 = (9\times 11\times 13/6)^{1/2}$. The same angular average occurs in the matrix element $(F_3|H'|F_0)$. The resulting (negative) energy shift is termed $-\epsilon$:

$$E''(0,0,\tfrac{1}{2},0) = -\epsilon, \tag{36}$$

and ϵ serves as a unit in the following discussion.

The first two excited states, Σ and Y_1^*, with $L=1$, $l=i=1$, $j=\frac{1}{2}$ and $\frac{3}{2}$, respectively, and with eigenfunctions

[6] From (4) and (10), it follows, for $\alpha^2 \neq 0$ and $\eta^2 \ll r_0^2$, that $\lambda_0\lambda_1\lambda_2\lambda_3 < 0$; no root can cross the zero line.

[7] It is even quite uncertain whether l remains a good quantum number in the transition region. However, the one-to-one correspondence of the l values on either side is fairly obvious.

$$F_1=c_1f(r)r^{-1}q_{i\rho},\quad c_1=3,$$

each mix with 2 states, $L'=2$ and 4:

$$F_2=c_2f(r)r^{-2}\Delta_{i\rho},\quad c_2=(9\times11/2)^{1/2},$$

$$F_4=c_4f(r)[r^{-4}q_{i\rho}\Delta-(1/13)r^{-2}\Delta_{i\rho}],\qquad c_4=(11\times15)^{1/2}(39/10),$$

($\Delta_{i\rho}=\partial\Delta/\partial q_{i\rho}$). The level shifts $L=1$, due to the two admixtures, are found to be $-(9/13)\epsilon$ and $-(25/39)\epsilon$, respectively; they add up to

$$E''(1,1,j,1)=-(4/3)\epsilon, \tag{37}$$

both for $j=\frac{1}{2}$ and $j=\frac{3}{2}$. Since these two levels are equally depressed, and more strongly so than the ground state, the relative spacing

$$\frac{E(Y_1^*)-E(\Sigma)}{E(\Sigma)-E(\Lambda)}=\tfrac{1}{2}(1+\tfrac{1}{18}\epsilon+\cdots)$$

increases (though not strongly) with increasing α^2.

Of the next set of states, $L=2$, we consider in detail only the two lowest ones. First, $l=i=2$, $j=\frac{3}{2}$. As a typical eigenfunction we choose

$$F_{22}=c_{22}f(r)r^{-2}(q_{i\rho}q_{j\sigma}+q_{i\sigma}q_{j\rho}),\quad c_{22}=(9\times11/2)^{1/2},$$

with $i\neq j$, $\rho\neq\sigma$. This mixes with $L'=3$ and $L'=5$ states:

$$F_{32}=c_{32}f(r)r^{-3}(q_{i\rho}\Delta_{j\sigma}+\Delta_{i\rho}q_{j\sigma}+q_{i\sigma}\Delta_{j\rho}+\Delta_{i\sigma}q_{j\rho}),\qquad c_{32}=(9\times11\times13/12)^{1/2},$$

$$F_{52}=c_{52}\{c_{22}^{-1}F_{22}r^{-3}\Delta-(1/15)c_{32}^{-1}F_{32}\},\qquad c_{52}=(9\times11\times13\times17/69)^{1/2}(15/2).$$

The resulting shifts are $-(11/10)\epsilon$ and $-(759/1785)\epsilon$, adding up to

$$E''(2,2,j,2)=-(363/238)\epsilon. \tag{38}$$

This downward shift is even larger than that of the $L=1$ levels, (37).

Now we come to the interesting state $L=l=2$, $i=0$, $j=\frac{3}{2}$. For $\alpha=0$, this level is degenerate with the one just discussed, because i does not enter in the eigenvalue formula (I,57). The mixing state functions F_{L0} are obtained from the F_{L2} ($i\neq j$), essentially, by a contraction with respect to the charge index. Note that $F_{30}=0$, because $\sum_\rho q_{i\rho}\Delta_{j\rho}=0$ for $i\neq j$. As a consequence, the level shift is caused by the $L'=5$ admixture only, and it is correspondingly smaller:

$$E''(2,2,j,0)=-(55/119)\epsilon. \tag{39}$$

Here, the distance from the ground state [see (36)] increases with α^2, contrary to the lower lying levels. This agrees well with our findings in Sec. 3 for $\alpha^2\sim1$, where a low-lying state $i=2$ (with $l=2$, $j=\frac{3}{2}$) exists, but no corresponding $i=0$ state. Quite obviously, as α^2 increases through the transition region, the energies (38) and (39) must separate very rapidly.

We add a few brief comments regarding the next higher levels [see Table I in I]. The states $L=2$, $j=\frac{5}{2}$, are covered by Eqs. (38), (39), and the discussion above applies equally. Equation (39) may also be used for $L=2$, $l=0$, $i=2$, because the space and charge indices are interchangeable in the perturbation calculation. The states $L=2$, $l=i=1$ are the first to mix with lower states, viz., $L'=1$. The corresponding partial shift is $+(9/13)\epsilon$, as follows by interchange of L and L'. This tends to push these levels ($j=\frac{1}{2}$ and $\frac{3}{2}$) up, as it should be since they have no counterparts for $\alpha^2\sim1$. The same remark applies to the level $L=3$, $l=i=0$, whose coupling with the ground state $L'=0$ causes a partial shift upward ($+\epsilon$).

Needless to say, the results of this section should be considered in context with Eqs. (27) or (29) of Sec. 3 which describe the spectrum on the other side of the transition region and clearly suggest a qualitative interpolation between the two regimes.

5. CONCLUDING REMARKS

The main weakness of the strong-coupling method is that corrections for smaller (intermediate) coupling strengths are hard to evaluate. Such corrections would include the imaginary parts of the resonance energies, i.e., level widths and branching ratios of various decay modes. What the method can accomplish, however, is a systematic search, in the domain of its validity, for resonant states, and their classification by quantum numbers, including the orbital angular momentum l. This classification remains meaningful in the intermediate coupling range, at least for small and moderate l and i values. Larger values of l or i imply that there are more pions bound, and there may be as many as the rotational states inside the potential valley can accommodate. All number of pions bound are treated on equal footing, and this is very crucial in dealing with the higher states (whereas a "one-meson approximation" would *per se* discriminate against such states). Lowering the coupling strength causes these higher levels to move upward comparatively faster.

The merits of various g'/g ratios can hardly be objectively judged at this time. However, compared with the case of doublet symmetry, which can at best be an approximate symmetry only, the case $g'=0$ stands out by its simple appealing features. Several isobars with $i=0$ ($L=2,3,\cdots$) appear in the spectrum which one may attempt to relate to observed resonances. The objection might be made, arguing from a weak-coupling point of view, that $g'=0$ means absence of a direct $\sum\sum\pi$ coupling, and this would tend to suppress processes like $\Sigma^-+p\rightarrow\Sigma^0+n$, as against $\Sigma^-+p\rightarrow\Lambda+n$, whereas observations[8] seem to indicate a sizable branching ratio ($\sim\frac{1}{2}$ at low velocities). It is true that such a tendency shows up even in the strong-coupling theory, namely, if one calculates the hyperon-nucleon interaction mediated by the pion field, the highest order terms are found to contain no matrix elements for

[8] R. R. Ross, Bull. Am. Phys. Soc. **3**, 335 (1958).

transitions of the type $\Sigma^-+p\rightarrow\Sigma^0+n$ if $g'=0$[9]; in other words, the first Born approximation gives zero cross-section for such processes. On the other hand, just because the interaction is not weak, the Born expansion does not converge rapidly, and the higher order terms may well be large enough to account for the observations, even without invoking other kinds of mediating processes, like those involving K mesons, or those due to a small g' term (Sec. 4).[10]

[9] More precisely, this holds for all transitions which are first forbidden in the corresponding weak-coupling theory (e.g., elastic Λ-nucleon scattering). This similarity between strong-coupling and weak-coupling matrix elements for baryon-baryon interactions is easily derived (as in the nucleon-nucleon case) by considering the static self-energy of a pair of baryons kept at a given (not too small) distance.

[10] J. J. DeSwart and C. K. Iddings (to be published; I thank the authors for showing me their results and for valuable comments) have made a thorough numerical study of hyperon-nucleon interactions, based on a Schrödinger equation with pion-mediated potentials (including two-pion exchanges and repulsive cores), with the aim of finding out what values of the two coupling constants $f_{\Lambda\Sigma}$ (related to our g) and $f_{\Sigma\Sigma}$ ($\sim$our g') give the best fit to all experimental data now available. They conclude that $f_{\Sigma\Sigma}=0$ is not ruled out, and even favored by some data, including those of reference 8.

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Elastic Scattering of Lambda Hyperons from Protons*

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The Λ-proton elastic scattering cross section has been measured to be 20 ± 5 mb. This value represents an average over the momentum interval 150–1500 MeV/c. It is based on 26 events observed in a propane bubble chamber. The Λ hyperons were produced by the interactions of 1.15-BeV/c K^- mesons in the propane.

INTRODUCTION

IT is of value to experimentally determine the cross sections for the various reactions involving hyperons and nucleons in that these data may provide some criteria for judging the validity of the various baryon-nucleon symmetry schemes.[1] Of the various hyperon-nucleon reactions possible, Λ-proton elastic scattering is the most amenable to experimental investigation. Three measurements of the cross section for this process have been reported in the literature at this date. Crawford *et al.*[2] have estimated a value of 40 ± 20 mb based on 4 events. Recently, Alexander *et al.*[3] reported a value of 22.3 ± 5.9 mb on the basis of 14 events, and Arbuzov *et al.*[4] have estimated the cross section to be 36 ± 14 mb from a sample of 20 events. The results to be reported in this paper, based on 26 events, are seen to be in agreement with these earlier results.

PROCEDURE

The 30-in. propane bubble chamber used in this experiment has been described in considerable detail elsewhere.[5] The chamber was placed in a magnetic field of 13 kG and exposed to the 1.15-BeV/c K^- meson beam[6] at the Lawrence Radiation Laboratory Bevatron. A total of about 105 000 stereo-pairs of photographs were obtained during the run. Of these, 103 000 pairs were scanned for this experiment.

Scanning Procedure and Data Analysis

The scanning procedure followed in this experiment provided a means for the systematic detection of two types of kinematical configurations. Of primary interest were the elastic scattering configurations. These involved three vertices: the K^- beam interaction responsible for the production of the Λ via $K^-+N\rightarrow\Lambda+\pi$; the Λ-proton elastic scattering vertex, and the Λ-decay reaction $\Lambda\rightarrow p+\pi^-$. Also of interest were the two vertex configurations indicating K^- production of Λ followed by Λ decay without intervening interaction of the Λ prior to decay. There were too many examples of these latter configurations to make individual analysis of all of them feasible. Consequently, a count of them was kept and a smaller sample of them prepared for detailed analysis—a procedure yielding all of the information needed to provide the normalization for the cross-section calculation.

Events located in the scan were measured on digitizing devices and analyzed using the Fog IV Data Reduction System.[7] Included in the analysis were least-square fitting calculations in which the configurations were constrained to fit hypothetical event interpretations by requiring that the interactions conserve energy and momentum within the framework of the proposed interpretations.

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