# Modification of the One-Pion-Exchange Model for $\pi^- + p \to \pi^- + \pi^+ + n$†

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The one-pion-exchange model for the production process $\pi^- + p \to \pi^- + \pi^+ + n$ is modified to include a tensor component for the exchanged pion off its mass shell. This modification is suggested by the hypothesis of the Regge pole description for the pion. Experimental evidence for the deviation from the ordinary one-pion-exchange model can be understood in the present theory provided the final-state $\pi^-\pi^+$ pair contains an appropriate mixture of $J=0$ and $J=1$ ($\rho$ meson) amplitudes.

RECENTLY, Pickup, Robinson, and Salent[1] (PRS) applied the Treiman-Yang test[2] of one-pion exchange (OPE) to the production process $\pi^- + p \to \pi^- + \pi^+ + n$. For this process, OPE implies that, in the center-of-mass system of the final $\pi^-\pi^+$ pair, all cross sections should be independent of the angle between the plane defined by the momenta of the incident pion and the scattered $\pi^+$. We denote the four-momenta of the incident $\pi^-$ ,the scattered $\pi^-$, the scattered $\pi^+$, the target proton, and the scattered neutron by $k_1$, $k_2$, $k_3$, $p_1$, and $p_2$, respectively. The angle between the two planes described above is then given by $\cos\alpha = (\hat{k}_1 \times \hat{k}_3) \cdot (\hat{k}_1 \times \hat{p}_1)$, where the momenta are unit three-vectors in the final $\pi^-\pi^+$ center-of-mass system.

For incident pions of 1.25-BeV laboratory kinetic energy, the analysis of PRS divides into two categorees: (1) events inside the $\rho$ peak of the final $\pi^-\pi^+$ pair with four-momentum transfer squared $\Delta^2 > 0.03$ (BeV$/c)^2$ ($\simeq 15$ in pion units) and events outside the $\rho$ peak; (2) events inside the $\rho$ peak with $\Delta^2 < 0.03$ (BeV$/c)^2$. For class (1), there appears to be some $\alpha$ dependence. However, for these events, one does not expect OPE to be the dominant process in any case. We do not attempt to give a theoretical account for this result.[1] For class (2), PRS found that: (a) The asymmetry parameter associated with the angular distribution of the outgoing $\pi^-$ relative to the incident $\pi^-$ integrated over the $\rho$ peak is $0.40 \pm 0.09$ with $\alpha < 90°$ and $0.08 \pm 0.09$ with $\alpha > 90°$; (b) the ratio of all events with $\alpha < 90°$ to all events with $\alpha > 90°$ is consistent with unity; and (c) the asymmetry parameter integrated over $\alpha$ is approximately 0.3 at the $\rho$ peak and remains positive throughout the resonance region. The result (a) is inconsistent with OPE. Result (b) implies that the dominant $\alpha$ dependence disappears when one integrates over the outgoing $\pi^-$ angular distribution. Result (c) provides information on the interference of the $\rho$ state with $J \neq 1\,\pi^-\pi^+$ production. The purpose of this paper is to give a simple theoretical model on which all three results can be understood.

In view of recent speculations on the Regge pole hypothesis,[3] it appears that OPE must be modified if one associates the exchanged pion with a Regge pole. Except at $\Delta^2 = -m_\pi^2$ (unphysical region), the "pion" pole would consist of a superposition of all even spin states. In our present model, we simply consider an admixture of a pseudotensor component with the pseudoscalar pion in the $\rho$-production process corresponding to the two diagrams shown in Figs. (1a) and 1(b). In addition to the $\rho$ production, we also include an $S$-wave background in the final $\pi^-\pi^+$ state [Fig. 1(c)]. For simplicity, we use the OPE approximation for the $S$-wave background term. With these matrix elements, we calculate the cross section by keeping the OPE contribution to the $\rho$-production process and the interference terms with the tensor component and with the $S$-wave background. As we see below, the PRS result (b) is trivially satisfied in our approximation. After adjusting the tensor coupling and the final-state $S$-wave amplitude to fit (a), result (c) also follows.

Aside from inessential over-all factors, the matrix element corresponding to the sum of the three diagrams is given by

$$
\begin{aligned}
M = \bar{u}(p_2)\gamma_5 \Big\{ & [\tfrac{1}{2}A_1(s_2)/(t-m_\pi^2)] \\
& \times (k_1)_\mu (g_{\mu\nu} - q_\mu q_\nu/m_\rho^2)(k_2-k_3)_\nu \\
& + \tfrac{1}{2}A_1(s_2)\bigg[ a(t)\bigg(\gamma_\mu P_\nu + \gamma_\nu P_\mu - \frac{2m}{t}k_\mu P_\nu - \frac{2m}{t}k_\nu P_\mu\bigg) \\
& + b(t)(k_\mu k_\nu/t + 3P_\mu P_\nu/P^2 - g_{\mu\nu})\bigg] \\
& \times [(k_1)_\mu (g_{\nu\alpha} - q_\nu q_\alpha/m_\rho^2)(k_2-k_3)_\alpha] \\
& + [\tfrac{1}{3}A_0(s_2) + \tfrac{1}{6}A_2(m_\rho^2)]\bigg(\frac{1}{t-m_\pi^2}\bigg)\Big\} u(p_1). \quad (1)
\end{aligned}
$$

Here, $q = (k_2+k_3)$, $P = (p_1+p_2)$, $s_2 = q^2$, $t = (p_1-p_2)^2 = -\Delta^2$, and $m$ is the nucleon mass. $A_I(s_2)$; $I=0, 1, 2$, are pion-pion scattering amplitudes with $l=1$ ($\rho$ meson) for $I=1$ and $l=0$ for $I=0, 2$. The factors $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$ are

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[1] E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters 9, 170 (1962).
[2] S. B. Treiman and C. N. Yang, Phys. Rev. Letters 8, 140 (1962).
[3] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1962).

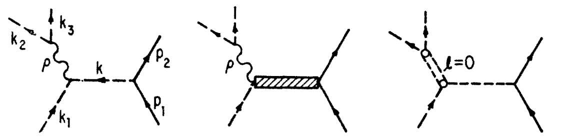


FIG. 1. (a) OPE contribution to the $\rho$-production process. (b) The exchange of a pseudotensor particle in the $\rho$-production process. (c) OPE contribution to the production of an $S$-wave $\pi^-\pi^+$ pair.

isospin projections of the $\pi^-\pi^+$ state. The form of the tensor coupling is determined by constructing symmetric, traceless, divergenceless, second-rank tensors out of the available four-vectors.[4] There are two unknown, but presumably smooth, functions $a(t)$ and $b(t)$ associated with the tensor coupling. However, only the term proportional to $b(t)$ contributes to the interference term with OPE when one evaluates the cross section for randomly polarized nucleons.

If we write the matrix element as $M=\bar{u}(p_2)Qu\,(p_1)$ then the differential cross section is simply given by the well-known formula

$$\begin{aligned}\sigma=&\tfrac{1}{2}\,\mathrm{Tr}[Q(\mathbf{p}_2+m)\gamma_0Q^\dagger\gamma_0(\mathbf{p}_1+m)]\\=&\tfrac{1}{2}[-2t/(t-m_\pi^2)^2]\{\tfrac{1}{4}|A_1|^2[k_1\cdot(k_2-k_3)]^2\\&+\tfrac{1}{2}|A_1|^2(t-m_\pi^2)b(t)[k_1\cdot(k_2-k_3)]\\&\times[(k\cdot k_1)(k\cdot k_2-k\cdot k_3)/t\\&+3(P\cdot k_1)(P\cdot k_2-P\cdot k_3)/P^2-k_1\cdot(k_2-k_3)]\\&+\mathrm{Re}[\tfrac{1}{3}A_1{}^*A_0+\tfrac{1}{6}A_1{}^*A_2][k_1\cdot(k_2-k_3)]\},\end{aligned}\quad(2)$$

where $\mathbf{p}\equiv\sum_\mu\gamma_\mu p_\mu$.

In the center-of-mass system of the final $\pi^-\pi^+$ pair, one can express all the scalar products in terms of five convenient variables:

$$\begin{aligned}s&=(p_1+p_2)^2,\\s_2&=(k_2+k_3)^2,\\t&=(p_1-p_2)^2,\\\cos\theta&=\hat{k}_1\cdot\hat{k}_2,\\\cos\alpha&=(\hat{k}_1\times\hat{k}_3)\cdot(\hat{k}_1\times\hat{p}_1).\end{aligned}$$

For example, the first term inside the curly brackets in (2) can be written as $|k_1k_2A_1\cos\theta|^2$, where $k_1$ and $k_2$ are now magnitudes of the three-momenta. Since we are not concerned with over-all factors, we can choose a convenient resonance formula for $A_1$

$$A_1=(3\Gamma/2k_1k_2)/[(m_\rho{}^2-s_2)-i\Gamma],\quad(4)$$

where the observed width is $\Gamma\simeq5m_\pi{}^2$.

Once the normalization of $A_1$ is fixed, the relation between $A_{0,2}$ and the $S$-wave phase shifts are also determined. In fact, we have chosen (4) so that $A_{0,2}=\exp(i\delta_{0,2})\sin\delta_{0,2}$.

We can now rewrite Eq. (2) as

$$\begin{aligned}\sigma=&[-t/(t-m_\pi)^2]\Gamma^2/[(m_\rho{}^2-s_2)^2+\Gamma^2]\\&\times\{(9/4)\cos^2\theta+(9/4)[b(t)(t-m_\pi{}^2)/t]\cos\theta\\&\times[(s_2-m_\pi{}^2+t)\cos\theta-3t(2s-2m^2-s_2-m_\pi{}^2-t)\\&\times(2p_1\cos\Phi\cos\theta/k_1-2p_1\sin\Phi\sin\theta\\&\times\cos\alpha/k_1+\cos\theta)/(4m^2-t)]\\&+\cos\theta[(m_\rho{}^2-s_2)(\sin2\delta_0+\tfrac{1}{2}\sin2\delta_2)/2\Gamma\\&+(\sin^2\delta_0+\tfrac{1}{2}\sin^2\delta_2)]\},\end{aligned}\quad(5)$$

where

$$\begin{aligned}k_1&=(E_1{}^2-m_\pi{}^2)^{1/2},\\p_1&=(W_1{}^2-m^2)^{1/2},\\E_1&=(s_2+m_\pi{}^2-t)/2s_2{}^{1/2},\\W_1&=(s-m_\pi{}^2-m^2+t)/2s_2{}^{1/2},\\\cos\Phi&=(m_\pi{}^2+m^2+2E_1W_1-s)/2k_1p_1.\end{aligned}\quad(6)$$

Our remaining task is to show that the differential cross section given by Eq. (5) is consistent with the PRS results (a), (b), and (c). For 1.25-BeV incident pions, $s=180$ in pion units. For $0<\Delta^2<0.3(\mathrm{BeV}/c)^2$, we find that the quantity inside the curly bracket in (5) is a rather smooth function of $t(-\Delta^2)$; therefore, we simply evaluate (5) with the average value $t=-7.5[\Delta^2=0.15\,(\mathrm{BeV}/c)^2]$. The dependence on $s_2$ is also unimportant within the $\rho$ peak $(24<s_2<34)$ except for the $S$-wave interference term proportional to $(m_\rho{}^2-s_2)$. We shall fix $s_2=29$ except for this term. Now, the only remaining variables in (5) are $\cos\theta$ and $\cos\alpha$. Again, aside from an over-all factor, Eq. (5) reads

$$\begin{aligned}\sigma=&\{(9/4)(1+2\beta)\cos^2\theta+4.76\beta\cos\theta\sin\theta\cos\alpha\\&+\cos\theta[(m_\rho{}^2-s_2)(\sin2\delta_0+\tfrac{1}{2}\sin2\delta_2)/2\Gamma\\&+(\sin^2\delta_0+\tfrac{1}{2}\sin^2\delta_2)]\},\end{aligned}\quad(7)$$

where $\beta=51.5\times b(-7.5)$.

Following the notation of PRS, we use the symbol $F$ for the integral of the cross section over the forward half of the $\cos\theta$ distribution $0<\theta<90°$, and $B$ for $90°<\theta<180°$. We also use the subscript $f$ and $b$ for $0<\alpha<90°$ and $90°<\alpha<180°$, respectively.

To compare our calculation with results of PRS, we first consider all events inside the $\rho$ peak, i.e., integrated cross sections over the range $24<s_2<34$. We find

$$\begin{aligned}F_f&=[0.75(1+2\beta)+0.794\beta+0.5\gamma],\\B_f&=[0.75(1+2\beta)-0.794\beta-0.5\gamma],\\F_b&=[0.75(1+2\beta)-0.794\beta+0.5\gamma],\\B_f&=[0.75(1+2\beta)+0.794\beta-0.5\gamma],\end{aligned}\quad(6)$$

where $\gamma=\sin^2\delta_0+\frac{1}{2}\sin^2\delta_2$.

It is easily seen from Eq. (8) that there is no forward-backward asymmetry in $\alpha$ if one integrates over all $\cos\theta$, i.e., $(F+B)_f=(F+B)_b$. Thus, result (b) of PRS is satisfied. In order to satisfy result (a) of PRS, we

[4] The "tensor" matrix element can be constructed by noting that the propagator for a spin 2 particle picks out that part of the vertex function $F_{\mu\nu}$ which is symmetric, traceless $(g^{\mu\nu}F_{\mu\nu}=0)$, and divergenceless $(k^\mu F_{\mu\nu}=0)$ (C. Fronsdal, private communication). For the construction of the propagator, see C. Fronsdal, Suppl. Nuovo Cimento 9, 416 (1958).

must adjust the two parameters $\beta$ and $\gamma$ so that

$$\begin{aligned} [(F-B)/(F+B)]_f &= 0.40 \pm 0.09, \\ [(F-B)/(F+B)]_b &= 0.08 \pm 0.09. \end{aligned} \tag{9}$$

This gives

$$\beta = 0.22_{-0.14}^{+0.23}, \quad \gamma = 0.52_{-0.18}^{+0.19}. \tag{10}$$

These values of $\beta$ and $\gamma$ are reasonable in the sense that the second-order terms in $\beta$ and $\gamma$ we have neglected in the calculation of the cross section are small and have little effect on the determination of $\beta$ and $\gamma$. In terms of phase shifts, the mean value $\gamma \simeq 0.52$ implies that the $S$-wave phase shifts should be in the neighborhood of $\pm 45°$. However, this should not be taken too seriously because the uncertainty of $\gamma$ is quite large. Furthermore, the background term could have been a superposition of $S$ and $D$ waves, etc., without altering the main features of our results.

We return now to examine the $s_2$ dependence of the asymmetry parameter integrated over all $\alpha$. We find

$$\begin{aligned} [(F-B)/(F+B)] = {} & [2\gamma + (m_\rho^2 - s_2) \\ & \times (\sin 2\delta_0 + \tfrac{1}{2}\sin 2\delta_2)/\Gamma]/[3(1+2\beta)], \end{aligned} \tag{11}$$

If we take the mean values of $\beta$ and $\gamma$ given by (10), the asymmetry parameter is 0.24 at the $\rho$ peak, consistent with result (c) of PRS. Furthermore, the asymmetry parameter remains positive throughout the resonance region provided $|\sin 2\delta_0 + \frac{1}{2}\sin 2\delta_2| < 1.04$. This seems to be a likely situation but, again, subject to the uncertainty in $\beta$ and $\gamma$.

In conclusion, we have found that a small mixture of a tensor component in the intermediate state of the $\rho$-production process together with a final-state pion-pion $S$-wave background is sufficient to account for all the features of the small $\Delta$ events of PRS in the neighborhood of the $\rho$ peak. The inclusion of a tensor component is suggested by the Regge pole hypothesis. However, any allowed two-particle intermediate state could have contained a $J=2$ term similar to what we have included and would also be consistent with our results. Another possible explanation of the PRS results has been suggested in connection with the $\rho$-$\omega$ mixing.[1] It seems to us that the $\rho$-$\omega$ mixing may be of secondary importance since it is of electromagnetic origin.

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# Radiative $\tau^+$ Decay*

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A clear example of a radiative $\tau^+$ decay, $K^+ \to \pi^+ + \pi^+ + \pi^- + \gamma$, has been found in nuclear emulsion. The photon energy is $34 \pm 1$ MeV.

An unusual $K^+$ decay event, believed to be $K^+ \to \pi^+ + \pi^+ + \pi^- + \gamma$, has been found during an examination of $K^+$ endings for Dalitz pairs. About 30 000 $K^+$ endings have been scanned to date in $G5$ nuclear emulsion exposed to a $K^+$ beam at the Bevatron of the Lawrence Radiation Laboratory of the University of California. Another unusual $K^+$ decay, interpreted as $K^+ \to \pi^+ + \pi^- + e^+ + \nu$, was found earlier as a by-product of the same scanning.[1]

The present event appears qualitatively as a "non-momentum-conserving" $\tau^+$ decay at rest (see Fig. 1). The primary, track 4, has $3.7 \pm 0.2$ times minimum grain density at 10.1-mm residual range, and is identified as a $K^+$ meson.[2] Tracks 1 and 2 end in $\pi$-$\mu$ decays and hence are identified as positive pion tracks. Track 3 ends in a zero-prong star, and hence is identified as a

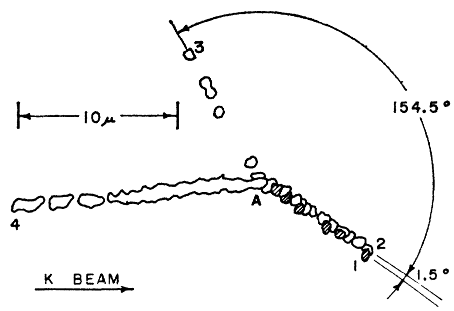


FIG. 1. Camera lucida drawing of the event. Track 4 is the primary, decaying at point $A$, and tracks 1 to 3 are the secondaries. The indicated angles are the projected angles in the plane of the emulsion.

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[1] E. L. Koller, S. Taylor, T. Huetter, and P. Stamer, Phys. Rev. Letters **9**, 328 (1962).

[2] The range-energy and grain count-energy curves of W. H. Barkas and D. M. Young, University of California Radiation Laboratory Report UCRL-2579 (Rev.) (unpublished), have been used throughout.