In the theory of nuclear matter, the diagonal element of the reaction matrix gives the energy of interaction.

$$\Delta E=\langle\Phi_0|V|\Psi\rangle/\langle\Phi_0|\Phi_0\rangle. \tag{15}$$

This, of course, can also be shown by direct calculation in this case. As $R\rightarrow\infty$ the energy shift, ΔE, can be related to the phase shift[2]

$$(k^2-k_0^2)/k \xrightarrow[R\rightarrow\infty]{} -2\delta/R,$$

which can be written in the form

$$(k_0-k^{\alpha,\beta})/\delta^{\alpha,\beta} \xrightarrow[R\rightarrow\infty]{} 1. \tag{16}$$

The quantity $(k_0-k^\alpha)R/\delta^\alpha$ is plotted in Fig. 3 as a function of R for $\epsilon=1$. The behavior is similar for other values of ϵ.

CONCLUSION AND SUMMARY

Explicit results have been obtained for a particle interacting with a potential in a box. The mixing parameter was considered in the limits $R\rightarrow\infty$ and $\epsilon\rightarrow 0$. It was found that if $\epsilon\rightarrow 0$ for R finite then $\eta_R^{\alpha,\beta}\rightarrow 0$. This is a direct result of the nondegeneracy of the levels in the presence of the box boundary condition. On the other hand, one finds that $\eta_R^{\alpha,\beta}\rightarrow$ const if $\epsilon\rightarrow 0$ for $R=\infty$ (i.e., scattering boundary conditions). This results from the degeneracy of the S and D levels for an infinite region. Thus, it is important to keep in mind when performing calculations involving infinite systems precisely which limit is the correct extrapolation for the problem in hand.

These calculations were carried out on the Minneapolis-Honeywell 800 digital computer at the Computer Sciences Laboratory of the University of Southern California.

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Elementary Particle Classification Based upon a Massless Dirac Field

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Elementary particles are viewed as linearized excitations of a nonlinear massless Dirac field, whose ground state is assumed to contain a finite density of field quanta. Nucleons and antinucleons are viewed as single-particle and single-hole excitations, and other elementary particles as space-symmetric correlations of 2, 3, or 4 particles and holes. Hyperons are viewed as composite. The strangeness quantum number is idenitfied with half the excess of particles over holes in the excitation. The classification has room for 29 particles: the known elementary particles plus doubly charged K and μ mesons and antiparticles, a strange charged spin 1 boson and antiparticle, and a boson quintet of strangeness and charge 0, ± 1, ± 2.

INTRODUCTION

ELEMENTARY particles can be viewed as excitations of a simple substrate. Heisenberg[1] has suggested that the substrate is a nonlinear spinor field, and Nambu[2] and the author[3] have suggested in addition that it resembles the superconducting fluid described by Bardeen, Cooper, and Schrieffer.[4] The collective excitations of an interacting massless Dirac fluid (a quantized nonlinear massless Dirac field) can be classified under the assumption that the interaction conserves the number of quanta of each type in the fluid. The resulting spectrum of elementary excitations bears a strong resemblance to the spectrum of elementary particles.

[1] H.-P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki, Z. Naturforsch. **14a**, 441 (1959). Earlier papers are referred to there.
[2] Y. Nambu, Phys. Rev. Letters 4, 380 (1960). See also Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
[3] J. C. Fisher, Bull. Am. Phys. Soc. **3**, 68 (1958).
[4] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **106**, 162 (1957).

MATHEMATICAL APPARATUS

The particle representation of a quantized massless Dirac field leads to a description of the possible states of the field in terms of two types of two-component particles. Each particle state can be characterized by a quantum number $t=\pm 1$ that denotes the type of particle, a quantum number $s=\pm 1$ that denotes whether its spin is parallel or antiparallel to its momentum, and a quantum number $q=(q_x,q_y,q_z)$ that denotes its momentum. The operator that creates a particle in state (t,s,q) is $c^\dagger(t,s,q)$, and its Hermitian adjoint is $c(t,s,q)$. The operators anticommute according to the relationships

$$[c^\dagger(t,s,q),\, c^\dagger(t',s',q')]_+=[c(t,s,q),\, c(t',s',q')]_+=0,$$
$$[c^\dagger(t,s,q),\, c(t',s',q')]_+=\delta_{tt'}\delta_{ss'}\delta(q-q').$$

The state with no particles is $|0\rangle$, and it satisfies the relationship

$$c(t,s,q)|0\rangle=0.$$

Any n-particle state of the field can be represented as a linear combination of states of the form

$$c^\dagger(t_1,s_1,q_1)c^\dagger(t_2,s_2,q_2)\cdots c^\dagger(t_n,s_n,q_n)|0\rangle,$$

and an arbitrary state can be represented as a linear combination of states with different values of n. The indices t_i and s_i are invariant under Lorentz transformations, and operators in different Lorentz frames are related through the equations

$$|q|^{1/2}U_Lc^\dagger(t,s,q)U_L^{-1}=|q'|^{1/2}c^\dagger(t,s,q'),$$
$$|q|^{1/2}U_Lc(t,s,q)U_L^{-1}=|q'|^{1/2}c(t,s,q'),$$

where U_L is a unitary operator that depends upon the Lorentz transformation.

There are 16 Lorentz-invariant operators of the form

$$\int d^3q\; c^\dagger(t,s,q)c(t',s',q).$$

Linear combinations of particular interest include

$$\tau_1=\tfrac{1}{2}\sum_{ts}\int d^3q\; c^\dagger(t,s,q)c(-t,\,s,\,q),$$

$$\tau_2=\tfrac{1}{2}\sum_{ts}(-it)\int d^3q\; c^\dagger(t,s,q)c(-t,\,s,\,q),$$

$$\tau_3=\tfrac{1}{2}\sum_{ts}t\int d^3q\; c^\dagger(t,s,q)c(t,s,q),$$

$$\sigma_1=\tfrac{1}{2}\sum_{ts}\int d^3q\; c^\dagger(t,s,q)c(t,\,-s,\,q),$$

$$\sigma_2=\tfrac{1}{2}\sum_{ts}(-is)\int d^3q\; c^\dagger(t,s,q)c(t,\,-s,\,q),$$

$$\sigma_3=\tfrac{1}{2}\sum_{ts}s\int d^3q\; c^\dagger(t,s,q)c(t,s,q),$$

$$X_3=\tfrac{1}{2}\sum_{ts}\int d^3q\; c^\dagger(t,s,q)c(t,s,q),$$

and products of particular interest include

$$\tau(\tau+1)=\tau_1^2+\tau_2^2+\tau_3^2,$$
$$\sigma(\sigma+1)=\sigma_1^2+\sigma_2^2+\sigma_3^2.$$

$$P_2=\sum_{tst's'}\int d^3qd^3q'\; c^\dagger(t,s,q)c^\dagger(t',s',q')c(t',s',q)c(t,s,q').$$

Note that the operator P_2 permutes the q indices of pairs of single-particle states, and the operators

$$\tau(\tau+1)-2X_3+X_3^2$$
$$\sigma(\sigma+1)-2X_3+X_3^2$$

permute the t and s indices of single-particle states. For example,

$$\begin{aligned}[\tau(\tau+1)-2X_3+X_3^2]&c^\dagger(t,s,q)c^\dagger(t',s',q')c^\dagger(t'',s'',q'')|0\rangle\\ =[c^\dagger(t',s,q)&c^\dagger(t,s',q')c^\dagger(t'',s'',q'')\\ &+c^\dagger(t'',s,q)c^\dagger(t',s',q')c^\dagger(t,s'',q'')\\ &+c^\dagger(t,s,q)c^\dagger(t'',s',q')c^\dagger(t',s'',q'')]|0\rangle.\end{aligned}$$

The operators X_3, τ_i, σ_i, $\tau(\tau+1)$, $\sigma(\sigma+1)$, P_2 all commute save

$$[\tau_i,\tau_j]=i\tau_k,\quad (i,\,j,\,k\text{ cyclic }1,\,2,\,3)$$
$$[\sigma_i,\sigma_j]=i\sigma_k,\quad (i,\,j,\,k\text{ cyclic }1,\,2,\,3)$$

so that X_3, τ_3, σ_3, $\tau(\tau+1)$, $\sigma(\sigma+1)$, P_2 can be diagonalized simultaneously.

GROUND STATE

Bardeen, Cooper, and Schrieffer[4] have shown that the ground state of a Fermi gas with attractive interactions contains strong pair correlations, and Blatt[5] has pointed out that it corresponds to a Bose-condensed sea of $c^\dagger(\uparrow,q_1)c^\dagger(\downarrow,q_2)$ molecules. Analogously, the ground state of a system containing equal densities of the four types of Dirac field quanta in attractive interaction should correspond to a Bose-condensed sea of $c^\dagger(1,1,q_1)\times c^\dagger(1,\,-1,\,q_2)c^\dagger(-1,\,1,\,q_3)c^\dagger(-1,\,-1,\,q_4)$ molecules. In a discrete momentum representation and in the center-of-mass frame of the condensed sea, the ground state then is

$$\begin{aligned}|G\rangle\sim[\sum_{q_1q_2q_3q_4}&h_{q_1q_2q_3q_4}\delta_{kr}(q_1+q_2+q_3+q_4)c^\dagger(1,1,q_1)\\ &\times c^\dagger(1,\,-1,\,q_2)c^\dagger(-1,\,1,\,q_3)c^\dagger(-1,\,-1,\,q_4)]^n|0\rangle,\end{aligned}$$

where the coefficients $h_{q_1q_2q_3q_4}$ are chosen to minimize the expectation of the energy $\langle G|H|G\rangle/\langle G|G\rangle$.

A more restricted form of $|G\rangle$ would be

$$\begin{aligned}|G\rangle=(\prod_{ts}&P_{nts})\\ &\times\prod_{sq}[(1-h_q)^{1/2}+h_q^{1/2}c^\dagger(1,s,q)c^\dagger(-1,\,-s,\,-q)]|0\rangle,\end{aligned}$$

where the projection operators P_{nts} pick out components with n particles of each type (t,s), introducing thereby a hint of four-particle correlation. Ignoring the projection operators for the moment, consider the normalized state

$$|G\rangle=\prod_{sq}[(1-h_q)^{1/2}+h_q^{1/2}c^\dagger(1,s,q)c^\dagger(-1,\,-s,\,-q)]|0\rangle,$$
$$\langle G|G\rangle=1.$$

Assume the parameters h_q to be determined so that $\langle G|(H-\mu N)|G\rangle$ is a minimum ($N=2X_3$ is the total particle-number operator), the parameter μ being a Lagrangian multiplier introduced to give the desired number of particles.

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[5] J. M. Blatt, Progr. Theoret. Phys. (Kyoto) **23**, 447 (1960).

COLLECTIVE EXCITATIONS

Anderson[6] has shown that the low-lying excitations of a pair-correlated Fermi gas can be obtained by contracting the equations of motion of simple operators in a way that takes account of the structure of the correlations. The commutator $[H,c^\dagger]$ contains terms of the form $c_1{}^\dagger c_2{}^\dagger c_3$ that come from $[H_{\text{int}},c^\dagger]$. Anderson contracts them to multiples of single creation and annihilation operators according to the prescription

$$c_1{}^\dagger c_2{}^\dagger c_3 \rightarrow \langle G|c_1{}^\dagger c_2{}^\dagger|G\rangle c_3 + c_1{}^\dagger\langle G|c_2{}^\dagger c_3|G\rangle - c_2{}^\dagger\langle G|c_1{}^\dagger c_3|G\rangle$$

(only exceptionally does more than one of these terms differ from zero) where $|G\rangle$ is the correlated ground state. In this way he linearizes the equations of motion of $c^\dagger$ and c (and of $c^\dagger c^\dagger$, $c^\dagger c$, and cc) while taking account of pair correlations through the expectations

$$\langle G|c^\dagger(\uparrow,q)c^\dagger(\downarrow,-q)|G\rangle = \langle G|c(\downarrow,-q)c(\uparrow,q)|G\rangle = [h_q(1-h_q)]^{1/2}.$$

In a system with four-particle correlations, one has to replace $c^\dagger c^\dagger c^\dagger c^\dagger$ by $\langle G|c^\dagger c^\dagger c^\dagger c^\dagger|G\rangle$ and $c^\dagger c^\dagger cc$ by $\langle G|c^\dagger c^\dagger cc|G\rangle$ in order to take account of the correlations. Terms $c^\dagger c^\dagger c$ arising from $[H_{\text{int}},c^\dagger]$ cannot be contracted in this way, and it is necessary to consider commutators of the form $[H_{\text{int}},[H_{\text{int}},c^\dagger]]$ which contain terms $c^\dagger c^\dagger c^\dagger cc$. These can be contracted according to the prescription

$$c_1{}^\dagger c_2{}^\dagger c_3{}^\dagger c_4 c_5 \rightarrow c_1{}^\dagger\langle G|c_2{}^\dagger c_3{}^\dagger c_4 c_5|G\rangle + c_2{}^\dagger\langle G|c_3{}^\dagger c_1{}^\dagger c_4 c_5|G\rangle + c_3{}^\dagger\langle G|c_1{}^\dagger c_2{}^\dagger c_4 c_5|G\rangle,$$

and in this way $[H,[H,c^\dagger]]$ can be linearized.

Approximate creation and annihilation operators for both single-particle and many-particle collective excitations can be obtained by linearizing the commutators $[H,[H,A]]$ where for single-particle excitations A is $c^\dagger$ or c and for many-particle excitations A is a linear combination of one of the forms $c^\dagger c^\dagger$, $c^\dagger c$, cc, $c^\dagger c^\dagger c^\dagger$, $c^\dagger c^\dagger c$, $\cdots$. For the operators A of interest, this procedure leads to a linearized commutator of the form

$$[H,[H,A]]_{\text{lin}} = (q^2+m^2)A,$$

where q is the net momentum change associated with A and where $m^2 \geq 0$ is a number that depends upon the form of the interaction and the nature of the operator A.

The equation $[H,[H,A]]_{\text{lin}} = (q^2+m^2)A$ suggests that there are two operators, A^+ and A^-, with eigenvalues $\pm(q^2+m^2)^{1/2}$, that satisfy the equations

$$[H,A^\pm]_{\text{lin}} = \pm(q^2+m^2)^{1/2}A^\pm.$$

These operators are

$$A^+ = A + (q^2+m^2)^{-1/2}[H,A],$$
$$A^- = A - (q^2+m^2)^{-1/2}[H,A],$$

using the rough linearization

$$[H,A^\pm]_{\text{lin}} = [H,A] \pm (q^2+m^2)^{-1/2}[H,[H,A]]_{\text{lin}} = \pm(q^2+m^2)^{1/2}A^\pm.$$

The operator A^+ adds energy to the system and plays the role of creation operator for a quantized excitation. The operator A^- subtracts energy and plays the role of annihilation operator for the corresponding antiexcitation.

TABLE I. Classification of space-symmetric states.

X_3	τ	τ_3	σ	σ_3	P_2	Number of components	Form of state
0	0	0	0	0	0	1	$1\|0\rangle$
$\frac{1}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	0	4	$c^\dagger\|0\rangle$
1	1	1, 0, −1	0	0	2	3	$c^\dagger c^\dagger\|0\rangle$
	0	0	1	1, 0, −1		3	
$\frac{3}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	6	4	$c^\dagger c^\dagger c^\dagger\|0\rangle$
2	0	0	0	0	12	1	$c^\dagger c^\dagger c^\dagger c^\dagger\|0\rangle$

TABLE II. Operators generating space-symmetric states. [The operator $P(q_1q_2\cdots q_n)$ means Σ(all permutations of q_1, q_2, $\cdots q_n$).]

1
$c^\dagger(1,1,q)$
$c^\dagger(1,-1,q)$
$c^\dagger(-1,1,q)$
$c^\dagger(-1,-1,q)$

$P(q_1q_2)c^\dagger(1,1,q_1)c^\dagger(1,-1,q_2)$
$P(q_1q_2)[c^\dagger(1,1,q_1)c^\dagger(-1,-1,q_2)+c^\dagger(-1,1,q_1)c^\dagger(1,-1,q_2)]$
$P(q_1q_2)c^\dagger(-1,1,q_1)c^\dagger(-1,-1,q_2)$

$P(q_1q_2)c^\dagger(1,1,q_1)c^\dagger(-1,1,q_2)$
$P(q_1q_2)[c^\dagger(1,1,q_1)c^\dagger(-1,-1,q_2)+c^\dagger(1,-1,q_1)c^\dagger(-1,1,q_2)]$
$P(q_1q_2)c^\dagger(1,-1,q_1)c^\dagger(-1,-1,q_2)$

$P(q_1q_2q_3)c^\dagger(1,1,q_1)c^\dagger(1,-1,q_2)c^\dagger(-1,1,q_3)$
$P(q_1q_2q_3)c^\dagger(1,1,q_1)c^\dagger(1,-1,q_2)c^\dagger(-1,-1,q_3)$
$P(q_1q_2q_3)c^\dagger(-1,-1,q_1)c^\dagger(-1,1,q_2)c^\dagger(1,-1,q_3)$
$P(q_1q_2q_3)c^\dagger(-1,-1,q_1)c^\dagger(-1,1,q_2)c^\dagger(1,1,q_3)$

$P(q_1q_2q_3q_4)c^\dagger(1,1,q_1)c^\dagger(1,-1,q_2)c^\dagger(-1,1,q_3)c^\dagger(-1,-1,q_4)$

SPACE-SYMMETRIC STATES

There are four types of single-particle states, $c^\dagger(t,s,q)|0\rangle$ with $t=\pm1$, $s=\pm1$, and 16 types of two-particle states $c^\dagger(t,s,q)c^\dagger(t',s',q')|0\rangle$. The two-particle states can be grouped in linear combinations of which ten are antisymmetric in the interchange of q and q' (the eigenvalue of P_2 is -2) and six are symmetric (the eigenvalue of P_2 is $+2$). As examples

$$P_2c^\dagger(1,1,q)c^\dagger(1,1,q')|0\rangle = -2c^\dagger(1,1,q)c^\dagger(1,1,q')|0\rangle,$$
$$P_2[c^\dagger(1,1,q)c^\dagger(1,-1,q')\pm c^\dagger(1,1,q')c^\dagger(1,-1,q)]|0\rangle = \pm2[c^\dagger(1,1,q)c^\dagger(1,-1,q') \pm c^\dagger(1,1,q')c^\dagger(1,-1,q)]|0\rangle.$$

The symmetric states are of particular interest, because particles in space-symmetric states have maximum

[6] P. W. Anderson, Phys. Rev. **112**, 1900 (1958).

probability of being in contact. With an attractive interaction, these states will have lowest energy. Four of the 64 types of 3-particle states are space-symmetric, as is one of the 256 types of 4-particle states.

The space-symmetric states can be classified by their eigenvalues of X_3, τ, τ_3, σ, σ_3, P_2. This classification, including the no-particle and one-particle states, is listed in Table I. Operators generating representative members of these states are given in Table II.

SPACE-SYMMETRIC EXCITATIONS

Each of the 16 types of operators in Table II generates a collective excitation. Let $A_\nu(q_1,q_2,\cdots,q_n)$ with $1\leqslant\nu\leqslant 16$ be one of the 16 types. Then linearization of the commutator $[H,[H,A_\nu]]$ gives a sum of operators of the same type,

$$[H,[H,A_\nu(q_1,q_2,\cdots,q_n)]]_{\mathrm{lin}} =\sum [\text{operators of the form } A_\nu(q_1',q_2',\cdots,q_n')].$$

Hence there exist linear combinations $\{A_\nu\}$ of the A_ν's with the property

$$[H,[H,\{A_\nu\}]]_{\mathrm{lin}}=\lambda^2\{A_\nu\}.$$

Let $\{A_\nu\}_{\mathrm{min}}$ stand for the linear combination with minimum λ^2. Corresponding to it there is the collective excitation creation operator

$$A_\nu^+=\{A_\nu\}_{\mathrm{min}}+\lambda_{\mathrm{min}}^{-1}[H,\{A_\nu\}_{\mathrm{min}}],$$

generating the linearized excited state $A_\nu^+|G\rangle$.

From among the 16 types of operators A_ν, the operator 1 regenerates the ground state, the operators $c^\dagger$ generate single-particle states, and the others generate excitations with space-symmetric correlations. These excitations are expected to have low energy and long life because they preserve the maximum correlation.

Because of the correlations in $|G\rangle$, the single-particle states (t,s,q) and $(-t, -s, -q)$ are either both occupied or both unoccupied. As a result, the states $c^\dagger(t,s,q)|G\rangle$ and $c(-t, -s, -q)|G\rangle$ both certainly have the single-particle state (t,s,q) occupied and the single-particle state $(-t, -s, -q)$ unoccupied. This means that collective excitation creation operators based upon the operators

$$P(q_1q_2)c^\dagger(1,1,q_1)c^\dagger(1, -1, q_2),$$

$$P(q_1q_2)[c^\dagger(1,1,q_1)c(-1, 1, -q_2) +c(-1, -1, -q_1)c^\dagger(1, -1, q_2)],$$

$$P(q_1q_2)c(-1, -1, -q_1)c(-1, 1, -q_2),$$

(for example) acting upon $|G\rangle$ all generate configurations with single-particle states $(1,1,q_1)$ and $(1, -1, q_2)$ both occupied in space-symmetric fashion. Hence to each of these operators there corresponds a space-symmetric excitation. The full set of space-symmetric excitations then is generated by operators of the forms listed in Table III. Those involving only $c^\dagger$'s are given in detail in Table II. The others can be obtained from those in Table II by making substitutions $c^\dagger(t,s,q)\rightarrow c(-t, -s, -q)$ as in the example for $P(q_1q_2)c^\dagger(1,1,q_1)c^\dagger(1, -1, q_2)$.

The operators generating space-symmetric states, and the excitations they generate, may be classified by quantum numbers X and X_3 as listed in Table IV. The quantum number X will be called the correlation number. It is half the (minimum) number of particles participating in the correlated excitation [and equivalently is half the (minimum) number of particles no longer participating in the ground-state correlation]. Its component X_3 is half the net increase in particle number associated with the excitation.

TABLE III. Operator forms capable of generating space-symmetric excitations.

		1		
	$c^\dagger$		c	
	$c^\dagger c^\dagger$	$c^\dagger c$	cc	
$c^\dagger c^\dagger c^\dagger$	$c^\dagger c^\dagger c$	$c^\dagger cc$	ccc	
$c^\dagger c^\dagger c^\dagger c^\dagger$	$c^\dagger c^\dagger c^\dagger c$	$c^\dagger c^\dagger cc$	$c^\dagger ccc$	$cccc$

TABLE IV. Classification of operator forms.

Operator form	X	X_3
1	0	0
$c^\dagger$	$\frac{1}{2}$	$\frac{1}{2}$
c		$-\frac{1}{2}$
$c^\dagger c^\dagger$	1	1
$c^\dagger c$		0
cc		-1
$c^\dagger c^\dagger c^\dagger$	$\frac{3}{2}$	$\frac{3}{2}$
$c^\dagger c^\dagger c$		$\frac{1}{2}$
$c^\dagger cc$		$-\frac{1}{2}$
ccc		$-\frac{3}{2}$
$c^\dagger c^\dagger c^\dagger c^\dagger$	2	2
$c^\dagger c^\dagger c^\dagger c$		1
$c^\dagger c^\dagger cc$		0
$c^\dagger ccc$		-1
$cccc$		-2

ASSOCIATION OF OPERATORS WITH OBSERVABLES

The operators now are associated with observables as indicated in Table V. These differ from the customary identifications where $2\tau_3$ is associated with charge and $X_3=N/2$ is not associated with strangeness. Assuming the operators X_3, τ_3, $\tau(\tau+1)$, $\sigma(\sigma+1)$, P_2 to commute with the energy-momentum four-vector (including the interaction portion), then one can diagonalize them all simultaneously and states can be classified by their eigenvalues. (It is unlikely that σ_3 will commute with any interaction capable of yielding excitations with nonzero rest energy. The multiplicity $2\sigma+1$, formerly associated with σ_3, then would correspond to the multiplicity of an intrinsic angular momentum component.)

ELEMENTARY PARTICLE CLASSIFICATION

Assuming an elementary particle to correspond to each type of space-symmetric excitation, and accepting

Table V. Operators and their associated observables.

Operator	Observable
X_3	Strangeness
$\tau(\tau+1)$	τ = isospin
$Q=X_3+\tau_3$	Charge (net number of particles with $t=1$)
$\sigma(\sigma+1)$	σ = spin
σ_3	Helicity
P_2	(Space symmetry, distinguishing nucleons, mesons, leptons, scalar bosons)

the correspondance between operators and observables given in the preceeding paragraph, a table of elementary particles can be constructed. Classes of particles may be distinguished by the eigenvalues of X and τ to which they belong, each class having multiplicity $(2X+1)(2\tau+1)$ not counting the spin multiplicity $(2\sigma+1)$.

In abbreviated form, the table of elementary particles is as listed in Table VI. (Here the quantum number X is used to distinguish types of particles instead of the equivalent quantum number P_2.)

The full classification is given in Table VII. There is room in this classification for all known elementary particles, if hyperons are viewed as bound states of nucleons and anti-K mesons. There is room in addition for the 11 other particles that appear in parentheses: a doubly charged K meson and antiparticle, a strange charged spin 1 boson and antiparticle, a doubly charged μ meson and antiparticle, and five bosons of strangeness and charge 2, 1, 0, -1, -2.

QUESTIONS (AND PARTIAL ANSWERS)

A number of questions must be answered if this classification is to prove useful.

1. Is it possible that doubly charged particles $K^{\pm 2}$, $\mu^{\pm 2}$, $G^{\pm 2}$ belonging to the classification could have escaped detection if they exist? If their lifetimes were exceedingly short, as might be required by the electrostatic forces that tend to disrupt them, it is possible that they could have escaped detection.

2. Hyperons do not appear as elementary particles. How are they to be viewed? Hyperons must be viewed as composite particles in this classification, composed of nucleons and anti-K-mesons. Six hyperons can be constructed from a nucleon and a single meson, as indicated in Table VIII. The hyperons in parentheses, associated with the doubly charged anti-K, have not been observed.

Table VI. Abbreviated table of elementary particles.

X	τ	σ	Type of particle	Number of particles
0	0	0	Vacuum	...
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	Nucleon	4
1	1	0	Meson	9
1	0	1	Vector boson	3
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	Lepton	8
2	0	0	Scalar boson	5
			Total number = 29	

Table VII. Elementary particle classification. X = correlation number, τ = isotopic spin, X_3 = strangeness, τ_3 = isotopic spin 3-component, σ = spin, and Q = charge.

X	τ	X_3	τ_3	σ	Q	Particle
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	p^+
			$-\frac{1}{2}$	$\frac{1}{2}$	0	n
		$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\bar{n}$
			$-\frac{1}{2}$	$\frac{1}{2}$	-1	p^-
1	1	1	1	0	2	(K^{+2})
			0	0	1	K^+
			-1	0	0	K^0
		0	1	0	1	π^+
			0	0	0	π^0
			-1	0	-1	π^-
		-1	1	0	0	$\bar{K}^0$
			0	0	-1	K^-
			-1	0	-2	(K^{-2})
1	0	1	0	1	1	(Γ^+)
		0	0	1	0	γ
		-1	0	1	-1	(Γ^-)
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2	(μ^{+2})
			$-\frac{1}{2}$	$\frac{1}{2}$	1	μ^+
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	e^+
			$-\frac{1}{2}$	$\frac{1}{2}$	0	ν
		$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\bar{\nu}$
			$-\frac{1}{2}$	$\frac{1}{2}$	-1	e^-
		$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	μ^-
			$-\frac{1}{2}$	$\frac{1}{2}$	-2	(μ^{-2})
2	0	2	0	0	2	(G^{+2})
		1	0	0	1	(G^+)
		0	0	0	0	(G^0)
		-1	0	0	-1	(G^-)
		-2	0	0	-2	(G^{-2})

3. There is so far perfect symmetry between $t=1$ and $t=-1$, so that "charged" and "neutral" particles of nonzero strangeness occur in degenerate multiplets. How is the splitting of mass eigenvalues by charge to be understood? A related question follows.

4. The quantity $X_3=N/2$, associated with strangeness, is conserved. How can one understand the apparent nonconservation of strangeness in weak interactions? Both the splitting of mass eigenvalues by

Table VIII. Construction of six hyperons from a nucleon and a single meson.

Nucleon	Anti-K-meson	Hyperon
n	(K^{-2})	(Σ^{-2})
n	K^-	Σ^-
n	$\bar{K}^0$	Σ^0, Λ
p^+	K^-	Σ^0, Λ
p^+	$\bar{K}^0$	Σ^+
p^+	(K^{-2})	(Σ'^-)

charge and the apparent nonconservation of strangeness can be understood in terms of a slight inequality in the densities of the two types of Dirac particles in the physical vacuum $|G\rangle$, the excess with $t=-1$ occurring as zero-momentum Bose-condensed pairs. The condensation of one such pair corresponds to a loss of one unit of X_3 and a gain of one unit of τ_3 by the excitations, and to the converse changes in the vacuum $|G\rangle$, thereby conserving only $(X_3+\tau_3)=Q$ in the excitations. Let B stand for the zero-momentum Bose-condensed "particle" carrying $X_3=1$, $\tau_3=-1$. Then all weak interactions involve B or $\bar{B}$, examples being β decay:

$$n=p^{+}+e^{-}+\bar{\nu}+B,$$

decay of the μ meson:

$$\mu^{-}=e^{-}+\nu+\bar{\nu}+\bar{B},$$

and decay of the charged π meson:

$$\pi^{+}=\mu^{+}+\bar{\nu}+\bar{B},$$

$$\pi^{+}=e^{+}+\nu+\bar{B}.$$

5. How are the violations of parity conservation in weak interactions to be understood? If the Bose-condensed molecules formed from the slight excess of particles with $t=-1$ have odd parity, the parity of the excitations will change when the strangeness changes. It is possible that the observed violations of parity conservation could have their origin here.

6. Why are baryons so heavy and leptons so light? Baryons are generated by the operators $c^{\dagger}$ and c, which destroy some of the four-particle correlation of the ground state and thereby produce a rest energy. At the other extreme, the excitations $c^{\dagger}c^{\dagger}c^{\dagger}c^{\dagger}$, $c^{\dagger}c^{\dagger}c^{\dagger}c$, $c^{\dagger}c^{\dagger}cc$, $c^{\dagger}ccc$, $cccc$ would be expected to have little or no rest energy because the lost correlation is regained in the internal motion of the excitation. The mesons ($c^{\dagger}c^{\dagger}$, $c^{\dagger}c$, cc) and leptons ($c^{\dagger}c^{\dagger}c^{\dagger}$, $c^{\dagger}c^{\dagger}c$, $c^{\dagger}cc$, ccc) would be expected to have intermediate rest energies, with leptons lighter than mesons. The analogous situation in the electrically neutral superconductor is as follows: The excitation made up of $c^{\dagger}$ and c has a rest energy, but the excitation made up of $c^{\dagger}c^{\dagger}$, $c^{\dagger}c$, and cc has not, and may be thought of as a moving correlated pair, as shown by Anderson.

7. No baryon quantum number has been introduced. How is the stability of nucleons to be understood? In this model it appears that the stability of nucleons must be a consequence of the dynamics. The leading term in the creation operator for a nucleon excitation with momentum q is of the form $c^{\dagger}(q)$, whereas the leading term in the creation operator for a lepton excitation with momentum q is

$$\int d^3q_1 d^3q_2\, \alpha(q_1,q_2) c^{\dagger}(q+q_1) c^{\dagger}(q_2) c(q_1+q_2).$$

The nucleon is related to a δ function in momentum space, and the lepton to a function $\alpha(q_1,q_2)$ that spreads through much of momentum space. The stability of the baryon may be related to the vanishingly small overlap of these functions in a large system. [*Note added in proof.* Conservation of P_2 may play a role in the stability of baryons.]

CONCLUSION

I find it suggestive that a Dirac gas should have this brief spectrum of simple collective excitations, exceeding the spectrum of recognized elementary particles by so small a margin and resembling it so closely. Although much more work is required in order to evaluate this elementary particle model, in particular the quantitative calculation of masses, interactions, and decay rates, I believe there are concepts of value in the classification as it now stands. The physical interpretation of strangeness, the suggestion that the physical vacuum be a gas of finite density, the suggestion that particle and antiparticle be related as $c^{\dagger}$ and c operating upon the physical vacuum rather than as the positive and negative energy solutions of the Dirac equation, the suggestion that the asymmetry between charged and neutral particles arises through an asymmetry of the physical vacuum and not of the Hamiltonian, the suggestion that the K and μ mesons are charge triplets each with a short-lived doubly charged member, all are of interest.

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