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The Classical Limit of the Quantum Theory of Radiation Damping

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The classical limit of the quantum theory of damping (without radiative corrections) has been studied for scattering procedures involving photons and neutral or charge-symmetric mesons [with $S(S)$, $PS(PV)$, $V(V)$, and $V(T)$ coupling] interacting with nucleons. In the case of dipole coupling the nuclear spin $\frac{1}{2}\hbar$ must be replaced by a classical intrinsic angular momentum, and classical rather than quantum mechanical averages over spin variables must be taken. The cross sections are then calculated taking classical limits at all stages rather than just in the summation over the final stages. Comparison is made with the corresponding results in the classical field and action-at-a-distance theories; the cross sections agree with those of the theory of action at a distance in all cases for which these results are available.

I. INTRODUCTION

A PARTICLE emitting radiation (electromagnetic or other) is generally considered to experience a radiation force. The interpretation of this force differs depending on the point of view taken, that of field theory or that of action at a distance.[1] The first field-theoretical interpretation of the electromagnetic radiation reaction force, the Lorentz theory of the electron,[2] considered it as part of the force of the charge on itself. This theory was quite compatible with experiment for small electron accelerations, and emitted wave lengths large in comparison with the electron radius. However, the total self-force diverged in the limit of a point electron, and led to other difficulties if a finite electron was assumed.

Lorentz's derivation was restricted to low velocities; however, this limitation was overcome by Schott[3] who obtained a relativistically invariant set of equations of motion. An alternative derivation of Schott's result was given by Dirac[4] who proposed to treat the electron as a point singularity in the electromagnetic field and obtained the equations of motion of the electron by requiring conservation of energy and momentum for the system of field and singularities. The method of Dirac was extended by Bhabha,[5,6] Harish-Chandra,[7] and Le Couteur[8] to include the case of nucleons in mesic fields.[9] In all these calculations retarded interactions were used exclusively, although Dirac stated that his calculations were symmetric in time; it was later shown, however, that this time symmetry is only apparent.[10]

The restriction to retarded potentials introduces a time asymmetry not contained in the underlying field equations. Time-symmetric fields can be used, however, both in field theory[10] and in the action-at-a-distance theory developed by Fokker,[11] but the resulting equations appear to be unable to account for radiation. However, it was suggested by Wheeler and Feynman[12] that these equations should be supplemented by a condition of "complete absorption";[13] the resulting theory gives a description of radiation equivalent to that provided by the Lorentz-Dirac equations.

The extension of action-at-a-distance theory to include the motion of nucleons in mesic fields was carried out by several authors.[14–18] Mesic fields do not propagate with the velocity of light and thus nucleons could "catch up" with their own radiated fields; the equations of motion of field theory thus contain terms involving the past (for retarded interactions) or entire (for symmetric interactions) motion. Such terms must be deleted as a matter of principle in action-at-a-distance theory, and thus the equations of motion of the two theories are not equivalent. The resulting differences between their predictions for meson scattering were investigated by Mehl and Havas[19–21] with the intent of establishing a basis for a future experimental decision between the two theories, which cannot be made on the basis of electrodynamics.[10]

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[1] For a general discussion of this problem see P. Havas, in Argonne National Laboratory Summer Lectures on Theoretical Physics, 1958 ANL-5982 (unpublished), p. 124.

[2] H. A. Lorentz, *Collected Papers* (M. Nijhoff, The Hague, 1936), Vol. II, pp. 281 and 343.

[3] G. A. Schott, *Electromagnetic Radiation* (Cambridge University Press, New York, 1912); Phil. Mag. **29**, 49 (1915).

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[18] R. C. Majumdar, S. Gupta, and S. K. Trehan, Progr. Theoret. Phys. (Kyoto) **12**, 31 and 697 (1954).

[19] C. Mehl and P. Havas, Phys. Rev. **91**, 393 (1953).

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[21] C. Mehl, Lehigh University thesis, 1954 (unpublished); C. Mehl and P. Havas (to be published).

A quantum theory of radiation damping applicable to scattering problems was developed by Heitler,[22] Gora,[23] and Wilson.[24] The classical limit ($\hbar \rightarrow 0$) of the cross sections for meson-nucleon scattering obtained by this theory was compared with the classical field-theoretical results of Bhabha[5,6] and Le Couteur[8] by Vachaspati.[25] He noted that there was some similarity in form in the case of charge scattering, but much less for dipole scattering, where even constant factors disagreed. We noted, on the other hand, that the classical limit of the Heitler results did resemble much more closely the results of Mehl and Havas. This suggested (in the absence of a complete quantum theory of action at a distance) that a detailed comparison of the classical limit of the meson-scattering cross sections obtained by the Heitler theory of radiation damping with the corresponding action-at-a-distance results should be undertaken. This comparison indeed showed that the cross sections agreed, provided the limit was properly defined; in the following we present a brief account of these calculations.[26]

II. THE QUANTUM THEORY OF RADIATION DAMPING

According to Heitler's theory[18] a transition from state A to state B is described by the integral equation

$$U_{AB}=H_{AB}-i\pi H_{AC}U_{CB}\rho d\Omega. \tag{1}$$

The solution U_{AB} of Eq. (1) is related to the transition probability per unit time w_{AB} by

$$w_{AB}=(2\pi/\hbar)|U_{AB}|^2\rho, \tag{2}$$

H_{AB} is the first nonvanishing matrix element, ρ is the density of states, and $d\Omega$ the differential solid angle. Summations over repeated indices are understood as well as integrations over differentials. In the following ϕ_{out} and ϕ_{in} stand for the Schroedinger wave functions of outgoing and incoming waves, respectively, and P for the Cauchy principal value.

A derivation based on a method developed by Dirac[27] in scattering theory was given by Ma and Hsüeh,[28] who used the stationary method of perturbation theory; they obtained

$$U_{AB}=H_{AB}+H_{AC}U_{CB}\rho[P/(E_B-E_D) -i\pi\delta(E_B-E_D)]dE_D d\Omega. \tag{3}$$

The analysis of Dirac shows that the term in brackets is necessary if U_{AB} is to represent a scattered wave, and Blatt[29] in further discussion of the bracket quantity showed that the term $P/(E_B-E_D)$ alone leads to $\frac{1}{2}(\phi_{\text{out}}+\phi_{\text{in}})$ at large distances. The term in the δ function then corresponds to $\frac{1}{2}(\phi_{\text{out}}-\phi_{\text{in}})$. Ma and Hsüeh neglect the term $P/(E_B-E_D)$ in Eq. (3), which then reduces to Eq. (1). In a classical field-theoretical derivation of the Heitler equation, Blatt[29] showed explicitly that it is one-half the difference between the retarded and advanced potential of the scattered wave that acts on the source.

The original Heitler theory arbitrarily rejected higher order effects ("round-about transitions") which involved emission and subsequent reabsorption of quanta by the same particle. These self-action effects resulted in diverging integrals which at that time had no physical interpretation. In action-at-a-distance theory, on the other hand, self-action is rejected *a priori*. Furthermore, the Wheeler-Feynman condition[12] leads to a force acting on the "emitter" by the "absorber" determined by one-half the difference between the retarded and advanced potential in action-at-a-distance theory, while the same expression is regarded as the force on the particle by the radiation field in radiation damping theory. Thus, the original Heitler theory has some similarities with the theory of action-at-a-distance.

A more informative comparison of the two theories would be possible if there were a complete quantum theory of action at a distance, or if the classical Heitler equation, as derived by Blatt, could be derived from the action-at-a-distance formalism. Although some work on a quantum theory of action at a distance has been undertaken,[30] the formalism has not been developed sufficiently to allow the study of problems corresponding to mesic interactions. Attempts to derive the classical Heitler equation from action at a distance have not been successful until now. Thus, it seemed that a direct comparison of the predictions of the two theories would be the most promising way to establish the degree of similarity.

In addition to Vachaspati,[25] partial comparisons of the Heitler theory with classical field theory were also indicated by Bhabha,[5] Le Couteur,[8] and Majumdar.[31] The results were inconclusive, and in the case of Vachaspati and Majumdar, some of the formulas seem to be in error. All of these comparisons share a common inadequacy in that the methods employed in obtaining a classical limit in the case of dipole coupling still involve quantum mechanical summations. Therefore, we have recalculated the results of these authors and also extended the comparisons to other processes.

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[23] E. Gora, Acta Phys. Polon. **7**, 159 (1938).
[24] A. H. Wilson, Proc. Cambridge Phil. Soc. **37**, 301 (1941).
[25] Vachaspati, Phys. Rev. **80**, 973 (1950).
[26] For details see J. E. Chatelain, Lehigh University thesis, 1957 (unpublished).
[27] P. Dirac, *Quantum Mechanics* (Oxford University Press, Oxford, 1948), 3rd ed., p. 198.
[28] S. T. Ma and C. F. Hsüeh, Proc. Cambridge Phil. Soc. **40**, 167 (1940).
[29] J. M. Blatt, Phys. Rev. **72**, 466 (1947).
[30] H. J. Groenewold, Koninkl. Ned. Akad. Wetenschap. Proc. **52**, 133 and 226 (1949); G. Ludwig, Z. Naturforsch. **5a**, 637 (1950); L. Foldy, Phys. Rev. **122**, 512 (1961).
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III. CROSS SECTIONS AND CLASSICAL SUMS

From Eq. (2) we can obtain the cross sections in the usual manner, using the density of final states ρ and the incident intensity I

$$\rho = K\omega v/(2\pi)^3\hbar c^2, \quad I=\hbar Kc^2/\epsilon v \tag{4}$$

$$\rho=\omega^2 v/(2\pi)^3 c^3\hbar, \quad I=c/v \tag{5}$$

for mesons and photons, respectively. The notation used here and in subsequent formulas is explained in the Appendix.

Dirac (spin $\frac{1}{2}$) particles can interact with the meson field by mesic charge coupling or by dipole coupling. In approaching the classical limit of radiation damping cross sections we have to distinguish these two possibilities. In the case of charge coupling the classical limit ($\hbar \rightarrow 0$) of the Dirac particle is a particle with no angular momentum. However, in the classical theory the mesic dipole moment is proportional to the angular momentum. Thus, for a Dirac particle with dipole coupling, we must take as a classical analog a particle with an intrinsic angular momentum $I=\frac{1}{2}\hbar$ (see Bhabha[6]).

Both in the solution of the damping equation (1) and in the calculation of the total scattering cross section from (4) and (5) we encounter the same type of summations. In the case of dipole coupling we will have occasion to average and sum over expressions containing the Pauli spin operator $\boldsymbol{\sigma}$. The quantum mechanical summations and averages over all the angles between a given vector and the spin operator $\boldsymbol{\sigma}$ are different, however, because of the discrete nature of $\boldsymbol{\sigma}$, from the corresponding classical results obtained by considering $\boldsymbol{\sigma}$ as an ordinary unit vector. In comparisons of field-theoretical cross sections with those of radiation damping, Le Couteur[8] and Vachaspati[25] account for differences only in the summations occurring in the integration of differential cross sections, but not in the solution of Eq. (1).

The summations over all directions of a unit vector $\mathbf{k}$ occurring most frequently are

$$\int \boldsymbol{\sigma}\cdot\mathbf{k}\; \boldsymbol{\sigma}\cdot\mathbf{k}\, d\Omega = 4\pi \text{ (Q.M.)}; \ \tfrac{4}{3}\pi \text{ (Cl.)} \tag{6}$$

$$\int \boldsymbol{\sigma}\cdot\mathbf{k}\; \mathbf{k}\cdot\mathbf{A}\, d\Omega = \tfrac{4}{3}\pi\boldsymbol{\sigma}\cdot\mathbf{A} \text{ (Q.M. and Cl.)} \tag{7}$$

$$\int \boldsymbol{\sigma}\cdot\mathbf{k}\; \boldsymbol{\sigma}\cdot\mathbf{A}\; \boldsymbol{\sigma}\cdot\mathbf{k}\, d\Omega = -\tfrac{4}{3}\pi\boldsymbol{\sigma}\cdot\mathbf{A} \text{ (Q.M.)}; \ \tfrac{4}{3}\pi\boldsymbol{\sigma}\cdot\mathbf{A} \text{ (Cl.)} \tag{8}$$

$$\begin{aligned}\int \boldsymbol{\sigma}\cdot\mathbf{k}\; \boldsymbol{\sigma}\cdot\mathbf{B}\; \boldsymbol{\sigma}\cdot\mathbf{A}\; \boldsymbol{\sigma}\cdot\mathbf{k}\, d\Omega &= \tfrac{4}{3}\pi(2\,\boldsymbol{\sigma}\cdot\mathbf{A}\,\boldsymbol{\sigma}\cdot\mathbf{B}+\boldsymbol{\sigma}\cdot\mathbf{B}\,\boldsymbol{\sigma}\cdot\mathbf{A}) \text{ (Q.M.)};\\ &= \tfrac{4}{3}\pi(2\,\boldsymbol{\sigma}\cdot\mathbf{A}\,\boldsymbol{\sigma}\cdot\mathbf{B}-\boldsymbol{\sigma}\cdot\mathbf{B}\,\boldsymbol{\sigma}\cdot\mathbf{A}) \text{ (Cl.)}.\end{aligned} \tag{9}$$

IV. MATRIX ELEMENTS AND RESULTS

Now we consider scattering of photons and charged and neutral mesons, as well as charge exchange scattering by nucleons. We distinguish two different physical situations. In the first (case I) we consider the nucleon as the source of an electromagnetic and of a neutral meson field, and in the second (case II) as the source of a charge-symmetric meson field. We do not consider combinations of case I and case II here as these cross sections have not been calculated as yet in classical action-at-a-distance theory (approximate quantum mechanical radiation damping results were obtained by Heitler[32]).

Case I. We consider the following couplings: $S(S)$, $PS(PV)$, $V(V)$, and $V(T)$. For scalar and vector coupling the scattering is due to the motion of the nucleon, while for tensor and pseudovector coupling the scattering is due to the nucleon spin.

The scattering problem is initially formulated in terms of the usual relativistic Hamiltonian of a nucleon interacting with a quantized photon-meson field with the above-mentioned interactions (in the quantum theory of radiation damping all processes have to be considered together). For a quantized field the energy is $\hbar\omega$ and the limit $\hbar\rightarrow 0$ may be regarded as the limit $\hbar\omega=\epsilon$, $\epsilon\rightarrow 0$, or as a low energy limit.[33]

Since we are interested in classical results we use the low-energy (velocity) limit for nucleons. The desired matrix elements are obtained by performing a Foldy-Wouthuysen transformation of the Hamiltonian to the first order in reciprocal nucleon mass.[26] The resulting first-order matrix elements, together with the possible Feynman diagrams, yield the following compound matrix elements for scalar and vector coupling

$$\begin{aligned}
H_{fi} &= (2\pi\hbar^4 c^2 g^2 K^2/M\epsilon^3)\mathbf{n}_f\cdot\mathbf{n}, && S(S)\\
H_{LL} &= (2\pi\hbar^2 g^2\mu^2 c^4/M\epsilon^3)\mathbf{n}_f\cdot\mathbf{n}_i, && V(V)\\
H_{TL} &= -(i2\pi\hbar^2 g^2\mu c^2/M\epsilon^2)\mathbf{j}_f\cdot\mathbf{n}_i, && V(V)\\
H_{TT} &= (2\pi\hbar^2 g^2/M\epsilon)\mathbf{j}_f\cdot\mathbf{j}_i, && V(V)\\
H_{LP} &= (2\pi\hbar^2 ge\mu c^2/M\epsilon^2)\mathbf{n}_f\cdot\mathbf{J}_i, && V(V)\\
H_{TP} &= (2\pi\hbar^2 ge/M\epsilon)\mathbf{j}_f\cdot\mathbf{J}_i, && V(V)\\
H_{PP} &= (2\pi\hbar^2 e^2/M\epsilon)\mathbf{J}_f\cdot\mathbf{J}_i, && V(V)
\end{aligned} \tag{10}$$

and for tensor and pseudovector coupling

$$\begin{aligned}
H_{fi} &= (2\pi f^2\hbar^2 c^2 K^2/\chi^2\epsilon^2)(\boldsymbol{\sigma}\cdot\mathbf{n}_f\,\boldsymbol{\sigma}\cdot\mathbf{n}_i-\boldsymbol{\sigma}\cdot\mathbf{n}_i\,\boldsymbol{\sigma}\cdot\mathbf{n}_f), && PS(PV)\\
H_{fi} &= (2\pi f^2\hbar^2 c^2 K^2/\chi^2\epsilon^2)(\boldsymbol{\sigma}\cdot\mathbf{j}_f\times\mathbf{n}_f\,\boldsymbol{\sigma}\cdot\mathbf{j}_f\times\mathbf{n}_i\\
&\qquad -\boldsymbol{\sigma}\cdot\mathbf{j}_i\times\mathbf{n}_i\,\boldsymbol{\sigma}\cdot\mathbf{j}_f\times\mathbf{n}_f), && V(T)\\
H_{fi} &= (2\pi\hbar^2 c^2/\epsilon^2)(\boldsymbol{\sigma}\cdot\mathbf{A}\,\boldsymbol{\sigma}\cdot\mathbf{B}-\boldsymbol{\sigma}\cdot\mathbf{B}\,\boldsymbol{\sigma}\cdot\mathbf{A}), && V(T)
\end{aligned} \tag{11}$$

where $\mathbf{A}$ or $\mathbf{B}=Kg\mathbf{j}$ or $ke\mathbf{J}$.

[32] W. Heitler, Proc. Roy. Irish Acad. 49A7, 101 (1943). An exact solution of the damping equation is given in reference 26, Appendix B.

[33] See, e.g., W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), 3rd ed.

The solution of the damping equation readily follows if one substitutes

$$U_{fi}=X_{fi}H_{fi}+Y_{fi}H_{if} \tag{12}$$

in the damping equation and then solves the resulting system of linear equations for the coefficients X and Y [26]; the results are special cases of case II discussed below.

The total cross sections obtained from Eqs. (6) and (5) by the procedure indicated for $S(S)$ and $V(V)$ agree with the results of Havas[20] in action-at-a-distance theory and thus are not presented explicitly. If the photon coupling constant is equated to zero, the results agree with those of Mehl and Havas[9] for $V(V)$ scattering of neutral mesons. Our results are also in agreement with Mehl and Havas for $V(T)$ and $PS(PV)$ scattering of neutral mesons.

Case II. For a nucleon which is a source of a charge-symmetric field, scattering and charge exchange scattering have to be considered together. In the first approximation scattering can be accomplished without nucleon recoil. Therefore, the matrix elements can be obtained from the usual relativistic Hamiltonian by taking the limits for the Dirac operators

$$\begin{gathered}\boldsymbol{\alpha}\rightarrow 0,\quad \beta\rightarrow 1,\quad \boldsymbol{\gamma}\rightarrow 0,\\ \gamma_5\rightarrow 0,\quad \beta\boldsymbol{\sigma}_{\mathrm{Dirac}}\rightarrow\boldsymbol{\sigma}_{\mathrm{Pauli}}.\end{gathered} \tag{13}$$

The resulting first-order matrix elements, together with the Feynman diagrams, yield the following compound matrix elements in the case of vector mesons[26] $[V(T)]$:

$$\begin{aligned}
&H_{LL}=2\pi c^2g^2\hbar^2K^2/\epsilon^2\chi^2,\\
&H_{TL}\text{ or }H_{LT}=(2\pi c^2fgK^2\hbar^2/\epsilon^2\chi^2)\boldsymbol{\sigma}\cdot\mathbf{j}_{i\text{ or }f},\\
&H_{TT}=(2\pi c^2f^2\hbar^2K^2/\epsilon^2\chi^2)\boldsymbol{\sigma}\cdot\mathbf{j}_f\,\boldsymbol{\sigma}\cdot\mathbf{j}_i,\\
&H_{LL_0}=H_{L_0L}=4\pi c^2gg_0\hbar^2K^2/\epsilon^2\chi^2,\\
&H_{LT_0}\text{ or }H_{T_0L}=(4\pi c^2gf_0\hbar^2K^2/\epsilon^2\chi^2)\boldsymbol{\sigma}\cdot\mathbf{j}_{0i\text{ or }f},\\
&H_{TL_0}\text{ or }H_{L_0T}=(4\pi c^2g_0f\hbar^2K^2/\epsilon^2\chi^2)\boldsymbol{\sigma}\cdot\mathbf{j}_{i\text{ or }f},\\
&H_{TT_0}\text{ or }H_{T_0T}=(2\pi c^2f_0f\hbar^2K^2/\epsilon^2\chi^2)(\boldsymbol{\sigma}\cdot\mathbf{j}_f\boldsymbol{\sigma}\cdot\mathbf{j}_{0i}\\
&\qquad+\boldsymbol{\sigma}\cdot\mathbf{j}_{0i}\boldsymbol{\sigma}\cdot\mathbf{j}_f)\text{ or }(\boldsymbol{\sigma}\cdot\mathbf{j}_{0f}\boldsymbol{\sigma}\cdot\mathbf{j}_i+\boldsymbol{\sigma}\cdot\mathbf{j}_i\boldsymbol{\sigma}\cdot\mathbf{j}_{0f}),\\
&H_{L_0L_0}=H_{L_0T_0}=H_{T_0L_0}=0,\\
&H_{T_0T_0}=(2\pi c^2f_0^2\hbar^2K^2/\epsilon^2\chi^2)(\boldsymbol{\sigma}\cdot\mathbf{j}_{0f}\boldsymbol{\sigma}\cdot\mathbf{j}_{0i}-\boldsymbol{\sigma}\cdot\mathbf{j}_{0i}\boldsymbol{\sigma}\cdot\mathbf{j}_{0f}).
\end{aligned} \tag{14}$$

The solution of the damping equation, though somewhat more involved than in case I, is obtained by an analogous procedure. This problem was solved by Heitler[32] for the quantum mechanical case; the solution to the damping equation presented below is the classical limit of the Heitler result, but can only be obtained by recalculation using "classical sums" as discussed above. Putting

$$U_{fi}=(2\pi c^2\hbar^2K^2/\epsilon^2\chi^2)(X_{fi}A_fA_i+Y_{fi}A_iA_f), \tag{15}$$

where

$$A=g,\ g_0,\ f\boldsymbol{\sigma}\cdot\mathbf{j},\text{ or }f_0\boldsymbol{\sigma}\cdot\mathbf{j}_0,$$

we get the following results:

$$\begin{aligned}
&Y_{TT_0}=Y_{T_0T}=1/(1+\tfrac{4}{3}Qf_0^2)=-Y_{T_0T_0}\\
&\text{all other }Y_{fi}=0;\\
&X_{LL}=X_{TL}=[1+2(2Qg_0^2+\tfrac{4}{3}Qf_0^2)]W^{-1},\\
&X_{T_0L}=X_{L_0L}=2W^{-1}=X_{LL_0}=X_{TL_0},\\
&X_{L_0L_0}=X_{T_0L_0}=4(Qg^2+\tfrac{2}{3}Qf^2)W^{-1},\\
&X_{LT}=X_{TT}=\{1+4Qg_0^2\\
&\qquad+[\tfrac{4}{3}Qf_0^2+(32/9)Q^2f_0^4]/(1+\tfrac{4}{3}Qf_0^2)\}W^{-1},\\
&X_{L_0T}=[2-\tfrac{4}{3}Qf_0^2(2Qg^2+\tfrac{4}{3}Qf^2)/(1+\tfrac{4}{3}Qf_0^2)]W^{-1},\\
&X_{T_0T}=\frac{1+(8/3)Qf_0^2+(Qg^2+\tfrac{2}{3}Qf^2)(1+4Qg_0^2)}{1+\tfrac{4}{3}Qf_0^2}W^{-1},\\
&X_{L_0T_0}=[4Qg^2+\tfrac{4}{3}Qf^2\\
&\qquad+\tfrac{4}{3}Qf_0^2(2Qg^2+\tfrac{4}{3}Qf^2)/(1+\tfrac{4}{3}Qf_0^2)]W^{-1},\\
&X_{LT_0}=[2-\tfrac{2}{3}Qf^2-(8/3)Q^2f^2g_0^2-(16/9)Q^2f_0^2f^2\\
&\qquad+\tfrac{4}{3}Qf_0^2/(1+\tfrac{4}{3}Qf_0^2)]W^{-1},\\
&X_{T_0T_0}=\{4Qg^2+\tfrac{4}{3}Qf^2+1/(1+\tfrac{4}{3}Qf_0^2)[1-Qg^2-\tfrac{2}{3}Qf^2\\
&\qquad-2Qg_0^2(2Qg^2+\tfrac{4}{3}Qf^2)]\}W^{-1},\\
&X_{TT_0}=[1+Qg^2+4Q^2g^2g_0^2+(8/3)Q^2g^2f_0^2\\
&\qquad+\tfrac{4}{3}Qf_0^2/(1+\tfrac{4}{3}Qf_0^2)]W^{-1},
\end{aligned} \tag{16}$$

where

$$W=1-\tfrac{2}{3}Qf^2-Qg^2-(2Qg^2+\tfrac{4}{3}Qf^2)(2Qg_0^2+\tfrac{4}{3}Qf_0^2)$$

and

$$Q=-iK^3/\omega\hbar\chi^2.$$

The cross sections can be obtained from this as before, but are not given explicitly because of the lengthy expressions involved.

A direct comparison of the cross sections calculated with the above solutions with the corresponding results of classical action-at-a-distance theory is not possible at the present time because of the absence of such calculations including charge exchange. However, if we put $f_0=g_0=0$, our results agree with those of Mehl[21] and Majumdar *et al.*[18] for the scattering of charged mesons.

The results for charged scalar and pseudoscalar mesons follow directly from the above results. We need only limit our solutions to either g and g_0 or f and f_0 coupling and replace $\tfrac{2}{3}f$ (or f_0) by $\tfrac{1}{3}f$ (or f_0); the factor 2 is the result of summing over two states of transverse polarization. We again get agreement with the action-at-a-distance results[18,21] for $g_0=f_0=0$.

VI. DISCUSSION

The scattering cross sections which we have calculated employing the classical limit of the radiation damping theory as described agree with the results of classical action-at-a-distance theory in all cases where comparison is possible.[34]

The broad area of agreement of the two theories would tend to indicate that, indeed, there exists general correspondence at least in the approximation considered.

The classical action-at-a-distance cross sections in both the neutral and charge-symmetric theories are obtained assuming small amplitude fields and small nucleon vibrations and recoil velocity. The correspondence with radiation damping theory cannot be extended to high nucleon recoil velocities, as the classical limit of this theory is restricted to low energy fields. The above assumptions decouple the spin and isotopic spin in the classical charge-symmetric theory and thus the results for the scattering of charged mesons resemble the quantum mechanical ones for charged, rather than charge-symmetric, theory. These assumptions also exclude the possibility of charge exchange. The results in radiation damping theory do involve higher powers in the coupling constants than the classical calculations and are therefore presumably more accurate. To increase the accuracy of the calculations any further would require the use of radiative corrections. With these corrections we might expect the theory to correspond more closely to classical field theory than to action-at-a-distance theory because of the explicit appearance of self-actions, although the equations of the latter theory also contain terms of the same form as the finite classical field-theoretical self-action, but with different interpretation. This question is currently being investigated.

On the other hand, the development of a complete action-at-a-distance quantum theory would allow direct comparison with the quantum theory of radiation damping, without necessitating a study of the classical limit. Conversely, the similarities revealed between the Heitler theory and the classical theory of action at a distance could be exploited to anticipate the results of a future quantum theory of action at a distance by calculations based on the quantum theory of radiation damping.

[34] The classical action-at-a-distance cross sections for $PV(PV)$ and $PV(T)$ have recently been obtained by A. D. Craft, Lehigh University thesis, 1959 (unpublished), and A. D. Craft and P. Havas (to be published). The corresponding quantum mechanical calculations are in progress.

APPENDIX: NOTATION

The following is a list of those symbols used whose meaning is not obvious or explicitly explained in the text. We have also used g and f as the mesic charge and dipole coupling constants, respectively, for the scalar, pseudoscalar, and vector meson theories; this is not intended to imply that the coupling constants of these theories are equal, but as there is no mixing of these theories there is no chance for confusion.[35]

χ $\mu c/\hbar$ (reciprocal Compton wavelength of meson);
ϵ Meson energy or photon energy;
E Nucleon energy;
K Meson wave number;
k Photon wave number;
μ Meson mass;
M Nucleon mass;
$\mathbf{j}$ Transverse polarization vector for mesons;
$\mathbf{n}$ Longitudinal polarization vector for mesons;
$\mathbf{J}$ Transverse polarization vector for photons;
V Normalization volume;

Subscripts:
i Initial state;
f Final state;
0 Neutral meson (used only in charge exchange calculations);
T Transverse meson;
L Longitudinal meson;
P Photon.

[35] The notation differs slightly from that used in reference 26; also, the definition of χ given there should be replaced by the one used here.