Studies of Low-Lying Levels of Erbium and Tungsten Isotopes with (d,p) Reactions*

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Natural metal targets of erbium and tungsten and separated isotope targets of W^{182} and W^{183} were bombarded with an energy-analyzed 15-MeV deuteron beam and energy spectra of outgoing protons from the stripping reaction were taken using a precision magnetic analyzer and photographic plates. Ground-state rotational bands of even-even nuclei Er^{168} and W^{184} were observed in the highest energy region of the respective spectra. Relative intensities and angular distributions for different rotational levels agreed reasonably well with the predictions from the stripping reaction theory using the Nilsson eigenfunction for the captured neutron orbit. The next highest energy proton groups were assigned to the intrinsic two-quasi-particle or phonon excitation levels of the above even-even nuclei, on the basis of peak positions, yields, and angular distributions. Transition rates to the so-called γ-vibrational states were found quite high. Several prominent peaks with smaller Q values were interpreted as due to those levels of the odd-A isotopes which were associated with the Nilsson eigenstates involving low orbital angular momenta. The absolute cross sections for almost all transitions studied agreed with the predicted ones, using the distorted-wave Born approximation and the pairing interaction theory, within an accuracy of a factor of two or three.

I. INTRODUCTION

IN the quasi-molecular nuclear model only the motion of the nucleons which are weakly bound, and thus associated with the excitation of low-lying levels, is described by the shell-model wave function, while the collective motion of the bulk of nucleons in the nucleus is treated as the oscillation of a liquid drop. If the nucleus has a spheroidal equilibrium shape the angular momentum components along the symmetry axis of both the individual particle and the collective motion are approximately constants of motion. The complete wave function is, thus, represented by the following formula[1–3]:

$$\Psi(IMK\Omega)=[(2I+1)/16\pi^2]^{1/2}\phi_{K\Omega}{}^{I}(\beta,\gamma)$$
$$\times\{\chi_\Omega D_{MK}{}^{(I)}(\theta_i)+(-)^{I+K-\Omega-\frac{1}{2}A}\chi_{-\Omega}D_{M-K}{}^{(I)}(\theta_i)\}. \quad (1)$$

χ_Ω is the shell-model wave function for the particle motion in the deformed potential, specified by the sum Ω of the angular momentum components of the individual particles along the symmetry axis. $\phi_{K\Omega}{}^{I}(\beta,\gamma)$ is the wave function of surface oscillation which is a function of the deformation parameters β and γ and specified by the total angular momentum I of the whole system, its component K along the symmetry axis, and by Ω. Finally $D_{MK}{}^{(I)}(\theta_i)$ is the wave function expressing the rotation of a symmetric top, which is a function of the Euler angles θ_i determining the orientation of the nucleus in the fixed coordinate system and specified by the quantum number I, its component M along the z axis of the fixed-coordinate system and its component, K. The wave function (1) is symmetrized to be invariant under reflection in the equatorial plane.

The particle wave function χ_Ω in formula (1) can be constructed with the single-particle wave functions given by Nilsson.[4] A recent quasi-particle shell-model theory, taking into account the pairing interaction, allows an accurate estimate of the single-particle state configuration of the nuclear levels.[5] The ground states of even-even nuclei are interpreted as the quasi-vacuum state in which all nucleons occupy the single-particle levels in pairs ($+\Omega$ and $-\Omega$). The pairing interaction causes mixing of the different states with the particles coupled to spin zero. One such state, the ground state, is pushed far down. The low-lying intrinsic states of the odd-A (odd-N in our case) nucleus are interpreted as the one-quasi-particle state, in which one odd nucleon occupies a particular orbit and the others occupy other orbits in pairs. Pairing interaction causes a similar configuration mixing except for the orbit occupied by the odd particle. The lowest intrinsically excited states of the even-even nuclei are generated by breaking up a pair of nucleons in the ground state (two-quasi-particle state). The excitation energy is approximately equal to the sum of one-quasi-particle energies corresponding to the respective orbits occupied by the two odd particles.

The deuteron stripping reaction is useful for investigating the individual particle states in the nucleus. Its validity for such heavy nuclei as the rare earths has been strengthened by the recent progress in the distorted-wave Born approximation calculation of the yield and the angular distribution.[6] The theory of

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[1] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **26**, No. 14 (1952).

[2] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **27**, No. 16 (1953).

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[4] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 16 (1955).

[5] S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **32**, No. 16 (1961).

[6] W. Tobocman, Phys. Rev. **115**, 99 (1959); R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory report ORNL 3240, 1962 (unpublished); R. H. Bassel, R. M. Drisko, G. R. Satchler, and E. Rost, Phys. Rev. **128**, 2693 (1962).

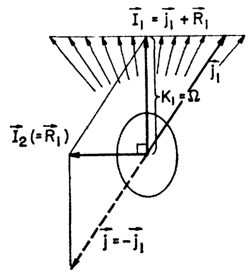


FIG. 1. A vector diagram illustrating the angular momentum coupling in the stripping reaction on the deformed nuclei. The initial state is assumed to be an odd-A nucleus ground state specified by $I_1 = K_1 = \Omega$. The total angular momentum j and the orbital angular momentum l of the odd nucleon in the initial nucleus fluctuate in both magnitude and direction with time, keeping the projection of $\mathbf{j}$ onto the symmetry axis constant. The manner of fluctuation is specified by the expansion coefficients $C_{jl}(\Omega)$ of the Nilsson eigenfunctions. Due to the recoil from the odd nucleon, the angular momentum of the rotational motion of the rest of the nucleus $\mathbf{R}$ fluctuates in the equatorial plane so that the total angular momentum of the system $\mathbf{I}_1 (= \mathbf{j} + \mathbf{R})$ is conserved. In time average $\mathbf{R}$ vanishes for the ground state. If a nucleon is captured and couples to make a pair with the odd nucleon in the initial nucleus so that the instantaneous angular momentum $\mathbf{j}_1$ of the odd nucleon is cancelled by that of the captured nucleon, the residual angular momentum $\mathbf{I}_2$ is represented by the rotational angular momentum $\mathbf{R}_1$ at the instant of capture. A selection rule for the total angular momentum of the captured nucleon, $j \geq \Omega$ can be inferred from this diagram. The transition in which a nucleon is captured by the even-even nucleus can be illustrated in a similar way.

stripping reaction on deformed nuclei has been treated by Satchler[3] using the wave function shown in formula (1). Angular momentum coupling in the stripping reaction on a rotational nucleus is illustrated in Fig. 1. It explains why transitions to the higher rotational states is possible by means of the capturing process which is essentially a single-particle phenomena. The differential cross sections for the cases of interest in the present experiments are given by the following formulas:

(a) For the even-even target nucleus (initial-state spin $I_1 = 0$, its component along the symmetry axis $K_1 = 0$; final-state spin I_2, its component along the symmetry axis $K_2 = \Omega_2$)

$$d\sigma/d\omega = 2\langle\phi_2|\phi_1\rangle^2 C_{j=I_2,l}^2(\Omega_2)\sigma_l(\theta)S(\Omega_2). \quad (2)$$

(b) For the odd-N, even-Z target nucleus (initial-state spin I_1, $K_1 = \Omega_1$; final-state spin I_2, $K_2 = \Omega_2$)

$$\frac{d\sigma}{d\omega} = g^2\langle\phi_2|\phi_1\rangle^2 \sum_l \sum_{j=l\pm\frac{1}{2}} \langle I_1\, j \mp K_1\, \Omega_2 \pm \Omega_1 | I_2 K_2\rangle^2 \times C_{jl}^2(\Omega_2 \pm \Omega_1)\sigma_l(\theta)S(\Omega_2 \pm \Omega_1), \quad (3)$$

$g = \sqrt{2}$ if $K_1 = \Omega_1 = 0$ or $K_2 = \Omega_2 = 0$

and

$$g = 1 \text{ otherwise,}$$

where the orbit in which the neutron is captured is specified by the quantum number Ω. The sign $\pm$ in formula (3) corresponds to the alternative cases of $\Omega = \Omega_2 + \Omega_1$ or $\Omega = |\Omega_2 - \Omega_1|$. $C_{jl}(\Omega)$ is the coefficient of expansion of the Nilsson eigenfunction (Ω) in terms of the eigenfunctions, specified by j and l, in the spherical potential well. The actual values of $C_{jl}(\Omega)$ are obtained from Nilsson's article,[4] assuming a proper deformation parameter of the nuclear potential well. $\sigma_l(\theta)$ is the differential cross section of capturing a nucleon in an orbit of angular momentum l in the above-mentioned hypothetical spherical potential well except for a statistical factor $(2I_2+1)/(2I_1+1)$. $S(\Omega)$ is the intrinsic spectroscopic factor and $\langle\phi_2|\phi_1\rangle$ is an overlap integral between the initial and final vibrational wave functions.

The intrinsic spectroscopic factor $S(\Omega)$ depends on the configuration of the single-particle states in the initial or final nuclear levels and can be calculated using the quasi-particle theory taking into account the pairing interaction[7];

(a) for the transition in which the captured nucleon couples with the odd nucleon in the initial state to make a pair,

$$S(\Omega) = V_f^2(\Omega); \quad (4)$$

(b) for the transition in which the captured nucleon does not make a pair,

$$S(\Omega) = U_i^2(\Omega), \quad (5)$$

where $V_f^2(\Omega)$ and $U_i^2(\Omega)$ represent the probabilities of the relevant single-particle state Ω being occupied by the nucleon pair in the final nucleus and unoccupied in the initial nucleus, respectively.

The overlap integral $\langle\phi_2|\phi_1\rangle$ vanishes if the surface vibrational states of the initial and final nuclei are not the same. However, actual state wave functions are not expected to be pure since nondiagonal matrix elements, which are ignored in the approximate wave function (1), cause mixing of the states of different K and Ω. It is shown that these matrix elements connect the states which are different in K and/or Ω by at most two units.[1] Therefore, the ground-state wave function of the odd-A nucleus is expected to be contaminated with the surface vibrational states; and hence, the transitions from this initial state to the vibrational states of the even-even nucleus are possible.

Another way of treating the surface vibrational state is to use a complete particle picture as was done for the spherical nucleus. In this theory the surface vibrational state is expressed as a superposition of the two-quasi-particle states. Similarly, for instance, the γ-vibrational state of the deformed nucleus is expected to be composed of the even-parity two-quasi-particle states mostly of $\Omega = 2$. The stripping transition from the odd-A nucleus to the vibrational state of the even-even nucleus is thus the formation of some of the relevant two-quasi-particle states by adding a particle

[7] S. Yoshida, Phys. Rev. **123**, 2122 (1961).

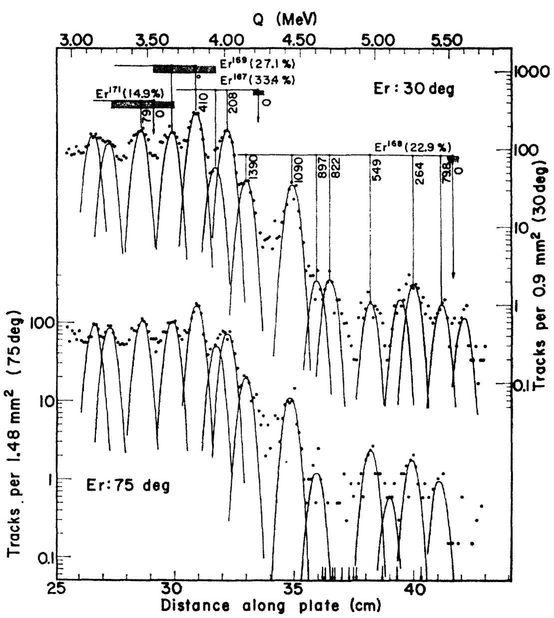


FIG. 2. Energy spectra of protons from the 15-MeV deuteron stripping reaction on a natural erbium target. Relative positions of the ground-state peaks for the different isotopes, predicted from the atomic masses, are indicated with hatch marks. Arrows put along the abscissa indicate that no track was counted in the corresponding intervals.

to a one-quasi-particle state. The contribution of each two-quasi-particle state to the reaction yield can be calculated from formula (3) except for the mixing rate.

II. EXPERIMENTAL

Targets of metal foils of natural erbium (evaporated, 8.6 mg cm^{-2} thick) and natural tungsten (rolled, 16.7 mg cm^{-2} thick) were bombarded with an energy-analyzed 15-MeV deuteron beam from the University of Pittsburgh cyclotron. The energy spectra of the protons from deuteron stripping were taken at twelve angles between 9° and 90°, using a magnetic analyzer and photographic plates. In order to prevent all particles except protons from being detected by the photographic plates an absorber was placed in front of the photographic plates. In order to reduce the spread in the proton energy the target was placed in such an oblique angle, with respect to the incident beam, that the total of the energy loss due to deuterons and protons was constant, irrespective of the depth of the reaction taking place in the target.[8] In this way the apparent energy spread could be reduced about a factor of four compared to the energy loss of the deuteron beam. Typical spectra obtained are shown in Figs. 2 and 3. Half-widths of the peaks in the erbium and tungsten spectra are approximately 80 and 120 keV, respectively. The peak width was nearly constant at all angles. From the shapes of prominent peaks which were supposed to be single, the standard peak form (Gaussian) was determined and spectra through the whole energy region were decomposed into a superposition of peaks of this shape.

For identifying the origin of the observed peaks, spectra were taken at 38° with powder targets of separated isotopes W^{182} and W^{183} (several mg cm^{-2} thick). The spectra are shown in Figs. 3 and 4.

III. SUMMARY OF THE DATA FROM OTHER SOURCES AND THE THEORETICAL PREDICTIONS

A. Erbium

Among the six stable isotopes, Er^{166}, Er^{167}, Er^{168}, and Er^{170} have comparable abundances. The abundances of the other two are small enough to be ignored in their contribution to the reaction yield. Since there is a difference in Q values of more than 1 MeV between the odd-A and the even-even nuclei, we expect the proton spectrum in the highest energy region to be due to Er^{168}.

Figure 5 shows the plots of the quasi-particle energy and the parameter U^2 for each quasi-particle state calculated as a function of the atomic mass number in the region in which we are concerned.[9,10] In the low

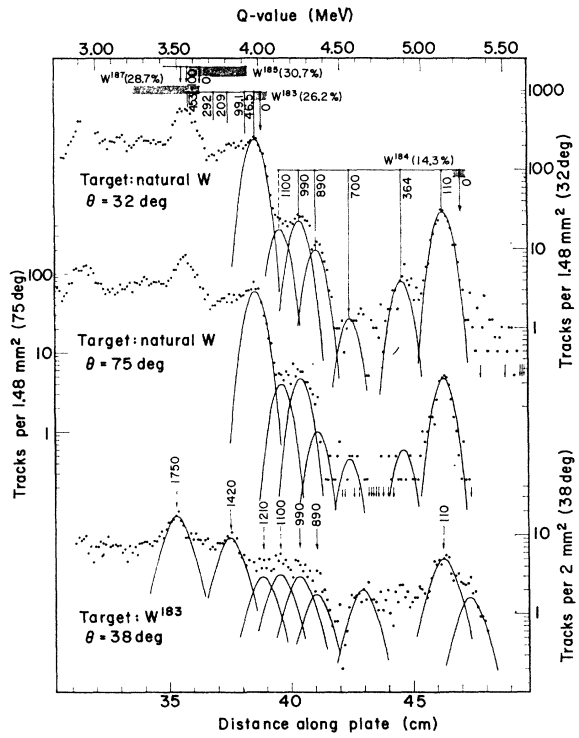


FIG. 3. Energy spectra of protons from the 15-MeV deuteron stripping reaction on the tungsten nuclei. Spectra from a natural tungsten target taken at angles of 32 and 75 deg are shown on the top and middle, respectively, and that from a W^{183} separated isotope target at 38 deg on the bottom. Relative positions of the ground-state peaks for the different isotopes, predicted by the atomic masses, are indicated with hatch marks.

[8] B. L. Cohen, Rev. Sci. Instr. **30**, 415 (1959).

[9] The quasi-particle calculations were performed by Dr. S. Yoshida.

[10] J. J. Griffin and M. Rich, Phys. Rev. **118**, 850 (1960).

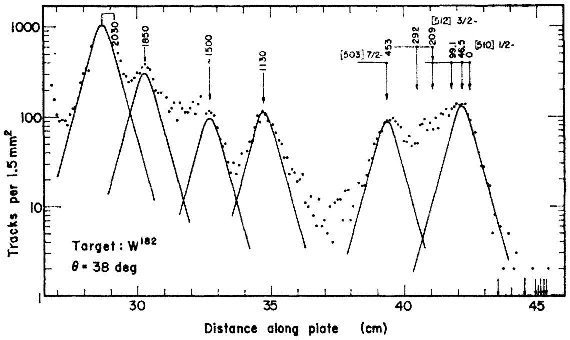


FIG. 4. An energy spectrum of protons from the 15-MeV deuteron stripping reaction on the W^{182} separated isotope target taken at an angle of 38 deg.

excitation region studied in the present experiment (<2 MeV) about ten states occur, but the important ones which act as the particle states (the states appearing on the left side of the minimum point on the quasi-particle energy curve) with a large component of low orbital angular momentum ($l=1$ to 3) are only two: $[521]\frac{1}{2}-$ and $[512]\frac{5}{2}-$. In Table I is listed available information on the nuclear levels studied, with predictions of the stripping reaction yields from formulas (2) and (3), and comparisons with the measured values. Assignments of the Nilsson eigenstates to the one-quasi-particle states of the odd-A nucleus are mostly taken from an article by Mottelson and Nilsson.[11]

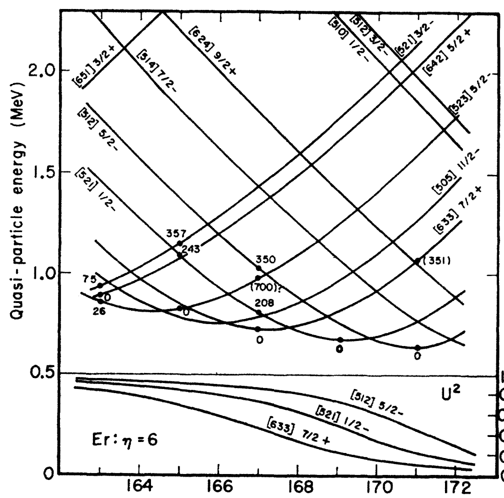


FIG. 5. Plots of the quasi-particle energies and the parameters U^2 as a function of the mass number ($=N+68$) for the erbium isotopes. The calculation in the quasi-particle theory is based on the 23 single-particle states mostly involved in the shell between $N=82$ and 126. Energy values of the single-particle states were primarily referred to the articles by Nilsson,[4] Griffin and Rich,[10] and Nilsson and Prior[5] assuming $\eta=6$; however, those of a few states were readjusted by trial and error so that more precise reproduction of the observed level scheme for each isotope is obtained. Small change of a particular single-particle energy value produces a shift of the corresponding quasi-particle energy curve without an appreciable change of the curve shape and changes of the curve positions for the other single-particle states. Since the value of the parameter U^2 is mainly determined by the position of the point relative to the minimum point of the quasi-particle energy curve, the calculated U^2 value is considered accurate if the corresponding excitation energy of each isotope is reproduced correctly. The dots along the curves indicate the intrinsic levels for each nuclide observed in experiments and the attached figures are observed excitation energies.

B. Tungsten

Among five stable isotopes, W^{182}, W^{183}, W^{184}, and W^{186} are important with comparable weights. This element also includes only one odd-A isotope so that the low-excitation region of W^{184} can be studied without interference from other isotopes. By inspecting the diagram of quasi-particle energy (Fig. 6), we find that

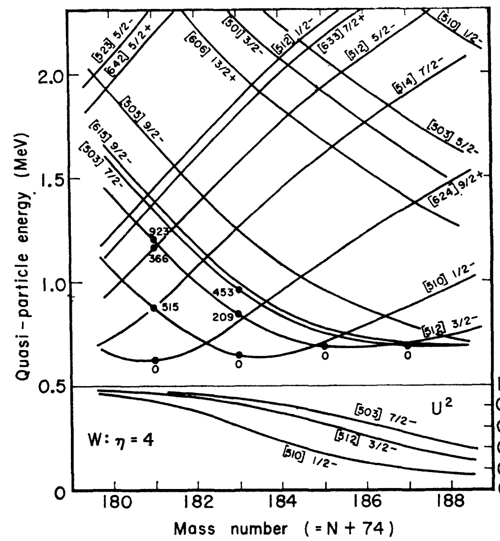


FIG. 6. Plots of the quasi-particle energies and the parameters U^2 as a function of the mass number ($=N+74$) for the tungsten isotopes. The calculation was made on the same basis as used for the erbium isotopes. The nuclear deformation parameter was assumed to be $\eta=4$ for all isotopes. Good reproduction of the level schemes for all isotopes and accurate values of U^2 cannot, then, be expected because the deformation changes considerably from isotope to isotope.

the following three neutron orbits are important; $[510]\frac{1}{2}-$, $[512]\frac{3}{2}-$, and $[503]\frac{7}{2}-$. Energies of other states including a large component of low orbital angular momentum are more than 1 MeV higher than the above levels, and thus need not be considered. Four levels originating from the levels belonging to the higher shell of $i_{11/2}$ and $g_{9/2}$, which are not taken into account in the present quasi-particle energy calculation, may occur in the lower energy region, but their low-l components are too weak to contribute appreciably to the reaction yield. Information and theoretical predictions on the tungsten isotopes are listed in Table II.

[11] B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. Skrifter 1, No. 8 (1959).

TABLE I. Summary of the information on the levels of erbium isotopes studied, the theoretical predictions of the stripping reaction cross sections, and the comparison of the observed cross sections with the predicted values.

Reaction and target nucleus[a]	Final state K_2 or Ω_2, π		I_2	Excitation energy[a] (keV)	Predicted cross section in terms of the single-particle stripping cross sections $\sigma_l(\theta)$ $(\langle\phi_2\vert\phi_1\rangle S(\Omega)=1)$	Intrinsic spectroscopic factor $S(\Omega)$	Ratio of the observed to predicted cross sections $(\langle\phi_2\vert\phi_1\rangle=1)$
$Er^{167}(d,p)Er^{168}$;	quasi-vacuum	0+	0	0	$0.0002\sigma_4$	$V_f^2=0.67$	
$Q=5.53\pm0.04$ MeV.			2	79.8	$0.030\sigma_4+0.001\sigma_6$		$4.0\ \pm1.0$
Initial state:			4	264.5	$0.034\sigma_4+0.276\sigma_6$		$4.0\ \pm1.0$[b]
$[633]\frac{7}{2}+$.			6	549	$0.006\sigma_4+0.520\sigma_6$		$4.0\ \pm1.0$[b]
$K_1=\Omega_1=I_1=\frac{7}{2}$.			8		$0.001\sigma_4+0.130\sigma_6$		
I.A. (isotopic			≥10		0		
abundance of the	γ vibration	2+	2	822			
target nuclide)			3	897			
$=22.9\%$			4	996			
	$([633]\frac{7}{2}+, [521]\frac{1}{2}-)$	3−	3	(1190)	$0.233\sigma_1+0.077\sigma_3+0.007\sigma_5$	$U_i^2=0.73$	
			4		$0.040\sigma_1+0.151\sigma_3+0.043\sigma_5$		
	$([633]\frac{7}{2}+, [521]\frac{1}{2}-)$	4−	4	(1190)	$0.267\sigma_1+0.142\sigma_3+0.018\sigma_5$	$U_i^2=0.73$	1.35 ± 0.20
			5		$0.008\sigma_1+0.208\sigma_3+0.062\sigma_5$		
	$([633]\frac{7}{2}+, [512]\frac{5}{2}-)$	1−	1		$0.069\sigma_3+0.001\sigma_5$	$U_i^2=0.86$	
			2		$0.113\sigma_3+0.047\sigma_5$		
			3	(1390)	$0.252\sigma_3+0.036\sigma_5$		
			4		$0.180\sigma_3+0.056\sigma_5$		
			5		$0.076\sigma_3+0.026\sigma_5$		
	$([633]\frac{7}{2}+, [512]\frac{5}{2}-)$	6−	6	(1390)	$0.403\sigma_3+0.032\sigma_5$	$U_i^2=0.86$	$2.2\ \pm0.2$
			7		$0.034\sigma_3+0.078\sigma_5$		
	$([633]\frac{7}{2}+, [624]\frac{9}{2}+)$	1+	1		0	$U_i^2=0.96$	
			2		$0.002\sigma_6$		
			3		$0.095\sigma_6$		
			4		$0.215\sigma_6$		
	$([633]\frac{7}{2}+, [642]\frac{5}{2}+)$	1+	1			$U_i^2=0.06$	
			2		$0.001\sigma_2$		
			3		$0.001\sigma_2+0.003\sigma_4$		
$Er^{164}(d,p)Er^{165}$;	$[523]\frac{5}{2}-$		$\frac{5}{2}$	0	$0.150\sigma_3$	$U_i^2=0.46$	
$Q=$unknown. Initial			$\frac{7}{2}$	77	$0.152\sigma_3$		
states: quasi-vacuum			$\frac{9}{2}$		$1.58\sigma_5$		
$K_1=\Omega_1=I_1=0$.			11/2		$0.122\sigma_5$		
I.A.$=1.56\%$	$[521]\frac{1}{2}-$		$\frac{1}{2}$	243	$0.298\sigma_1$	$U_i^2=0.88$	
			$\frac{3}{2}$	298	$0.050\sigma_1$		
			$\frac{5}{2}$	296	$0.364\sigma_3$		
			$\frac{7}{2}$		$0.460\sigma_3$		
	$[512]\frac{5}{2}-$		$\frac{5}{2}$		$0.020\sigma_3$	$U_i^2=0.93$	
			$\frac{7}{2}$		$1.572\sigma_3$		
$Er^{166}(d,p)Er^{167}$;	$[633]\frac{7}{2}+$		$\frac{7}{2}$	0	$0.002\sigma_4$	$U_i^2=0.59$	
$Q=4.22\pm0.04$ MeV.			$\frac{9}{2}$	78	$0.140\sigma_4$		
Initial state:			11/2	172	$0.030\sigma_5$		
quasi-vacuum.			13/2		$1.832\sigma_5$		
$K_1=\Omega_1=I_1=0$.	$[521]\frac{1}{2}-$		$\frac{1}{2}$	208	$0.498\sigma_1$	$U_i^2=0.80$	$2.2\ \pm0.2$
I.A.$=33.4\%$			$\frac{3}{2}$	264	$0.050\sigma_1$		
			$\frac{5}{2}$	262	$0.364\sigma_3$		
			$\frac{7}{2}$		$0.460\sigma_3$		
	$[512]\frac{5}{2}-$		$\frac{5}{2}$	~350	$0.020\sigma_3$	$U_i^2=0.89$	
			$\frac{7}{2}$	(410)	$1.572\sigma_3$		1.85 ± 0.20[c]
	$[523]\frac{5}{2}-$		$\frac{5}{2}$	~700	$0.150\sigma_3$	$U_i^2=0.24$	
			$\frac{7}{2}$		$0.152\sigma_3$		
			$\frac{9}{2}$		$1.580\sigma_3$		
			11/2		$0.122\sigma_5$		

[a] Energy values in parentheses have been determined in the present work.
[b] It is assumed that $\sigma_6\sim0.1\sigma_4$.
[c] Corresponding peaks overlap each other on the spectra, so comparison with the theoretical prediction was made for the sum of the intensities.

TABLE I (*continued*)

Reaction and target nucleus[a]	Final state K_2 or Ω_2, π	I_2	Excitation energy[a] (keV)	Predicted cross section in terms of the single-particle stripping cross sections $\sigma_l(\theta)$ $(\langle\phi_2\vert\phi_1\rangle S(\Omega)=1)$	Intrinsic spectroscopic factor $S(\Omega)$	Ratio of the observed to predicted cross sections $(\langle\phi_2\vert\phi_1\rangle=1)$
$Er^{168}(d,p)Er^{169}$;	$[521]\frac{1}{2}-$	$\frac{1}{2}$	0	$0.498\sigma_1$	$U_i^2=0.62$	1.85 ± 0.20[c]
$Q=3.73\pm0.20$ MeV.		$\frac{3}{2}$		$0.050\sigma_1$		
Initial state:		$\frac{5}{2}$		$0.0364\sigma_3$		
quasi-vacuum.		$\frac{7}{2}$		$0.460\sigma_3$		
$K_1=\Omega_1=I_1=0$.		$\frac{9}{2}$		$0.534\sigma_5$		
I.A.=27.1%	$[633]\frac{7}{2}+$	$\frac{7}{2}$		0	$U_i^2=0.33$	
		$\frac{9}{2}$		0		
	$[512]\frac{5}{2}-$	$\frac{5}{2}$	(80)	$0.020\sigma_3$	$U_i^2=0.81$	
		$\frac{7}{2}$	(160)	$1.572\sigma_3$		$1.6\ \pm0.2$
		$\frac{9}{2}$		$0.284\sigma_5$		
	$[514]\frac{7}{2}-$	$\frac{7}{2}$		$0.084\sigma_3$	$U_i^2=0.92$	
		$\frac{9}{2}$		$1.854\sigma_5$		
$Er^{170}(d,p)Er^{171}$;	$[512]\frac{5}{2}-$	$\frac{5}{2}$		$0.020\sigma_3$	$U_i^2=0.60$	
$Q=3.47\pm0.21$ MeV		$\frac{7}{2}$	(~80)	$1.572\sigma_3$		~3.5[c]
(3.53 ± 0.04).		$\frac{9}{2}$		$0.284\sigma_5$		
Initial state:	$[514]\frac{7}{2}-$	$\frac{7}{2}$		$0.084\sigma_3$	$U_i^2=0.85$	
quasi-vacuum.		$\frac{9}{2}$		$1.845\sigma_5$		
$K_1=\Omega_1=I_1=0$.	$[521]\frac{1}{2}-$	$\frac{1}{2}$	(~100)	$0.498\sigma_1$	$U_i^2=0.34$	~3.5[c]
I.A.=14.9%		$\frac{3}{2}$		$0.050\sigma_1$		
		$\frac{5}{2}$		$0.364\sigma_3$		
		$\frac{7}{2}$		$0.460\sigma_3$		
		$\frac{9}{2}$		$0.534\sigma_5$		

IV. SINGLE-PARTICLE DIFFERENTIAL CROSS SECTIONS IN THE DISTORTED-WAVE BORN APPROXIMATION

Figure 7 shows the single-particle differential cross sections $\sigma_l(\theta)$ for the relevant cases, which were calculated using the distorted-wave Born approximation.[12] Both sets of curves for erbium and tungsten are very similar to each other except that the peak positions for the tungsten curves are shifted one or two degrees to high angles compared to the corresponding erbium curves and that the cross sections at the peaks for the former curves are 20% less than those for the latter. The ratio of the peak cross sections of the different l curves are almost the same for the both sets of curves. The angular distributions for tungsten are more emphasized in the backward angles compared to those for erbium. The dependence on the Q value is not very sensitive so that the same set of curves may be used for cases with different Q values, if they are not very different from 4 MeV.[13] The uncertainty of the optical potential parameters, specially in the deuteron optical potential, and that of the cutoff radius R_N of the harmonic oscillator wave function for a captured neutron may easily lead to a factor of two change in the cross section. However, these uncertainties should not change relative cross sections by much.

[12] These calculations were performed by courtesy of the Oak Ridge National Laboratory Group.

[13] From the similar results of distorted-wave calculation for Pb, including sets of different Q values, it is found that an increase of Q by 1 MeV produces a decrease of cross sections by about 30% for the high-l (3 to 4) transitions and less for the low-l transitions.

V. ANALYSIS OF THE SPECTRA OF ERBIUM ISOTOPES

A. Ground-State Rotational Band of the Even-Even Nucleus Er^{168}

In the region of the spectra where the ground-state rotational band of Er^{168} is expected, many weak peaks appear, especially at forward angles ($<20°$). By investigating whether the peak position shifts with the reaction angle or not, the peaks due to erbium isotopes could be distinguished from impurity peaks. Mass numbers of the impurity elements were estimated to be 40 to 50. The samples of erbium used was 99.9% pure as to impurities of the rare-earth elements, and the concentration of tantalum, the only possible non-rare-earth impurity of mass number larger than 100, was less than 1%. Furthermore, the Q value of this element was low enough not to interfere with the energy region in which we are concerned. Therefore, no measurable contributions to the reaction yield were expected from heavy impurity elements.

Three peaks, corresponding to the first through the third excited states of the ground-state rotational band of Er^{168}, were identified from the known level spacings. The peak corresponding to the ground state is not

TABLE II. Summary of the information on the levels of tungsten isotopes studied, the theoretical predictions of the stripping reaction cross sections, and the comparison of the observed cross sections with the predicted values.

Reaction and target nucleus	Final state K_2 or Ω_2, π		I_2	Excitation energy (keV)	Predicted cross section in terms of the single-particle stripping cross sections $\sigma_l(\theta)$ $(\langle\phi_2\|\phi_1\rangle S(\Omega)=1)$	Intrinsic spectroscopic factor $S(\Omega)$	Ratio of the observed to predicted cross sections $(\langle\phi_2\|\phi_1\rangle=1)$
$W^{183}(d,p)W^{184}$;	quasi-vacuum	0+	0	0	$0.004\sigma_1$	$V_f^2=0.54$	<5
$Q=5.23\pm0.04$ MeV.			2	111.20	$0.440\sigma_1+0.318\sigma_3$		0.76 ± 0.04
Initial state:			4	364.04	$0.098\sigma_3+0.074\sigma_5$		1.90 ± 0.20
$[510]\frac{1}{2}-$.			6		$0.008\sigma_5$		
$K_1=\Omega_1=I_1=\frac{1}{2}$.			≥ 8		0		
I.A. (isotopic	γ vibration	2+	2	903			
abundance of the target nuclide)	$([510]\frac{1}{2}-, [512]\frac{3}{2}-)$	1+	1		0	$U_i^2=0.82$	
$=14.3\%$			2		$0.027\sigma_1+0.494\sigma_3$		
			3		$0.264\sigma_3$		
			4		$0.009\sigma_3$		
	$([510]\frac{1}{2}-, [512]\frac{3}{2}-)$	2+	2		$0.108\sigma_1+0.124\sigma_3$	$U_i^2=0.82$	
			3		$0.619\sigma_3$		
			4		$0.019\sigma_3+0.037\sigma_5$		
	$([510]\frac{1}{2}-, [503]\frac{7}{2}-)$	3+	3		$0.802\sigma_3$	$U_i^2=0.88$	
			4		$0.115\sigma_3+0.003\sigma_5$		
	$([510]\frac{1}{2}-, [503]\frac{7}{2}-)$	4+	4		$0.916\sigma_3+0.001\sigma_5$	$U_i^2=0.88$	
			5		$0.019\sigma_5$		
$W^{180}(d,p)W^{181}$;	$[624]\frac{9}{2}+$		$\frac{9}{2}$	0	$\propto\sigma_4$	$U_i^2=0.67$	
$Q=4.53\pm0.17$ MeV.	$[512]\frac{5}{2}-$		$\frac{5}{2}$	366	$\propto\sigma_3$	$U_i^2=0.12$	
Initial state: quasi-vacuum.	$[510]\frac{1}{2}-$		$\frac{1}{2}$	515	$0.011\sigma_1$	$U_i^2=0.91$	
$K_1=\Omega_1=I_1=0$.			$\frac{3}{2}$	565	$0.882\sigma_1$		
I.A.$=0.14\%$			$\frac{5}{2}$	618	$0.644\sigma_3$		
	$[512]\frac{3}{2}-$		$\frac{3}{2}$	928	$0.202\sigma_1$	$U_i^2=0.96$	
$W^{182}(d,p)W^{183}$;	$[510]\frac{1}{2}-$		$\frac{1}{2}$	0	$0.008\sigma_1$	$U_i^2=0.76$	
$Q=3.956\pm0.010$			$\frac{3}{2}$	46.5	$0.898\sigma_1$		1.3 ± 0.1
MeV. Initial state:			$\frac{5}{2}$	99.1	$0.636\sigma_3$		
quasi-vacuum.			$\frac{7}{2}$	207.0	$0.196\sigma_3$		
$K_1=\Omega_1=I_1=0$.			$\frac{9}{2}$	308.9	$0.148\sigma_5$		
I.A.$=26.2\%$	$[512]\frac{3}{2}-$		$\frac{3}{2}$	208.8	$0.216\sigma_1$	$U_i^2=0.89$	
			$\frac{5}{2}$	291.7	$1.484\sigma_3$		1.3 ± 0.1
			$\frac{7}{2}$	412.1	$0.050\sigma_3$		
			$\frac{9}{2}$		$0.246\sigma_5$		
	$[503]\frac{7}{2}-$		$\frac{7}{2}$	453	$1.832\sigma_3$	$U_i^2=0.92$	1.3 ± 0.1
			$\frac{9}{2}$		$0.014\sigma_5$		
			11/2		$0.154\sigma_5$		
$W^{184}(d,p)W^{185}$;	$[512]\frac{3}{2}$		$\frac{3}{2}$	0	$0.228\sigma_1$	$U_i^2=0.74$	
$Q=3.72\pm0.18$ MeV.			$\frac{5}{2}$		$1.490\sigma_3$		
Initial state:			$\frac{7}{2}$		$0.044\sigma_3$		
quasi-vacuum.			$\frac{9}{2}$		$0.200\sigma_5$		
$K_1=\Omega_1=I_1=0$.	$[510]\frac{1}{2}-$		$\frac{1}{2}$		$0.008\sigma_1$	$U_i^2=0.46$	
I.A.$=30.7\%$			$\frac{3}{2}$		$0.908\sigma_1$		
			$\frac{5}{2}$		$0.646\sigma_3$		
			$\frac{7}{2}$		$0.188\sigma_3$		
			$\frac{9}{2}$		$0.144\sigma_5$		
	$[503]\frac{7}{2}-$		$\frac{7}{2}$		$1.832\sigma_3$	$U_i^2=0.82$	
			$\frac{9}{2}$		$0.014\sigma_5$		
			11/2		$0.154\sigma_5$		
$W^{186}(d,p)W^{187}$;	$[512]\frac{3}{2}-$		$\frac{3}{2}$		$0.228\sigma_1$	$U_i^2=0.51$	
$Q=3.40\pm0.21$ MeV.			$\frac{5}{2}$		$1.490\sigma_3$		
Initial state:			$\frac{7}{2}$		$0.440\sigma_3$		
quasi-vacuum.			$\frac{9}{2}$		$0.200\sigma_5$		
$K_1=\Omega_1=I_1=0$.	$[510]\frac{1}{2}-$		$\frac{1}{2}$		$0.008\sigma_1$	$U_i^2=0.26$	
I.A.$=28.7\%$			$\frac{3}{2}$		$0.908\sigma_1$		
			$\frac{5}{2}$		$0.646\sigma_3$		
			$\frac{7}{2}$		$0.188\sigma_3$		
			$\frac{9}{2}$		$0.144\sigma_5$		
	$[503]\frac{7}{2}-$		$\frac{7}{2}$		$0.183\sigma_3$	$U_i^2=0.64$	
			$\frac{9}{2}$		$0.014\sigma_5$		
			11/2		$0.154\sigma_5$		

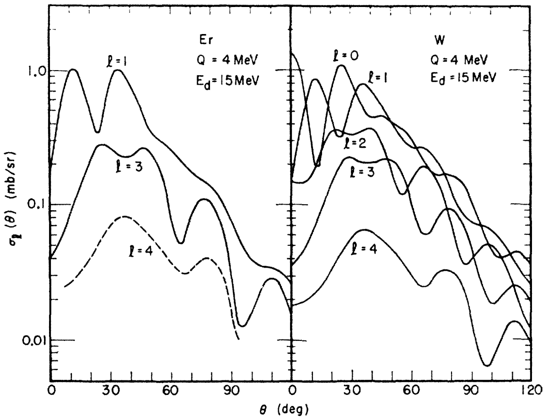


FIG. 7. Single-particle differential cross sections $\sigma_l(\theta)$, calculated by the distorted-wave Born approximation method, for the 15-MeV deuteron stripping reactions on erbium and tungsten nuclei assuming $Q=4$ MeV. The following parameters were used: Nuclear radius for a captured neutron $R_N(\mathrm{Er})=8.7$ F and $R_N(\mathrm{W})=8.8$ F; Saxon well for Er+d, $V=52$ MeV, $W=10$ MeV, $R=1.56A^{1/3}$ F, $a=0.61$ F; for Er+p, $V=48.0$ MeV, $W=9.0$ MeV, $R=1.30A^{1/3}$ F, $a=0.50$ F; for W+d, $V=48.5$ MeV, $W=9.0$ MeV, $R=1.52A^{1/3}$ F, $a=0.63$ F; for W+p, $V=48.5$ MeV, $W=11.3$ MeV, $R=1.30A^{1/3}$ F, $a=0.50$ F; radius for nuclear charge $R_c=1.30A^{1/3}$ F. The $l=4$ curve for erbium was not calculated, but was drawn analogously to the $l=4$ curve for tungsten assuming the same yield ratio for the different l values.

excited appreciably in conformity to the prediction. Yields and angular distributions of the three peaks are shown in Fig. 8. These curves show a feature of high angular momentum transfer ($l\geq 4$), that is, a sharp cutoff of the intensity at forward angles less than 30°, followed by a broad peak. Relative yields of the different rotational levels seem to agree approximately with the prediction in Table I. The observed yield values, however, are a factor of four larger than the predictions.

B. One-Quasi-Particle States of the Odd-*A* Nuclides Er^{167}, Er^{169}, and Er^{171}

Using the difference of the Q values determined from recent accurate mass measurements,[14] the peak position of the Er^{167} ground state could be predicted with reference to the Er^{168} ground-state position within an uncertainty of about 60 keV. Intense peaks appear corresponding to the excitation energy of 208 keV and 410 keV of Er^{167}. These are assigned to the lowest state of the $[521]\frac{1}{2}-$ band and the first excited state of the $[512]\frac{5}{2}-$ band, respectively. Table I predicts that these two levels should have the highest yields. Angular distributions for these levels seem to be compatible with the transfer angular momentum $l=1$ and 3 in the respective transitions; the angular distribution for the 208-keV level has a peak at the angular region less than 20 degrees which characterizes the $l=1$ transition, but the angular distribution for the 410-keV level does not (see Figs. 7 and 8). Predicted yields for these levels are of comparable magnitude; however, the observed yield for the 410-keV level is considerably higher than for the 208-keV level. This discrepancy suggests that some other peak of the comparable intensity overlaps the 410-keV level peak. Inspection of Table I shows that the only possible candidate is the ground state of Er^{169}, which belongs to the $[521]\frac{1}{2}-$ band. This position for the Er^{169} ground-state peak matches the Q-value assignment from the atomic masses.

The other strong peak expected in Er^{169}, belonging to $[512]\frac{5}{2}-$ band, can be assigned to a peak at $Q=3.66$ MeV. Since this peak appears 160 keV above the ground state, and the rotational excitation energy of this level is about 80 keV, assuming the level spacing to be the same as in the corresponding rotational band in the neighboring odd-A nuclide Yb^{173}, the lowest level energy of the $[512]\frac{5}{2}-$ band is found to be about 80 keV. This agrees with the calculated one-quasi-particle energy (see Fig. 5). The angular distribution has a feature of the $l=3$ transition. The yield is reasonable (see Table I).

The next peak at $Q=3.45$ MeV might be assigned to levels of Er^{171}, since this represents an almost unique choice which locates the Er^{171} ground state within the uncertainty width of the predicted peak position. The quasi-particle energy diagram (Fig. 5) shows that in

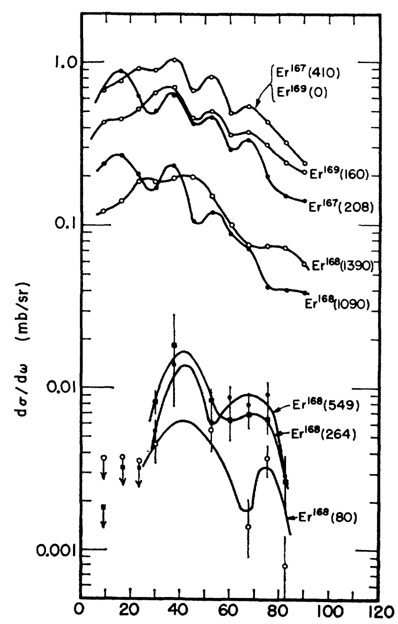


FIG. 8. Angular distributions and yields of the deuteron stripping reactions on the erbium nuclei. The numbers on each curve indicate the mass number of the final nucleus and the excitation energy of the final state in keV. Intensities at angles less than 30 deg in the three lower angular distributions, which correspond to the transitions to the ground-state rotational band of Er^{168}, could not be determined, being masked by the impurity peaks as mentioned in the text. However, for the two curves for 80- and 264-keV excited states, estimated upper limits of the intensities were put at 0.004 mb/sr or less; the curves fall rapidly at angles less than 30 deg.

[14] V. B. Bhanot, W. H. Johnson, Jr., and A. O. Nier, Phys. Rev. **120**, 235 (1960).

this nuclide the $[521]\frac{1}{2}-$ band is about 100 keV higher than the ground-state band $[512]\frac{5}{2}-$, and thus, the lowest state of the former closely overlaps the first excited state of the latter. The angular distribution of the intensity of the peak at $Q=3.45$ MeV seems to be a mixture of $l=3$ and $l=1$ curves (not shown in Fig. 8) and, thus, is consistent with the assumption that these levels are present. However, the yield calculated with this group of levels is not large enough to explain a bump on the spectra near $Q=3.45$ MeV. It is probable that two-quasi-particle levels of Er^{168}, for example the $([633]\frac{1}{2}+, [510]\frac{1}{2}-)$ band, fill up this gap.

C. Intrinsically Excited States of Er^{168}

Corresponding to the excitation energies of 1090 and 1390 keV of Er^{168}, a pair of peaks appear with comparable intensities. These intensities are much greater than that of the ground-state rotational band, meaning that low angular momentum transfer ($l=1$ or 3) participates in the transition. A possibility that these peaks may be due to the levels of Er^{165} or Er^{163} can be excluded because of their too small isotopic abundances. Thus, it seems to be reasonable to interpret them as the lowest intrinsically excited states of Er^{168}. Figure 5 shows that two-quasi-particle states $([633]\frac{7}{2}+, [521]\frac{1}{2}-)$ and $([633]\frac{7}{2}+, [512]\frac{5}{2}-)$ should have the lowest excitation energies of approximately 1400 keV and these are the two-quasi-particle states expected to be most strongly excited among others. The angular distribution for the 1090-keV level consists mostly of a $l=3$ component so that it is ascribed to $([633]\frac{7}{2}+, [512]\frac{5}{2}-)$ $\Omega=1$ or 6, negative parity, while the angular distribution for the 1390-keV level consists mostly of a $l=1$ component so that it is assigned to the state $([633]\frac{7}{2}+, [521]\frac{1}{2}-)$ $\Omega=3$ or 4, negative parity (see Table I). The predicted yields for these transitions based on the above assumptions agree reasonably well with the observed yields.

D. Vibrational State of the Even-Even Nucleus Er^{168}

A rotational band for the γ-vibrational state of Er^{168} has been found with the excitation energies 822, 897, and 996 keV.[15] In our proton spectra a few weak peaks appear in the corresponding positions with small intensities of the same order as for the ground-state band. These peaks are possibly due to the γ-vibrational levels of Er^{168}. However, another possibility that these may be levels of Er^{165} cannot be eliminated. Regularity of the atomic mass in this region predicts that the Q value of the reaction $Er^{164}(d,p)Er^{165}$ should be about 500 keV higher than that of $Er^{166}(d,p)Er^{167}$, so that a ground-state peak for the Er^{165} is expected in the energy region corresponding to about 800-keV excitation of Er^{168}. Predicted yields for the ground-state band are a little less than the observed peak intensities.

As mentioned in the introduction, transition to the vibrational state is made possible *via* the two-quasi-particle states of even parity. In the case of the transition to the Er^{168} the target nucleus holds an odd particle in the even-parity orbit, then the capturing orbit should be also of even-parity. In the nuclear shell in which we are concerned a few even-parity orbits, which originate from a single-particle state of $i\,13/2$, are available. From the calculated one-quasi-particle energy values one finds that the two-quasi-particle states which have the composition mentioned above do not occur within 1 MeV from the vibrational levels in question. Furthermore, predicted yields for the lowest relevant two-quasi-particle states $([633]\frac{7}{2}+, [624]\frac{9}{2}+)$ and $([633]\frac{7}{2}+, [642]\frac{5}{2}+)$, etc., are much smaller than the observed yields. Thus it does not seem to be possible to explain the observed peak intensities as due to the vibrational state only.

VI. ANALYSIS OF THE SPECTRA FOR TUNGSTEN ISOTOPES

A. Ground-State Rotational Band of the Even-Even Nucleus W^{184}

Transition to the ground-state rotational band of the even-even nucleus W^{184} is strong enough to allow a definite identification, as expected from the prediction. Angular distributions and yields of the first and second excited states are shown in Fig. 10. The corresponding theoretical curves are also plotted in the same figure. The angular distribution forms for these two transitions are fitted quite well by the $l=1$ and 3 curves, respectively, in conformity to the predictions. The yields also agree fairly well with the predictions. However, relative yield of the first to second excited states is a factor of 2.5 smaller than the predicted value (see Table II).

No sign of excitation of the lowest level of the band is seen. The experimentally determined upper limit of yield is about 5% of that of the first excited state (the prediction is 1%).

B. Vibrational States of the W^{184}

In the region of excitation energy from 800 to 1000 keV for W^{184} a group of peaks appears. Since no peak is expected in this region from the other isotopes and the spectrum from the W^{183} separated isotope target (Fig. 3) shows the corresponding peaks with the proper intensities (relative to the ground-state band peak), these are definitely ascribed to the levels of W^{184}. From the excitation energy values these are found to be the γ-vibrational band (Table II). The yields and the angular distributions of these peaks are shown in Fig. 9. All the angular distributions have the forms for the $l=1$ or 3 transitions and the yields are about 0.5 in the unit of the single-particle cross section $\sigma_3(\theta)$ given in Fig. 7.

[15] K. P. Jacob, J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. **117**, 1102 (1960).

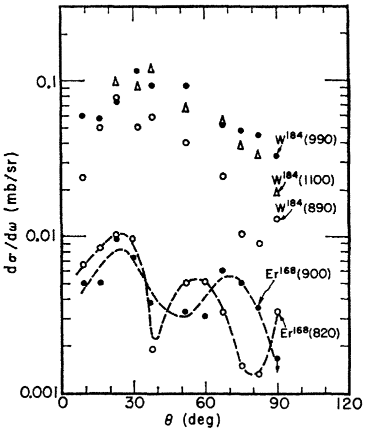


FIG. 9. Angular distributions and yields for the deuteron stripping reactions for the levels of Er^{168} and W^{184} which are supposed to be γ-vibrational states.

The vibrational levels of the W^{184} nucleus are reached from the W^{183} ground state through the formation of the two-quasi-particle states, in which the $[510]\frac{1}{2}-$ orbit and some odd-parity one are combined. A few two-quasi-particle states of this composition occur with specially low excitation energies of about 1.5 MeV:

$$([510]\tfrac{1}{2}-, [512]\tfrac{3}{2}-)_{\Omega=1 \text{ or } 2}$$

and

$$([510]\tfrac{1}{2}-, [503]\tfrac{7}{2}-)_{\Omega=3 \text{ or } 4}.$$

Other two-quasi-particle states which might contribute to the vibrational state are considerably removed from the energy region of the γ-vibrational state. For rotational bands of the states above mentioned, calculations predict mostly $l=3$ angular distributions with quite high reaction yields, which are of the same order as the observed ones for the band of the γ-vibrational state (see Table II). Therefore, it seems to be very probable that the two-quasi-particle state $([510]\frac{1}{2}, [512]\frac{3}{2}-)_{\Omega=2}$ is a main component of the γ-vibrational state in question and explain the most of the observed transition rates. If Ω is not a very good constant of motion, considerable contributions to the yield from other lowest two-quasi-particle states are possible.

C. Other Excited States of the W^{184}

The proton spectrum from the W^{183} separated isotope target (Fig. 3) shows many levels above 1 MeV. Prominent low-lying ones appear at about 1420 and 1750 keV. These are probably interpreted as two-quasi-particle states or their mixtures. Transition rates to these levels are considerably higher than expected from a single two-quasi-particle state.

D. One-Quasi-Particle States of the Odd-A Nuclides; W^{183}, W^{185}, and W^{187}

Peak positions for the levels of W^{183} relative to the ground-state rotational band of W^{184} can be located with an accuracy of 60 keV using the Q values determined from the mass data.[14] The prediction in Table II is that the transition to the lowest level of the ground state $[510]\frac{1}{2}-$ band is too weak to be measurable, while that to the first excited state is strong enough. The predicted position for the latter coincides exactly with a peak on the edge of a high-intensity yield bump rising above the vibrational state peaks of W^{184}. The angular distribution of this peak, shown in Fig. 10, is a typical $l=1$ curve. The yield is consistent with the prediction (see Table II).

From the comparison between the proton spectrum from the W^{182} separated isotope target (Fig. 4) and that from the natural tungsten target (Fig. 3), it is found that the reaction yield above the ground state within several hundred keV is mostly due to the levels of W^{183}. According to the prediction in Table II these peaks should consist of the bands due to $[510]\frac{1}{2}-$, $[512]\frac{3}{2}-$, and $[503]\frac{7}{2}-$ orbits. The intensity ratio of the peak corresponding to the 453-keV level of the $[503]\frac{7}{2}-$ band and the 46.6-keV peak of the ground-state $[510]\frac{1}{2}-$ band, determined from Fig. 4, agrees very well with the prediction (see Table II).

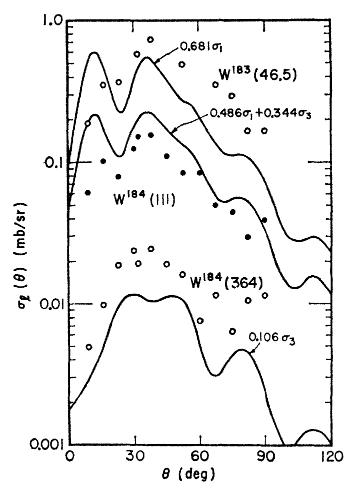
In the spectrum in Fig. 4 four large peaks appear below about 2 MeV with the excitation energies of 1130, $\sim$1500, 1850, and 2030 keV. From the quasi-particle energy diagram (Fig. 6), these are presumably due to intrinsic states of $[501]\frac{3}{2}-$, $[503]\frac{5}{2}-$, and $[501]\frac{1}{2}$.

A bump at $Q\sim3.5$ MeV is ascribed to the W^{185}, in which the important levels are concentrated near the ground state (see the quasi-particle diagram and Table II). The valley between two bumps at $Q\sim3.5$ and 2.9 MeV are, thus, mostly filled up with the peaks for the W^{187}.

VII. SUMMARY OF THE ANALYSIS AND DISCUSSION

(1) Theoretical predictions of the yields and angular distributions of the stripping reactions are obtained for the transitions to the different rotational levels of the final nucleus, assuming the unified nuclear model wave

FIG. 10. Angular distributions and yields of the deuteron stripping reactions on the tungsten nuclei. The numbers on each angular distribution indicate the mass number of the final nucleus and the excitation energy of the final state in keV.



functions for the initial and final states. The Nilsson eigenfunctions were used for the intrinsic states of the nucleons in the outer shell, which are treated as particles. The single-particle differential cross sections, on which the predictions are based, were obtained from the distorted-wave Born approximation calculations (Fig. 7). The intrinsic spectroscopic factors were calculated for each transition from the quasi-particle theory of the pairing interaction (Figs. 5 and 6). The predictions are shown in Tables I and II for the erbium and tungsten cases, respectively.

(2) The low-lying one-quasi-particle states of the odd-A nuclei, which were expected to be most strongly excited, were located on the proper positions of the proton spectra with reasonable intensities and angular distributions. Relative intensities of the different levels of the rotational bands seemed to conform to the predictions. Thus, the assignment of the $[521]\frac{1}{2}-$ and $[512]\frac{5}{2}-$ orbits to levels of erbium nuclides and $[510]\frac{1}{2}-$ and $[503]\frac{7}{2}-$ orbits to levels of tungsten nuclides have been confirmed. Predicted cross sections generally agreed with observed values within an error of a factor of two for both erbium and tungsten.

(3) Transitions to the ground-state rotational band of even-even nuclei Er^{168} and W^{184} were also reproduced by the theory with an accuracy of the same order. Thus, the assignment of the $[633]\frac{7}{2}+$ and $[510]\frac{1}{2}-$ orbits to the ground states of the target nuclei in the respective cases were justified. In the latter case, yields of the different rotational levels were measured with considerably high accuracy. They agreed with the predictions in general trend (Fig. 10); however, the ratio of the observed to predicted cross sections for the first rotational excited state was definitely too low (factor of about 2) compared to the ratios generally observed in this work.

(4) A few levels of even-even nuclei, appearing above the ground-state rotational bands, were interpreted as the lowest intrinsic two-quasi-particle states. Two levels of Er^{168} were reasonably well fitted in the yield and the angular distribution by assuming the single two-quasi-particle states of odd parity which were expected to appear lowest (see Table I).

(5) Transitions to the so-called γ-vibrational states of even-even nuclei Er^{168} and W^{184} were observed. Experimental evidences were poor for the former nucleus, while well established for the latter. Transition rates to a few rotational levels of the vibrational state were of the same order as to the ground-state rotational band of the same nucleus. If we interpret this transition as the capture of a nucleon into a hole in a body representing the final nucleus W^{184}, the high transition rates as mentioned above mean that different vibrational states of the surface and different single-particle states of the odd nucleon are mixed extremely strongly in the initial nucleus W^{183}. Such a strong mixing destroys the usefulness of the strong coupling wave function of the form (1). This might be due to the use of too crude a model in which the residual interactions between the nucleons are not expressed correctly.

The quasi-particle theory involving the long-range $Q-Q$ interaction is expected to allow a more elaborate treatment of the vibrational state and to be suitable for the present analysis purpose. The γ-vibrational state is considered to be some linear combination of the positive-parity two-quasi-particle states mostly of $\Omega=2$. In the case of W^{184} a few lowest two-quasi-particle states, one of which is a $\Omega=2$ state, are supposed to constitute the main parts of the γ-vibrational state in question. Calculated yields for the individual two-quasiparticle state are of the same order as the observed ones for the γ-vibrational state. Therefore, the observed rather high transition rate to the γ-vibrational state seems to be consistent with the particle picture of the vibrational state.

ACKNOWLEDGMENTS

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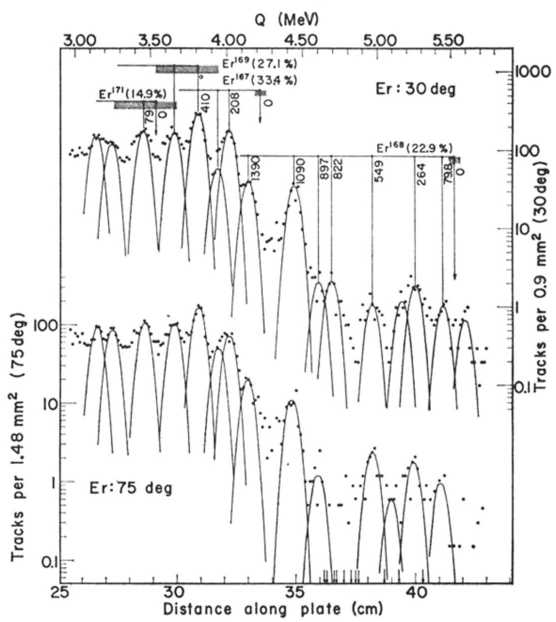

FIG. 2. Energy spectra of protons from the 15-MeV deuteron stripping reaction on a natural erbium target. Relative positions of the ground-state peaks for the different isotopes, predicted from the atomic masses, are indicated with hatch marks. Arrows put along the abscissa indicate that no track was counted in the corresponding intervals.

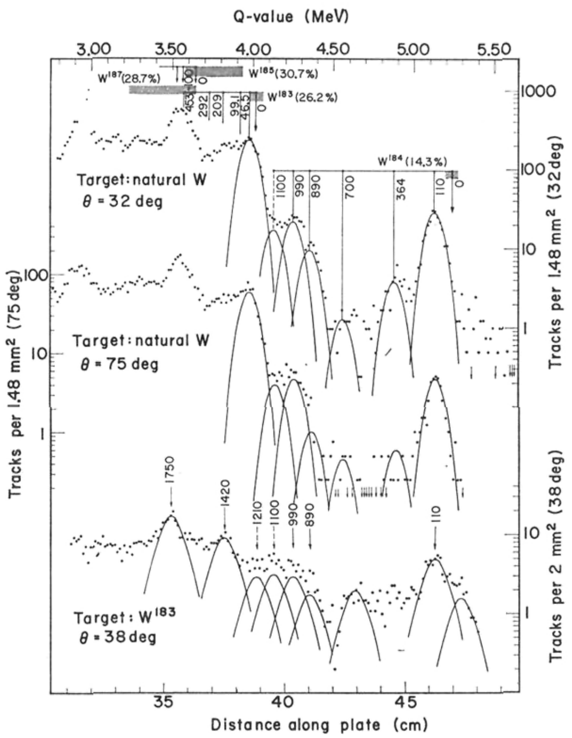

FIG. 3. Energy spectra of protons from the 15-MeV deuteron stripping reaction on the tungsten nuclei. Spectra from a natural tungsten target taken at angles of 32 and 75 deg are shown on the top and middle, respectively, and that from a W^{183} separated isotope target at 38 deg on the bottom. Relative positions of the ground-state peaks for the different isotopes, predicted by the atomic masses, are indicated with hatch marks.