with the statistical theory by Hauser and Feshbach[24] suggest the assignment $3/2^-$, with $5/2^-$ not completely excluded. Assuming the assignment for the 750-keV state to be $3/2^-$, which is in agreement with the two independent experiments, we again have a retarded $E2$ transition in Cr^{51}, its transition rate reduced by more than a factor of 10 compared to the extreme single-particle estimate. If, however, one considers Cr^{51} as resulting from a neutron hole in Cr^{52}, the ground state is then expected to have a reasonable amplitude of $\{[p(7/2)^4]_0, [n(7/2)^7]_{7/2}\}_{7/2}$, and the $3/2^-$ state should have a reasonable amplitude of $\{[p(7/2)^4]_2, [n(7/2)^7]_{7/2}\}_{3/2}$. The $E2$ transition between the two components as suggested by de-Shalit (private communication) should proceed at essentially the same reduced rate as the 1.45-MeV transition in Cr^{52}. That would lead to a lifetime of the order of 10^{-11} sec, a value about 1000 times smaller than our result, which apparently indicates that either the above expected components may not be present, or that some strange cancellations may take place among contributions from various components. It has also been suggested that a strong coupling of the valence neutrons and protons which are observed in the cases of Ti^{47} and Mn^{55} nuclei[25] may account for the retardation of the observed transitions.

[24] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).

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Low-Energy $\Sigma^- - p$ and $\Sigma^- - d$ Interactions and the Pion-Hyperon Coupling Constants*

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Using a static hyperon-nucleon potential, we calculate the fraction, $\Sigma^0/(\Sigma^0+\Lambda)$, for the low-energy reactions: $\Sigma^- + p \rightarrow n + \Sigma^0$ or Λ and $\Sigma^- + d \rightarrow 2n + \Sigma^0$ or Λ. The parameters in this potential are the pion-hyperon coupling constants, $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$, and the cores in the $^3S_1$ and $^1S_0$ states. We restrict these parameters to values which will reproduce the ΛN scattering lengths. Both even and odd Σ parity are considered. We cannot find agreement for global symmetry, but small $f_{\Sigma\Sigma}$ ($\lesssim 0.1$), in particular unitary symmetry, agrees very well with experiment. Solutions can also be found for the case of odd Σ parity.

1. INTRODUCTION AND DISCUSSION

In a previous paper[1] we considered the low-energy ΛN interactions for either Σ parity. We tried to see to what extent the pion-hyperon coupling constants are determined by the Λ-N scattering lengths. These scattering lengths are estimated[2] from analyses[3,4] of hyperfragment data. We used static hyperon-nucleon potentials which took account of one- and two-pion exchange only. Thus, the exchange of K mesons and higher mass bosons is neglected. Since there are two pion-hyperon coupling constants,[5] $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$, and two ΛN scattering lengths (for the $^1S_0$ and $^3S_1$ states), it would appear that a solution is always possible. Complications arise because phenomenological hard cores must be included in the potential. We require the cores to be of the same order of magnitude as the nucleon-nucleon cores. This still allows a rather large region of possible solutions for the coupling constants. For example, with even parity, global symmetry,[6] as well as unitary symmetry,[7] are both possible solutions. Global symmetry was favored over unitary symmetry because it required the same cores in singlet and triplet states while for unitary symmetry ($f_{\Sigma\Sigma}=0$), the cores differed by 0.08 μ^{-1}, the triplet core being the larger.

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[5] For the correct definition of the different coupling constants used, see A, Eq. (22).

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[7] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

In this paper we will consider the reactions:

$$\Sigma^- + p \rightarrow \Sigma^0 + n + 3.16\ \text{MeV}, \quad \text{(Ia)}$$

$$\Lambda + n + 79.30\ \text{MeV}, \quad \text{(Ib)}$$

and

$$\Sigma^- + d \rightarrow \Sigma^0 + 2n + 0.94\ \text{MeV}, \quad \text{(IIa)}$$

$$\Lambda + 2n + 77.08\ \text{MeV}. \quad \text{(IIb)}$$

Experimentally the fraction r_R of Σ^0 hyperons is determined for Σ^- interactions at rest. The experimental results[8,9] are

$$r_R(\text{H}) = 0.33 \pm 0.05,$$

$$r_R(\text{D}) = 0.037 \pm 0.022.$$

In Sec. 2 we discuss the method which we will use in calculating the branching ratio for reaction (I). Our starting point is the potentials for the YN system as derived in A. Knowing the transition amplitudes for reaction (I), the branching ratio in reaction (II) can be calculated by using an impulse approximation. The formulas used, which are discussed in Sec. 3, are due to Neville[10] and apply only to the case of even Σ parity. In Secs. 5 and 6, we present our results for even and odd Σ parity.

We now state our main conclusions for the case of even Σ parity. The assumption of equal cores in the singlet and triplet states seems attractive. In A, fitting the ΛN scattering lengths with equal cores ($x_0 = x_1 = 0.35\ \mu^{-1}$) led naturally to global (PV) symmetry. The global symmetry values of the branching ratios for hydrogen and deuterium, calculated in this paper, are not in agreement with experiment. Although it appears to us unlikely that uncertainties in the potential can be responsible for this disagreement, we do not feel that global symmetry can be definitely excluded.

Because agreement with experiment is impossible using equal cores, we employed different cores in the $^3S_1$ and $^1S_0$ states. We are now able to fit the experimental ΛN scattering lengths and the branching ratios in hydrogen and deuterium for the range of coupling constants,[5] $-0.05 \lesssim f_{\Sigma\Sigma} \lesssim 0.10$ and $0.25 \lesssim f_{\Lambda\Sigma} \lesssim 0.35$. In particular the point of unitary (PS) symmetry ($f_{\Sigma\Sigma} = 0$, $f_{\Lambda\Sigma} = 0.276$) gives $r_R(\text{H}) = 0.36$ and $r_R(\text{D}) = 0.037$, in good agreement with experiment. There are other pieces of evidence in support of small values of $f_{\Sigma\Sigma}$. Calculations[11–13] indicate that the presence[14] of low-lying, $T=0$, pion-hyperon resonances are most easily explained by a value of $f_{\Sigma\Sigma}$ much smaller than the global symmetry value. Also, the small upper limit[15] for the branching ratio, $(\Sigma\pi)/(\Lambda\pi)$, in the decay of Y_1^* seems to indicate[16] small $f_{\Sigma\Sigma}$. Direct information concerning $f_{\Sigma\Sigma}$ is therefore of great interest. The low-energy cross sections for $\Sigma^+ p$ scattering will provide considerable information. Global symmetry predicts the cross section to be of the order of the low-energy cross section for pp scattering while unitary symmetry predicts a cross section an order of magnitude smaller.

For odd Σ parity, again we cannot obtain agreement with experiment using equal cores in the different spin states. With unequal cores, a fit is obtained for two regions of the $g_{\Lambda\Sigma} - f_{\Sigma\Sigma}$ plane. Our results shed no light on the problem of the relative Σ parity.

2. REACTIONS IN HYDROGEN

Calculations for the system of reactions (I) are most conveniently done using the "particle" basis[17] ($\Lambda n, \Sigma^0 n, \Sigma^- p$). The Schrödinger equation has the form

$$(p^2/2m + V + Mc^2 - E)\psi = 0, \quad (1)$$

where ψ is the wave function matrix; V is the potential matrix; and $p^2/2m$, Mc^2, and E are the diagonal kinetic energy, rest mass, and total energy matrices. Isospin is obviously not conserved in reaction (I) because of mass differences between the Σ^- and Σ^0 hyperon and between the proton and neutron. We shall take account of the kinematical effects of this mass difference by using the correct masses in the kinetic energy and in the rest mass terms of Eq. (1). We neglect the dynamical effects of isospin violations by assuming that the isospin is a good quantum number for the potential.

In A, Sec. 4, using the prescription of Brueckner and Watson, we have calculated the static hyperon-nucleon potential due to one- and two-pion exchange. If $\boldsymbol{\tau}/2$ is the isotopic spin operator for the nucleon (usually represented by the Pauli matrices), $\mathbf{I}$ is the isospin operator for the hyperons, and $\boldsymbol{\varrho}_1$ is an operator [defined in Eqs. (6a) and (7a) of A] giving transitions between the Λ and Σ hyperons, then the most general charge independent hyperon-nucleon potential can be written as

$$V = A_1 + A_2\mathbf{I}^2 + A_3\mathbf{I}\cdot\boldsymbol{\tau} + A_4\boldsymbol{\varrho}_1\cdot\boldsymbol{\tau}. \quad (2)$$

The A's are functions only of $\mathbf{r}$, $\mathbf{p}$, and the (ordinary) baryonic spins. With respect to the particle basis, the operators are[18]

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[17] In the particle basis we shall adhere to this order of states.
[18] With the help of the transformations (45) of B and the definitions of the operators in the isospin basis (Sec. 5 of A), one can easily obtain the above formulas.

$$\mathbf{I}^2=\begin{Vmatrix}0&0&0\\0&2&0\\0&0&2\end{Vmatrix},\quad \mathbf{I}\cdot\boldsymbol{\tau}=\begin{Vmatrix}0&0&0\\0&0&\sqrt{2}\\0&\sqrt{2}&-1\end{Vmatrix},$$

$$\mathbf{I}\cdot\boldsymbol{\rho}_1=\begin{Vmatrix}0&-1&\sqrt{2}\\-1&0&0\\\sqrt{2}&0&0\end{Vmatrix}. \quad (3)$$

For coupled channels, a generalization of the scattering length[2,19] is

$$A=-k^{-(l+\frac{1}{2})}Kk^{-(l+\frac{1}{2})}, \quad (4)$$

where k is a diagonal matrix with ii element the relative momentum in channel i and l is the orbital momentum in channel i. K is the reaction matrix. The A matrix has a finite limit at zero energy in the $\Sigma^- p$ channel.

As in all exothermic reactions, the cross sections for (I) show the characteristic $1/v$ dependence in the $\Sigma^- p$ channel. It is, therefore, convenient to calculate the limit of $k\sigma$ as $k\rightarrow 0$, where k is the Σ^--p relative momentum. These quantities can easily be expressed in terms of the A matrix. We define a transition matrix, T, which is finite at zero energy in the Σ^--p channel, by

$$T=-\frac{1}{1+iAk^{2l+1}}A. \quad (5)$$

The cross sections can then be written (assuming the initial state to be an S state):

$$k_i\sigma(i\rightarrow f)=4\pi k_f^{2l+1}|T_{fi}|^2, \quad (6)$$

where l is the orbital momentum in the final state.

When the Σ^- comes to rest in hydrogen, it forms a $\Sigma^- p$ atom, because of the Coulomb potential. These atoms decay by reactions (Ia) and (Ib), the other modes of decay being negligible. We shall assume that the $\Sigma^- p$ atoms are formed $\frac{1}{4}$ in singlet states, and $\frac{3}{4}$ in triplet states, that the total spin is a sufficiently good quantum number[20] so that there are no important transitions between singlet and triplet states during the de-excitation of the atom and that reaction (I) takes place only from S states. For an atom in a state of spin S, the fraction, r_S, decaying by (Ia) is expressed as

$$r_S=\frac{\sigma_S(\text{Ia})}{\sigma_S(\text{Ia})+\sigma_S(\text{Ib})}. \quad (7)$$

With the above assumptions, the total fraction r_R, of the $\Sigma^- p$ atoms decaying by reaction (Ia) is

$$r_R(\text{H})=\tfrac{1}{4}r_0+\tfrac{3}{4}r_1. \quad (8)$$

This is to be contrasted with the ratio in flight, r_F, for reaction (Ia):

$$r_F(\text{H})=\frac{\sigma_T(\text{Ia})}{\sigma_T(\text{Ia})+\sigma_T(\text{Ib})}, \quad (9)$$

where the total cross section, $\sigma_T(\text{Ia})=\frac{1}{4}\sigma_0(\text{Ia})+\frac{3}{4}\sigma_1(\text{Ia})$ and similarly for $\sigma_T(\text{Ib})$. It is worth noting that r_F and r_R are independent combinations of the cross sections.

3. REACTIONS IN DEUTERIUM

As in hydrogen, Σ^- stopped in deuterium are captured in atomic orbitals. These atoms decay by reactions (II). Again we assume that the spin states ($S=\frac{1}{2},\frac{3}{2}$) of the resulting $\Sigma^- d$ atoms are statistically populated, that the total spin is good,[20] and that reaction (II) takes place only from atomic S states. Analogous to the singlet and triplet ratios, r_0, r_1, for hydrogen, there are the doublet, $r_{1/2}$, and the quartet, $r_{3/2}$, ratios for deuterium. The total fraction, $r_R(\text{D})$, of the $\Sigma^- d$ atoms decaying by reaction (IIa) is

$$r_R(\text{D})=\tfrac{1}{3}r_{1/2}+\tfrac{2}{3}r_{3/2}. \quad (10)$$

The in-flight ratio, $r_F(\text{D})$, for reaction (IIa) is defined just as in $r_F(\text{H})$ for hydrogen (Eq. 9).

For the calculation of reaction (II), we use the formulas given by Neville.[10] He considers only the case of even parity and employs an impulse approximation in which reaction (I) takes place with the proton of the deuteron; the neutron of the deuteron being a spectator. As already noted, reactions (II) are assumed to take place from atomic S orbitals; due to the low momentum of the proton in the deuteron, one may assume that the Σ^- hyperon is also in an S state with respect to the proton. The final $\Sigma^0 n$ and Λn states are, therefore, limited to $^1S_0$, $^3S_1$, or $^3D_1$ states. The transition matrix for this $\Sigma^- p$ system is calculated in the same way as that for reactions (I) [Eqs. (1–5)]. We have, therefore, the same A matrix but the requirement of energy conservation makes a difference in the momentum matrix k. The relative momenta are chosen as

$$k=\begin{Vmatrix}k_\Lambda&0&0\\0&k_{\Sigma^0}&0\\0&0&k_{\Sigma^-}\end{Vmatrix}=\begin{Vmatrix}2.030&0&0\\0&0.2275&0\\0&0&0\end{Vmatrix}\mu. \quad (11)$$

This corresponds to over-all energy conservation in reaction (II) with the spectator neutron at rest in the final state. For the relevant T matrix elements (calculated in the manner just discussed), we shall employ the following notation:

$$\Sigma_1=T(\Sigma^- p(^1S_0)\rightarrow\Sigma^0 n(^1S_0)), \quad (12a)$$

$$\Sigma_3=T(\Sigma^- p(^3S_1)\rightarrow\Sigma^0 n(^3S_1)), \quad (12b)$$

$$\Lambda_1=T(\Sigma^- p(^1S_0)\rightarrow\Lambda n(^1S_0)), \quad (12c)$$

$$\Lambda_3=T(\Sigma^- p(^3S_1)\rightarrow\Lambda n(^3S_1)), \quad (12d)$$

$$\Lambda_D=T(\Sigma^- p(^3S_1)\rightarrow\Lambda n(^3D_1))/\lambda\!\!\!^{-}_\pi{}^2. \quad (12e)$$

All Σ's and Λ's defined in this way have the dimensions of a length.

Several factors governing reaction (II) require careful treatment; they are especially important for the rate of

[19] M. H. Ross and G. L. Shaw, Ann. Phys. (New York) 13, 147 (1961).

[20] This assumption is investigated quite extensively in reference 10.

the reaction to the $\Sigma^0 n$ final state. First, the binding energy of the deuteron reduces the available energy for the final states and, therefore, restricts the accessible phase space. This effect is particularly strong for the $\Sigma^0 n$ final state where the momentum of the Σ^0 is reduced from its value in reaction (Ia), 0.4172 μ, to a maximum of 0.2275 μ, the value given above. Second, the final state contains two identical fermions (neutrons); therefore the Pauli principle has to be taken into account. This restricts the final di-neutron states to singlet even parity, ${}^1E$, or triplet odd parity, ${}^3O$, states and, therefore, further reduces the accessible phase space. Again this reduces the rate leading to $\Sigma^0 nn$ relative to the rate leading to Λnn. Third, because of the small amount of energy available for reaction (IIa), only the ${}^1S_0$ di-neutron state is of any importance. The kinetic energy in the ${}^1S_0$ nn state is less than 0.94 MeV; therefore, the final-state wave function will be enhanced by the strongly attractive nn forces. A rough estimate of this effect can be made as follows: With final-state interactions, the nn wave function is roughly

$$\psi_{nn}(r)=e^{i\mathbf{k}\cdot\mathbf{r}}+(e^{i\delta}\sin\delta/k)[(e^{ikr}-e^{-\lambda r})/r],$$

where λ is related to the nn effective range and δ is the nn phase shift. The main contribution to the transition matrix element for the Σ^0 nn final state of reaction (II) comes roughly at values of $r\approx R$ where R is the deuteron "radius." Since $kR\ll 1$ at an nn energy of $\lesssim 0.9$ MeV and $\lambda R\gg 1$, the wave function can be approximated by

$$\psi_{nn}(R)\cong 1+\frac{e^{i\delta}\sin\delta}{kR}=1-\frac{a}{R(1+ika)}\cong 2+2i,$$

where a is the ${}^1S_0$ nn scattering length. Relative to the case of no final-state interactions ($\delta=0$), this gives an enhancement by a factor of about 8. Of course the actual calculation of the enhancement of the final nn state is more complicated than this simple estimate.

The result of Neville's calculation[10] may be expressed as

$$w_{1/2}(\Lambda)=4.41|\Lambda_3-\Lambda_1|^2+|\Lambda_3+3\Lambda_1|^2+65.6|\Lambda_D|^2, \quad (13a)$$

$$w_{1/2}(\Sigma^0)=0.522|\Sigma_3-\Sigma_1|^2+3.72\times10^{-4}|\Sigma_3+3\Sigma_1|^2, \quad (13b)$$

$$w_{3/2}(\Lambda)=16|\Lambda_3|^2+301|\Lambda_D|^2, \quad (13c)$$

$$w_{3/2}(\Sigma^0)=5.95\times10^{-3}|\Sigma_3|^2, \quad (13d)$$

where $w_J(Y^0)$ is a number proportional to the rate for the reaction $\Sigma^-+d\to Y^0+2n$ through the channel of total angular momentum J. The constant of proportionality is the same for all four w's. In (13a) and (13b), the first terms on the right refer to transitions to final singlet nn states and the second terms to final triplet nn states. In both cases, the final $Y^0 n$ system (n is *not* the spectator neutron) is in an S state. The third term in (13a) arises from transitions in which the final Λn system is in a ${}^3D_1$ state and the di-neutron in a triplet state. We note that transitions to final $\Sigma^0 n$ systems in ${}^3D_1$ states are neglected in (13b) because of the low final-state energy and that transitions to final Λn, ${}^3D_1$ states with the nn system in a singlet state are not present. The low energy available to the final $\Sigma^0 nn$ system gives numerical coefficients in (13b) which are much smaller than those of (13a). Transitions to a final $\Sigma^0 nn$ system where the di-neutron is in a triplet state are suppressed. This is because of the combined effect of the small energy available and the requirement that the lowest accessible nn states are ${}^3P$ states. For a final $\Sigma^0 nn$ system with ${}^1S_0$ nn states, the wave function enhancement previously discussed tends to increase the transition rate to these states. All these factors result in the extreme disparity of the two numerical coefficients in (13b). The first two terms on the right in (13c) and (13d) give the transition rates to systems where the $Y^0 n$ are in ${}^3S_1$ states and the di-neutron is in a triplet state. We note the absence of transitions to final states where $Y^0 n$ is in an S state and the di-neutron is in a singlet. The second member of (13c) is a combination of the transition rate to Λn in ${}^3D_1$, nn in singlet (relative strength $137|\Lambda_D|^2$) and the rate to Λn in ${}^3D_1$, nn in triplet (relative strength $164|\Lambda_D|^2$). As in (13b), final states with $\Sigma^0 n$ in a D state are neglected in (13d). The smallness of the numerical coefficient in (13d) is due to exactly the same factors which suppress transitions to $\Sigma^0 n$ S states, di-neutron in triplet state, total $J=\frac{1}{2}$ [the second term in (13b)].

Because $w_{3/2}(\Sigma^0)$ is so small, $r_{3/2}$ is very small and the ratio at rest, $r_R(D)$, is essentially $\frac{1}{3}$ $r_{1/2}$.

4. METHOD OF COMPUTATION

Specification of the $\Lambda\Sigma\pi$ and $\Sigma\Sigma\pi$ coupling constants and the hard cores defines the potentials. (As in A, we assume that the $NN\pi$ coupling is known.) Using the numerical integration as outlined by De Swart and Eberlein,[21] we solve the Schrödinger equation on an IBM 704. We obtain the A matrix for the system (Λn, $\Sigma^0 n$, $\Sigma^- p$) at zero $\Sigma^- p$ relative momentum. We include exactly the effects of the tensor force coupling to D waves when we compute the A and T matrices. Since the initial $\Sigma^- p$ is in an S state, the A and T matrices are 3×3 for the spin singlet case and 6×6 for the spin triplet case. With the formulas of the preceding section, we then compute the branching ratios for hydrogen and deuterium. We used the following numerical values:

$$\mu=138.1\ \mathrm{MeV}/c^2,\qquad M_\Lambda=1115.36\ \mathrm{MeV}/c^2,$$
$$M_p=938.21\ \mathrm{MeV}/c^2,\qquad M_{\Sigma^0}=1191.50\ \mathrm{MeV}/c^2,$$
$$M_n=939.51\ \mathrm{MeV}/c^2,\qquad M_{\Sigma^-}=1195.96\ \mathrm{MeV}/c^2,$$
$$f_{\mathrm{NN}}=0.28504,\qquad \mu^{-1}=1.4289\ \mathrm{F}.$$

[21] J. J. de Swart and P. J. Eberlein, University of Rochester Report-NYO-9030, 1960 (unpublished). J. J. de Swart and C. Dullemond, Ann. Phys. (N. Y.) 16, 263 (1961).

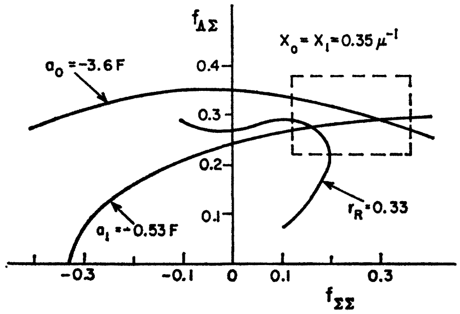


FIG. 1. Even Σ parity, equal singlet and triplet cores. The curve $r_R=0.33$ shows $f_{\Lambda\Sigma}$ vs $f_{\Sigma\Sigma}$ required to obtain the experimental branching ratio in reaction (I). Also given are $f_{\Lambda\Sigma}$ vs $f_{\Sigma\Sigma}$ for constant ${}^1S_0\,\Lambda N$ scattering length, $a_0=-3.6$ F, and for constant ${}^3S_1\,\Lambda N$ scattering length, $a_1=-0.53$ F. All cores have been put equal to $0.35\,\mu^{-1}$.

5. RESULTS FOR EVEN Σ PARITY

In A we have considered the Λ-nucleon scattering lengths as a function of the pion-hyperon coupling constants $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$. A major uncertainty in the calculation is the treatment of the repulsive cores. The simplest assumption we can make about these cores is that they are the same in all the channels (ΛN as well as ΣN) and all the states (${}^1S_0$ as well as ${}^3S_1$) and are equal to the commonly used core ($x_c=0.35\,\mu^{-1}$) in the nucleon-nucleon potentials.

When we fitted the ${}^1S_0\,\Lambda N$ scattering length, a_0, using a definite core ($0.35\,\mu^{-1}$), our solution, obtained in A, was a line in the $f_{\Lambda\Sigma}-f_{\Sigma\Sigma}$ plane. Likewise, fitting the ${}^3S_1\,\Lambda N$ scattering length, a_1, using the same core, gave as solution another line in this $f_{\Lambda\Sigma}-f_{\Sigma\Sigma}$ plane. Our aim now is to find, using the same cores, a third line in this plane, the values of which will fit the branching ratio, r_R, in hydrogen. The results are given in Fig. 1. From this graph, it is obvious that no point, ($f_{\Lambda\Sigma}$, $f_{\Sigma\Sigma}$), can be found, consistent with these three experimental quantities, r_R, a_0, and a_1. Even if we allow for the probable errors in the experimental quantities (see Fig. 2), this conclusion is unchanged.

The point of approximate global (PV) symmetry, $f_{\Lambda\Sigma}=f_{\Sigma\Sigma}=0.29$ with the cores $0.35\,\mu^{-1}$, fits the scattering lengths a_0 and a_1. For these values, the branching ratios are: $r_0=0.87$, $r_1=0.31$, $r_R(\mathrm{H})=0.45$, $r_{1/2}=0.57$, $r_{3/2}=0.76\times10^{-3}$, and $r_R(\mathrm{D})=0.19$. The large singlet ratio, r_0, is due to the strong $T=\frac{3}{2}$ part of the interaction. Although the isospin is not a good quantum number for the transition amplitudes, it is a rather good quantum number[22] for the scattering length matrix A. At this point of approximate global PV symmetry, the $T=\frac{3}{2}$, ${}^1S_0$ scattering length is large and positive, corresponding to a weakly bound $\Sigma^- n$ state.[23] This large scattering length results in a very large ${}^1S_0$ cross section for reaction (Ib) and, therefore, in a large branching ratio, r_0. The large branching ratio in deuterium is also due to this effect. This result is understandable because, as is shown in A, the singlet, $T=\frac{3}{2}$, hyperon-nucleon potential for global PV symmetry is equal to the singlet, $T=1$, nucleon-nucleon potential. The existence of a virtual bound state and its associated large scattering length has long been known for the ${}^1S_0\,NN$ system.

The branching ratios are much larger than the experimental ones for points in the neighborhood of global symmetry. Although we do not feel that global symmetry is definitely excluded, it appears to be rather difficult to obtain agreement with all the experimental results for points near the global symmetry values. The calculation of the potentials contains sufficient uncertainties (K-meson exchange, exchange of heavy mesons) to allow a weaker $T=\frac{3}{2}$, singlet potential. Thus, r_0 could possibly be lower than calculated. Also, a lower value of r_1 may not be incompatible with global PV symmetry. De Swart and Dullemond have obtained in B the ratio $r_1=0.2$. They used semi-phenomenological NN potentials to obtain YN potentials for global PV symmetry. The semiphenomenological potentials differ from ours in two respects. First, spin-orbit terms are present. Second, the inner region of the triplet potential is less attractive. (In B, the triplet ΛN scattering length, $a_1\approx0.1$ F.) If this second factor is responsible for the smaller r_1, then it appears to us very difficult to obtain a considerably smaller r_1 while maintaining $a_1=-0.53$ F as required by experiment. Reduction of both r_0 and r_1 seems to be required to obtain $r_R(\mathrm{H})=0.33$; thus, a fit to the experimental data, in the case of global symmetry, requires a fortunate combination of factors.

We have not shown in Figs. 1 or 2, the results for the branching ratio in deuterium, $r_R(\mathrm{D})$. As shown above, this branching ratio, at the point which fits the ΛN scattering lengths ($f_{\Lambda\Sigma}=f_{\Sigma\Sigma}=0.29$; equal cores, 0.35 μ^{-1}) is not consistent with the experimental value. As is the case with hydrogen, it seems difficult to obtain simultaneous agreement with the experimental ΛN scattering lengths and the branching ratio in deuterium using equal cores in both the singlet and triplet states.

Because we are unable to obtain agreement with the experimentally observed branching ratios and the ΛN scattering lengths, with one set of coupling constants and equal cores in singlet and triplet states, the next logical step, therefore, is to relax the requirement of

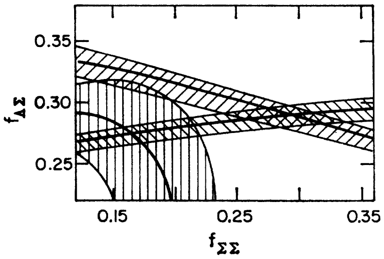


FIG. 2. Enlarged portion (dashed rectangle) of Fig. 1 showing the effect of the uncertainties in a_0, a_1, and r_R.

[22] See Sec. 8 of B.

[23] Analogous results were obtained in B, Sec. 4, using semi-phenomenological potentials.

TABLE I. Branching ratios for hydrogen and deuterium for even Σ parity.[a]

x_0	x_1	$f_{\Lambda\Sigma}$	$f_{\Sigma\Sigma}$	r_0	r_1	r_R(H)	r_F(H)	r_R(D)	r_F(D)
0.30	0.30	0.232	0.209	0.92	0.26	0.42	0.40	0.23	0.24
0.30	0.31	0.239	0.185	0.92	0.21	0.39	0.37	0.17	0.11
0.30	0.32	0.245	0.162	0.90	0.18	0.36	0.29	0.11	0.058
0.30	0.33	0.251	0.138	0.87	0.16	0.34	0.23	0.085	0.039
0.30	0.35	0.263	0.092	0.84	0.14	0.32	0.18	0.060	0.023
0.30	0.37	0.272	0.035	0.89	0.15	0.33	0.17	0.043	0.015
0.30	0.39	0.276	−0.040	0.94	0.18	0.37	0.19	0.030	0.012
0.30	0.40	0.275	−0.085	0.48	0.22	0.28	0.22	0.032	0.012
0.35	0.30	0.240	0.450	0.22	0.85	0.69	0.78	0.21	0.39
0.35	0.33	0.269	0.358	0.28	0.63	0.54	0.62	0.21	0.15
0.35	0.34	0.280	0.322	0.69	0.43	0.49	0.44	0.17	0.11
0.35[b]	0.35	0.290	0.290	0.87	0.31	0.45	0.39	0.19	0.15
0.35	0.36	0.298	0.260	0.93	0.25	0.42	0.43	0.24	0.24
0.35	0.37	0.308	0.230	0.93	0.21	0.39	0.38	0.16	0.10
0.35	0.38	0.315	0.203	0.92	0.18	0.37	0.31	0.11	0.055
0.35	0.40	0.328	0.150	0.88	0.16	0.34	0.23	0.067	0.027
0.35	0.42	0.341	0.086	0.88	0.15	0.33	0.19	0.044	0.016
0.35	0.43	0.345	0.055	0.89	0.16	0.34	0.18	0.038	0.013
0.35	0.44	0.349	0.027	0.93	0.16	0.35	0.18	0.034	0.011
0.40	0.40	0.356	0.384	0.81	0.37	0.48	0.40	0.17	0.12
0.25	0.30	0.198	0	0.94	0.15	0.35	0.16	0.043	0.015
0.28	0.35	0.245	0	0.97	0.16	0.36	0.17	0.039	0.014
0.30[c]	0.38	0.276	0	0.98	0.16	0.36	0.17	0.037	0.013
0.32	0.41	0.306	0	0.98	0.16	0.36	0.17	0.034	0.012
0.35	0.45	0.353	0	0.97	0.17	0.37	0.18	0.031	0.011

[a] x_0 and x_1 in μ^{-1}.
[b] Global *PV* symmetry.
[c] Unitary *PS* symmetry.

equal cores. We still require the cores in the Λ- and Σ-nucleon channels to be the same.[24] In our analysis in A we found that every point in the $f_{\Lambda\Sigma}-f_{\Sigma\Sigma}$ plane can be made to fit the Λ-nucleon scattering lengths provided we choose the right cores. In other words, for a fixed set of coupling constants, $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$, we can choose singlet and triplet cores, x_0 and x_1, so as to obtain the correct singlet and triplet ΛN scattering lengths, a_0 and a_1. We require the cores to be of the same order of magnitude as those used in nucleon-nucleon scattering, $0.30\,\mu^{-1}\lesssim x_c \lesssim 0.40\,\mu^{-1}$. This restricts the region of interest in the $f_{\Lambda\Sigma}-f_{\Sigma\Sigma}$ plane to the values $-0.1\lesssim f_{\Sigma\Sigma}\lesssim 0.5$, $0.25\lesssim f_{\Lambda\Sigma}\lesssim 0.35$. We have calculated the values of r_R in hydrogen and deuterium in this region, taking the cores at every point so as to fit the ΛN scattering lengths. The results are given in Tables I and II. Due to the relatively large error in the ratios r_R(H) and r_R(D) and the weak dependence of the calculated values of these ratios on the coupling constants, we shall not give specific points where the experimental values are obtained exactly. We give instead two lines marked "H" and "D" in Fig. 3. These lines correspond to $r_R(\mathrm{H})=0.38$ for "H" and $r_R(\mathrm{D})=0.06$ for "D." These are the lines at which the ratios are one standard deviation away from the experimental values. All points to the left of these lines (thus smaller values of $f_{\Sigma\Sigma}$), including the line $f_{\Sigma\Sigma}=0$, *do* fit the experimental ratios, within the errors. We have not calculated the ratios at many negative values of $f_{\Sigma\Sigma}$ because this requires the cores to be outside the range of those used in NN scattering and because the difference between the cores in singlet and triplet states becomes very large for negative $f_{\Sigma\Sigma}$. From this graph it

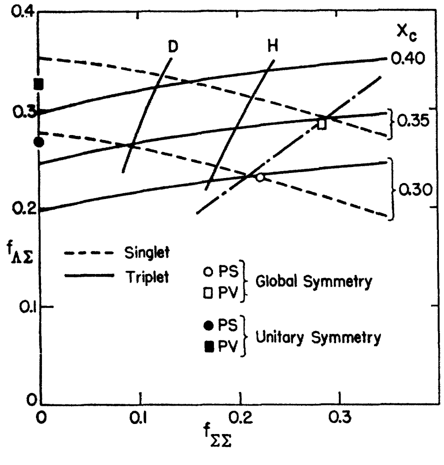


FIG. 3. Even Σ parity, variable cores. Points which fit the experimental branching ratio in hydrogen or deuterium must lie to the left of the line marked H or D. The region of $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$ which reproduces the experimental results includes the axis $f_{\Sigma\Sigma}=0$, and points slightly to the left of it. (See the discussion of Sec. 5.) At every point the cores, x_0 and x_1, in the singlet and triplet states have been chosen so as to obtain the correct ΛN scattering lengths. Values of these cores (in units of μ^{-1}) are indicated by the other lines. Also shown is the dash-dotted line of equal cores.

is clear that (with the previously given restrictions on the core) the only values of $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$, which are consistent with the ΛN scattering lengths and the ratios r_R(H) and r_R(D), lie in a region roughly bounded by $-0.05\lesssim f_{\Sigma\Sigma}\lesssim 0.1$ and $0.25\lesssim f_{\Lambda\Sigma}\lesssim 0.35$. It is interesting to note that the deuterium ratio gives the smallest upper limit on the cores. The region $f_{\Sigma\Sigma}\gtrsim 0.2$ shows exactly the same difficulties in fitting the branching ratios as have been discussed previously for the case of global symmetry.

Unitary symmetry (*PS* coupling)[7] is a particularly

TABLE II. Cross sections for the reaction (I) for even Σ parity.[a]

x_0	x_1	$k\sigma_0$ (Ia)	$k\sigma_0$ (Ib)	$k\sigma_0$ ($^3S_1$; Ia)	$k\sigma_1$ ($^3S_1$; Ib)	$k\sigma_1$ ($^3D_1$; Ib)
0.30	0.30	251	23.2	82.1	77.1	161
0.30	0.31	220	18.5	56.2	50.2	164
0.30	0.32	109	12.4	41.0	30.5	158
0.30	0.33	60.6	9.19	32.3	17.0	154
0.30	0.35	27.9	5.19	25.8	3.5	151
0.30	0.37	12.8	1.57	25.6	1.3	150
0.30	0.39	5.17	0.32	30.5	16.1	124
0.30	0.40	3.75	4.00	32.2	27.7	90.1
0.35	0.30	3.38	11.9	34.0	0.02	6.02
0.35	0.33	4.56	11.9	106.	40.8	21.8
0.35	0.34	28.6	12.8	103.	66.9	73.0
0.35[b]	0.35	113.	16.1	88.3	70.3	126.
0.35	0.36	275.	21.0	67.8	55.3	153.
0.35	0.37	219.	16.1	50.3	36.3	158.
0.35	0.38	126.	11.4	39.9	22.9	157.
0.35	0.40	52.2	6.90	28.8	7.2	150.
0.35	0.42	24.1	3.44	25.2	0.96	139.
0.35	0.43	17.1	2.06	24.4	1.23	132.
0.35	0.44	13.0	1.01	24.2	2.75	125.
0.40	0.40	51.4	12.1	85.5	58.1	92.4
0.25	0.30	4.7	0.30	27.9	7.1	152.
0.28	0.35	7.2	0.23	29.1	6.8	152.
0.30[c]	0.38	8.5	0.22	28.4	6.4	144.
0.32	0.41	9.3	0.24	26.0	5.5	133.
0.35	0.45	10.3	0.28	24.2	5.0	117.

[a] x_0 and x_1 in μ^{-1}, k in μ, and the cross sections in mb.
[b] Global *PV* symmetry.
[c] Unitary *PS* symmetry.

[24] This is not the only possibility. See the discussion of Sec. 8 of A.

interesting case which falls in the region where a fit can be obtained

$$f_{\Sigma\Sigma}=0, \quad f_{\Lambda\Sigma}=\tfrac{2}{3}\sqrt{3}[2M_N/(M_\Sigma+M_\Lambda)]f_{NN}=0.276,$$
$$x_0=0.30\,\mu^{-1}, \quad x_1=0.38\,\mu^{-1},$$
$$r_0=0.97, \quad r_1=0.16, \quad r_R(\mathrm{H})=0.36, \quad \text{and} \quad r_R(\mathrm{D})=0.037.$$

It must also be remarked that all points on the axis $f_{\Sigma\Sigma}=0$ fit both H and D branching ratios very well.

If the transition matrix elements for reactions (Ia) and (Ib) are taken as equal, the branching ratio for hydrogen is $\simeq 0.2$. (This is the so-called phase-space estimate.) It is perhaps surprising that for $f_{\Sigma\Sigma}=0$ a branching ratio much larger than phase space is obtained. This can be understood as follows: If $f_{\Sigma\Sigma}=0$, the reaction (Ib) is possible only by the one-pion-exchange part of the potential (OPEP), while the reaction (Ia) can only happen by means of the two-pion-exchange potential (TPEP). (We recall that our potentials neglect K mesons and the exchange of three or more pions.) In the singlet states, the OPEP is very weak while the TPEP, especially the crossed graph, is strong. Therefore, the cross section for reaction (Ib) should be much smaller than the cross section for reaction (Ia) and so the ratio for the singlet states, r_0, should be nearly one. These arguments do not hold for the triplet states because here both potentials, OPEP and TPEP are of comparable strength. It is, therefore, clear that relatively large branching ratios for Σ^0 hyperons can be found, despite the fact that $f_{\Sigma\Sigma}=0$.

6. RESULTS FOR ODD Σ PARITY

We shall follow exactly the same approach here as we used for even Σ parity; however, we shall not consider reaction (II). The results for odd Σ parity are shown in Fig. 4, where both the singlet and triplet cores are fixed at $0.35\,\mu^{-1}$. The three lines corresponding to fits to the $^1S_0$ ΛN scattering lengths, a_0, the $^3S_1$ ΛN

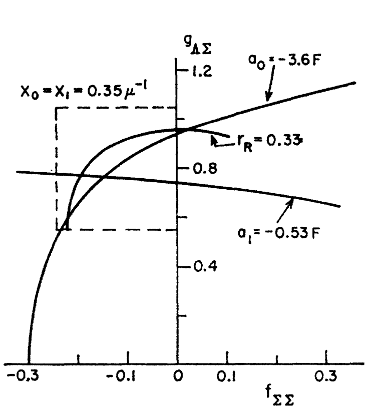
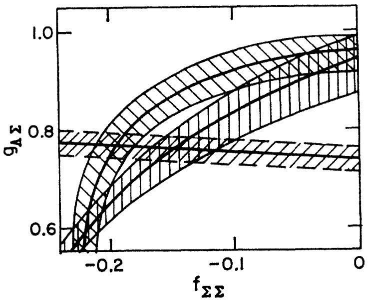


FIG. 4. Odd Σ parity, equal singlet and triplet cores. The curve $r_R=0.33$ shows $g_{\Lambda\Sigma}$ vs $f_{\Sigma\Sigma}$ required to obtain the experimental branching ratio in reaction (I). Also given are $g_{\Lambda\Sigma}$ vs $f_{\Sigma\Sigma}$ for constant $^1S_0$ ΛN scattering length, $a_0=-3.6$ F, and for constant $^3S_1$ ΛN scattering length, $a_1=-0.53$ F. All cores have been put equal to $0.35\,\mu^{-1}$.

FIG. 5. This enlarged portion (dashed square) of Fig. 4 shows the effect of the uncertainties in a_0, a_1, and r_R.



scattering length, a_1, and the branching ratio, $r_R(\mathrm{H})$, have no common point. Even if we allow for the probable error in the experimental quantities (see Fig. 5), there is still no solution, $g_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$, consistent with a_0, a_1, and $r_R(\mathrm{H})$. For completeness, we give the ratios for the point $g_{\Lambda\Sigma}=0.763$, $f_{\Sigma\Sigma}=-0.15$, $x_0=x_1=0.35\,\mu^{-1}$, which fits the ΛN scattering lengths:

$$r_0=0.065, \quad r_1=0.26, \quad r_R(\mathrm{H})=0.21.$$

We note that this calculated ratio, $r_R(\mathrm{H})$, is far outside the experimental errors.

As in the case of even Σ parity, the next logical step is to relax the requirement of equal cores. The results are given in Tables III and IV. In Fig. 6, we show the regions in the $g_{\Lambda\Sigma}-f_{\Sigma\Sigma}$ plane where the ΛN scattering lengths and the branching ratio in hydrogen are fitted. The heavy lines show the points at which $r_R(\mathrm{H})=0.33$ and the bands include the effect of the experimental error in $r_R(\mathrm{H})$.

Here the treatment of the cores is slightly more complicated than in the case of even Σ parity. The cores x_0 and x_1 are determined by the requirement that the

TABLE III. Branching ratios for hydrogen for odd Σ parity.[a]

x_0	x_1	$g_{\Lambda\Sigma}$	$f_{\Sigma\Sigma}$	r_0	r_1	r_R(H)	r_F(H)
0.25	0.25	0.575	−0.090	0.07	0.21	0.18	0.18
0.25	0.30	0.652	−0.046	0.09	0.22	0.19	0.19
0.25	0.35	0.733	0.015	0.15	0.27	0.24	0.24
0.25	0.40	0.820	0.125	0.31	0.46	0.42	0.42
0.25	0.45	0.899	0.293	0.60	0.99	0.89	0.94
0.30	0.25	0.585	−0.154	0.07	0.27	0.22	0.23
0.30	0.30	0.661	−0.116	0.07	0.23	0.19	0.20
0.30	0.35	0.750	−0.065	0.10	0.25	0.21	0.21
0.30	0.40	0.842	0.019	0.18	0.30	0.27	0.27
0.30	0.45	0.930	0.145	0.35	0.43	0.41	0.41
0.35	0.25	0.593	−0.222	0.54	0.36	0.40	0.43
0.35	0.30	0.674	−0.192	0.07	0.28	0.23	0.24
0.35	0.35	0.763	−0.150	0.07	0.26	0.21	0.22
0.35	0.40	0.858	−0.081	0.10	0.28	0.23	0.23
0.35	0.45	0.953	0.010	0.18	0.32	0.28	0.28
0.40	0.25	0.601	−0.306	0.25	0.56	0.48	0.48
0.40	0.30	0.683	−0.274	0.42	0.39	0.39	0.40
0.40	0.35	0.777	−0.235	0.06	0.30	0.24	0.26
0.40	0.40	0.874	−0.175	0.06	0.28	0.23	0.23
0.40	0.45	0.973	−0.103	0.10	0.31	0.25	0.26
0.45	0.25	0.610	−0.388	0.29	0.67	0.57	0.56
0.45	0.30	0.693	−0.364	0.32	0.54	0.48	0.48
0.45	0.35	0.795	−0.327	0.14	0.39	0.33	0.34
0.45	0.40	0.890	−0.280	0.06	0.32	0.25	0.27
0.45	0.45	0.980	−0.220	0.06	0.30	0.24	0.25

[a] x_0 and x_1 in μ^{-1}.

$^1S_0$ and $^3S_1$ ΛN scattering lengths agree with experiment. The ΛN states $^1S_0$ and $(^3S_1+^3D_1)$ are coupled to the $^3P_0$ and $(^1P_1+^3P_1)$ ΣN states, respectively. For the reactions (I), the $\Sigma^- p$ system is in $^1S_0$ or $(^3S_1+^3D_1)$ states and, therefore, the corresponding Λn states are $^3P_0$ or $(^1P_1+^3P_1)$. First, we shall still require that the cores are the same in both ΛN and ΣN channels. Second, we take the cores to be independent of the orbital angular momentum. Thus, we take the core in the singlet S ΣN state the same as the core in the singlet P ΣN state. Of course, the $^1P_1$ ΣN state is coupled to the $^3P_1$ ΣN state and to the $^3S_1+^3D_1$ ΛN states. Therefore, the core appropriate in the $^1S_0$ ΣN state should be the same as the core, x_1, in the $^3S_1$ ΛN state. Similarly, the core in the $^3S_1$ ΣN state is taken the same as the core, x_0, in the $^1S_0$ ΛN state.

If there is a dependence of the cores on the orbital angular momentum, the above reasoning does not hold and the cores present in the $^1S_0$ and $^3S_1$ ΣN states which are important for reaction (I) need not be the same as those determined from the ΛN scattering lengths in the $^3S_1$ and $^1S_0$ ΛN states. This difficulty was not present in the case of even Σ parity.

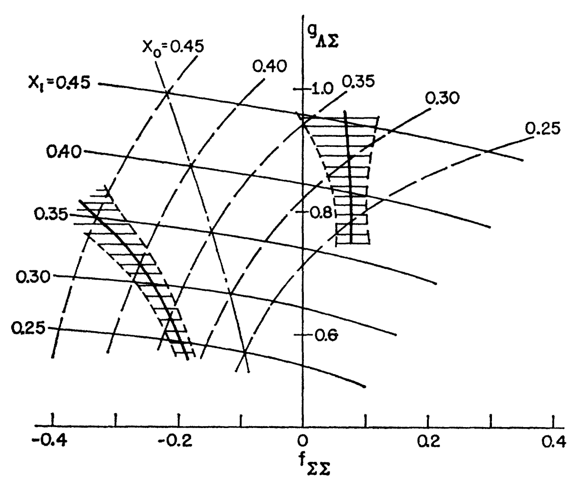


FIG. 6. Odd Σ parity, variable cores. Points which give the experimental branching ratio $r_R(\mathrm{H})=0.33$ in hydrogen fall on either of the two heavy lines. The effect of experimental uncertainty broadens these lines to give the shaded areas. As in the analogous case for even Σ parity, Fig. 3, the cores are chosen so as to obtain the correct ΛN scattering lengths. Values of the cores (given in units μ^{-1}) are indicated by the other lines. (See the discussion of Sec. 6.)

TABLE IV. Cross sections for the reaction (I) for odd Σ parity.[a]

x_0	x_1	$k\sigma_0$ (Ia)	$k\sigma_0$ (Ib)	$k\sigma_1$ ($^3S_1$; Ia)	$k\sigma_1$ ($^1P_1$; Ib)	$k\sigma_1$ ($^3P_1$; Ib)
0.25	0.25	5.0	65.3	18.9	40.2	30.8
0.25	0.30	5.9	56.4	15.4	23.7	30.6
0.25	0.35	8.5	47.0	16.0	13.0	29.6
0.25	0.40	16.9	37.1	24.6	4.3	24.4
0.25	0.45	43.6	29.5	150.	1.5	0.1
0.30	0.25	5.6	70.2	28.8	57.4	22.8
0.30	0.30	4.5	61.1	18.7	37.4	25.8
0.30	0.35	5.4	51.7	15.6	21.4	25.1
0.30	0.40	8.9	41.7	15.2	10.9	25.3
0.30	0.45	17.7	32.8	21.6	4.5	24.1
0.35	0.25	126.	107.	43.7	64.8	13.4
0.35	0.30	4.7	67.3	28.8	55.0	19.2
0.35	0.35	4.0	57.4	20.0	35.1	20.8
0.35	0.40	5.1	47.1	15.5	19.1	21.4
0.35	0.45	8.4	38.0	14.7	10.1	21.5
0.40	0.25	26.7	82.3	61.6	45.2	3.7
0.40	0.30	81.8	113.	45.8	63.0	10.7
0.40	0.35	4.1	64.3	29.5	52.6	16.1
0.40	0.40	3.6	53.0	19.2	31.6	18.5
0.40	0.45	4.6	43.3	15.7	17.6	18.3
0.45	0.25	39.0	93.5	76.8	37.0	1.6
0.45	0.30	40.1	86.3	64.8	52.7	4.1
0.45	0.35	12.4	79.2	46.3	62.4	9.6
0.45	0.40	3.6	61.0	29.9	50.6	14.3
0.45	0.45	2.9	50.4	20.6	32.2	16.0

[a] x_0 and x_1 in μ^{-1}, k in μ, and the cross sections in mb.

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