The passage from (A16) to (A17) follows using the formulas of II, Sec. 4.2.

(d) *The effect of the ω meson.* Treating the ω as the limiting case of the exchange of a narrow Breit-Wigner resonant form, with mass m_ω, we get

$$
{}_{[\omega]}\bar{p}_\alpha{}^{+}(w,y)=\delta\left(y-1-\frac{m_\omega{}^2}{2k^2}\right)\lambda_\alpha(w,y), \qquad {}_{[\omega]}\bar{p}_\alpha{}^{-}(w,y)=0, \tag{A20}
$$

with

$$
\begin{aligned}
\lambda_1(w,y)&=(1/24k^2)(4m^2-m_\omega{}^2-2w)g_{2\omega}{}^2,\\
\lambda_2(w,y)&=(1/6k^2)g_{2\omega}g_{1\omega},\\
\lambda_4(w,y)&=-(1/6k^2)g_{1\omega}{}^2,\\
\lambda_3&=\lambda_5=0,
\end{aligned} \tag{A21}
$$

where $g_{2\omega}$ and $g_{1\omega}$ are the renormalized, rationalized coupling constants of the ω to nucleons using the same type of coupling as in (A16). As mentioned earlier, we have used $g_{1\omega}\sim 2g_{1\rho}$, $g_{2\omega}=0$ in this paper.

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Use of Radiative π Decay to Limit the Neutrino Mass

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A limit on the μ-neutrino mass may be obtained by observing radiative π decay. If one measures the γ-ray energy, then a value within $\frac{3}{4}$ MeV of the maximum possible γ energy is required to limit the neutrino mass to 1 MeV. If one also measures the coincident μ momentum, then the γ energy must be within, say, 3 MeV of the maximum, and the μ momentum within 0.1 MeV/c of its maximum. "Useful" events for both processes occur once in about 10^8 π decays.

THE present limit on the mass of the μ neutrino is 3.6 MeV.[1] Barkas, Birnbaum, and Smith obtained this limit by measuring the μ momentum in ordinary π decay,

$$\pi\rightarrow\mu+\nu.$$

The limit is so poor because the μ momentum is insensitive to the mass of the highly relativistic neutrino. An obvious way to improve this result is to find a reaction which gives little energy to the neutrino. Since there are no known two-body decays with this property, the next best thing is to look at three-body decays with low-energy neutrinos. One such process is radiative π decay,

$$\pi\rightarrow\mu+\nu+\gamma,$$

in the kinematic region with low neutrino momentum. If we assume that, even for finite neutrino mass, the decay coupling is pseudoscalar and $(1-i\gamma_5)$, then the branching ratio, R, of radiative π decay to ordinary π decay has the same formal dependence on momenta and masses as in the zero-mass case:

$$
R=\frac{2\alpha}{\pi}\frac{1}{(1-\mu^2)^2}\int d\epsilon\int dE\left(\frac{p\cdot k}{l\cdot k}+B\frac{(\mathbf{l}\times\hat{k})^2}{(l\cdot k)^2}\right),
$$

where $M_\pi=1=c$; $\mu=$mass of μ; $\nu=$mass of ν; $\alpha=$fine structure constant$=1/137$; $A=\frac{1}{2}(1-\mu^2)=29.80\pm0.04$ MeV/c (see reference 1); $B=\frac{1}{2}(1-\nu^2-\mu^2)$; $k=(k_0,\mathbf{k})$, $p=(p_0,\mathbf{p})$, and $l=(l_0,\mathbf{l})$ are the four-momenta of γ, ν, and μ, respectively; $\hat{k}=\mathbf{k}/|\mathbf{k}|$; $E=p_0$ is the energy of the ν; $\epsilon=A-k_0$ is the difference between the maximum possible γ energy and the given γ energy.[2]

The above expression lacks a term, contributing appreciably near $k=0$, which would cancel the logarithmic infrared divergence. This is permissible because we shall only need the formula in the neighborhood of $|\mathbf{k}|=A$.

A more serious defect is the neglect of all structure in the π meson, as well as possible intermediate boson effects. We may estimate the error from these sources by assuming of Neville's dimensionless form factors[3] that $|h_1|\leqslant 1$ and $|h_2|\leqslant 1$. This results in contributions to the rate for slow neutrinos of about the same size as that computed here from "inner bremmstrahlung" (I.B.) alone. The structure amplitude might interfere destructively with the I.B. amplitude. In that case, the branching ratio for slow neutrinos could be much smaller than my estimate. While such a cancellation seems quite unlikely, it is possible.

If we correct for the intermediate boson alone, assuming it has a mass greater than the K meson, we have $h_1<1/10$, $h_2=0$,[3] which leads to a negligible correction.

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[1] W. H. Barkas, W. Birnbaum, and F. M. Smith, Phys. Rev. **101**, 778 (1956).

[2] This is consistent with Eqs. (2.7) and (2.8) of D. E. Neville, Phys. Rev. **124**, 2037 (1961).

[3] See reference in footnote 2.

To lowest order in ϵ, E, and ν, the branching ratio is

$$R=\frac{2\alpha}{\pi}\frac{1}{(1-\mu^2)^2}\int_{\bar{\nu}}^{\epsilon_{\max}} d\epsilon \int_{E_1}^{E_2} dE\, \frac{2(E-\epsilon)}{1-\mu^2},$$

where

$$\bar{\nu}=\mu\nu,$$
$$E_{2,1}=[(1+\mu^2)\epsilon\pm(1-\mu^2)(\epsilon^2-\bar{\nu}^2)^{1/2}]/2\mu^2.$$

Integrating to $\epsilon_{\max}$, we find

$$R=\frac{4}{3\pi}\frac{\alpha}{1-\mu^2}\frac{1}{\mu^4}(\epsilon^2-\bar{\nu}^2)^{3/2}. \tag{1}$$

This gives the branching ratio for γ rays above a chosen energy, $A-\epsilon$, when ϵ is small. The formula (1) for R is correct to lowest order in ϵ, ν, and $(\epsilon^2-\bar{\nu}^2)^{1/2}$. To that order, it is unchanged by addition of a $(1+i\gamma_5)$ term, even if derivative coupling is used instead of pseudoscalar coupling.

If we ask for the probability, per ordinary π decay, of observing a γ ray within 1 MeV of A, this is

$$R\approx(4/140)(1/140)^3\approx10^{-8}.$$

Such an observation would imply

$$\bar{\nu}\leq 1 \text{ MeV} \quad\text{or}\quad \nu\leq 1.3 \text{ MeV}.$$

The above value for R is confirmed by the following rough estimate:

The total radiative decay of the π is down at least by a factor of 3×10^{-4} from the nonradiative rate.[4] The differential branching ratio is proportional to $p\nu$ and ν, just from the neutrino projection operator for the final state. Assuming a neutrino mass of ≈ 1 MeV, and integrating up to $\epsilon\approx 1$ MeV, we have

$$R\approx(\text{total radiative decay})\ (\text{phase-space fraction})\ [O(p_\nu,\nu)]$$
$$\approx(3\times10^{-4})(\tfrac{1}{2}/\tfrac{1}{2}\times30\times4)(1/140)\approx10^{-8}.$$

In addition to looking at the γ energy, one might try to improve the limit on ν by looking at the μ angle, or energy:

To lowest order, the mass limit is now

$$\nu\leq\frac{1}{\mu}(\epsilon^2-\mu^2x^2)^{1/2},\quad x=\frac{1-\mu^2}{2}\tan\theta_{\gamma\mu}.$$

[4] W. F. Fry, Phys. Rev. **91**, 130 (1953).

Remembering that 1 MeV corresponds to 1/140 in these units, we see that a precision in $\theta_{\gamma\mu}$ of better than a degree is required to get useful information from the angle measurement.

If one could measure the magnitudes of both the μ and γ momenta in coincidence, i.e., if ϵ and $\delta=A-|\mathbf{l}|$ are known, then, to lowest order in all quantities, the limit is

$$\nu^2\leq\frac{4}{(1+\mu^2)^2}[(1+\mu^2)\epsilon\delta-\mu^2\delta^2]-x^2,$$

which is better than the previous case (only ϵ measured) if δ can be measured very well. For example, if we know

$$\epsilon\leq 3 \text{ MeV},\ \delta\leq 0.1 \text{ MeV}/c, \quad\text{and}\quad \theta=0,$$

then $\nu\leq 1$ MeV follows.

Again, a μ-γ angle measurement would improve the limit, but possibly the method of γ detection would be useless for angle determination.

The rate for useful simultaneous energy measurements, 5 in about 10^8 π decays, is slightly higher than before because of the relaxed limits on ϵ. It is only slightly higher because, as ϵ increases, the permissible μ-γ angles must be restricted, lest the neutrino get too much kinetic energy.

I have described two ways for finding a limit on the μ-neutrino mass by observing radiative π decay.

(1) The first way is to measure only the energy of the γ ray, and record the highest value observed. This requires a γ energy measurement within 0.76 MeV of the maximum possible γ energy, 29.80 ± 0.04 MeV, to determine $\nu\leq 1$ MeV. It has a calculated useful rate of 1 in about 10^8 π decays, ignoring all experimental difficulties.

(2) The second way is to measure both the γ and μ momenta simultaneously. Now an accuracy of only a few MeV in E_γ is required, but at the price of 0.1 MeV/c accuracy in the μ momentum. The "useful" rate would now be roughly 5 in 10^8 π decays.

Note that, although there is some uncertainty in the predicted rate because of the form factors, there would be no difficulty with interpreting an observed event, since the resulting neutrino mass limit depends only on kinematics.

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