$$F_1'[\lambda]=\lambda K_1F_1'[\lambda]+K_1F_1[\lambda].$$

The solution of this integral equation for F_1' is

$$F_1'[\lambda]=F_1^2[\lambda].$$

Using this in (A21) gives

$$\left(\frac{d}{d\lambda}\Delta^{-1}[\lambda]\right)_{st}=-\langle\bar{s}|\,(1+\lambda F_1[\lambda])^2|t\rangle,$$

and so (A22) may be written

$$\mathrm{Tr}F[\lambda]=\mathrm{Tr}F_1[\lambda]+\frac{d}{d\lambda}\ln\,\mathrm{Det}\Delta[\lambda]. \qquad \text{(A23)}$$

Using (A23) with (A16) and (A17), we have finally

$$D=D_1/\mathrm{Det}\Delta. \qquad \text{(A24)}$$

Equation (27) is a special case of this general relation.

PHYSICAL REVIEW VOLUME 130, NUMBER 2 15 APRIL 1963

Isotopic Spin in $K\to 3\pi$*

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Because recent data on $K_2^0\to\pi^+\pi^-\pi^0$ are at variance with the $\Delta T=1/2$ rule while the data on $K^+\to 3\pi$ are not, the charge space kinematics of $K\to 3\pi$ are re-examined. Matrix elements are assumed to be at most linearly dependent on the usual variables s_i, and it follows that only four of the seven possible 3π states can contribute to the decay. Of these states, two have $T=1$, the third has $T=2$ and the fourth $T=3$. The possible values of ΔT are $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, and accordingly, the most general interaction Hamiltonian is written as the sum of four parts $H_{n/2}$, each corresponding to $\Delta T=n/2$ ($n=1$, 3, 5, 7). It is then possible to express the matrix elements, rates and spectra of all the modes of $K\to 3\pi$ in terms of the reduced matrix elements of $H_{n/2}$ between the four 3π states and the K meson. The analysis reveals that, provided the branching ratio of $K_2^0\to 3\pi^0$ to $K_2^0\to\pi^+\pi^-\pi^0$ is $\frac{3}{2}$, the present data are consistent with an interaction Hamiltonian containing only $\Delta T=\frac{1}{2}$ and $\frac{3}{2}$, and a 3π final state of isotopic spin one.

INTRODUCTION

RECENT experiments on $K_2^0\to\pi^+\pi^-\pi^0$ indicate that while the slope[1] of the π^0 spectrum may be consistent with the $\Delta T=\frac{1}{2}$ rule,[2] the rate of decay[3] is not.[4] In the case of K^+ decay, however, the rates[3] and spectra[5,6] of the τ and τ' decay modes all seem to be consistent with the predictions of $\Delta T=\frac{1}{2}$.[2,4] Because of this discrepancy, it seems appropriate to give a systematic restatement of the charge space kinematics of $K\to 3\pi$.

Dalitz[4] has shown that the τ to τ' branching ratio depends not on ΔT being $\frac{1}{2}$, but rather on the isotopic spin of the final state being equal to one; and that if the interaction Hamiltonian contains both $\Delta T=\frac{1}{2}$ and $\Delta T=\frac{3}{2}$, the admixture of $\Delta T=\frac{3}{2}$ affects only the relative rates for $K^+\to 3\pi$ and $K_2^0\to 3\pi$. Similarly, Weinberg's relation[2] between the spectra of τ and τ' is, as we shall show below, a consequence only of the final state having $T=1$; and further, as regards the slopes, an admixture of $\Delta T=\frac{3}{2}$ will show up only in the slope of the $K_2^0\to\pi^+\pi^-\pi^0$ spectrum. Hence, even if the $\Delta T=\frac{1}{2}$ rule has to be abandoned, it may still be true that the final state of $K\to 3\pi$ has isotopic spin equal to one. Our analysis shows that such a conclusion is, in fact, consistent with the present data, provided the branching ratio of $K_2^0\to\pi^0\pi^0\pi^0$ to $K_2^0\to\pi^+\pi^-\pi^0$ is assumed to be $\frac{3}{2}$.

THE LINEAR APPROXIMATION

We use the linear approximation, which appears to be in good agreement with the τ and τ' experimental data, and write the matrix element for

$$K^\rho\to\pi_1^\alpha+\pi_2^\beta+\pi_3^\gamma,$$

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[2] S. Weinberg, Phys. Rev. Letters 4, 87, 585 (1960).

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[4] R. H. Dalitz, Rev. Mod. Phys. 31, 823 (1959).

[5] For $\sigma(+\,+\,-)$ see M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and R. D. Tripp, Nuovo Cimento 22, 1087 (1962); also L. T. Smith, D. J. Prowse, and D. H. Stork, Phys. Letters 2, 204 (1962); G. Goldhaber, S. Goldhaber, and T. O'Halloran (private communication).

[6] Our value of $\sigma(0\,0\,+)$ is calculated from the 119 events in the compilation of J. K. Bøggild, K. H. Hansen, J. E. Hooper, M. Scharff, and P. K. Aditya, Nuovo Cimento 19, 621 (1961).

as

$$M(\alpha\beta\gamma)=E(\alpha\beta\gamma)[1+\sigma(\alpha\beta\gamma)(s_3-s_0)]. \qquad (1)$$

Here α, β, γ denote the charges of the pions and

$$s_i=(K-k_i)^2,$$
$$\sum_{i=1}^3 s_i=3s_0=M_K^2+m_1^2+m_2^2+m_3^2. \qquad (2)$$

K, k_i are the four-momenta of the K meson and ith pion, respectively, and M_K, m_i their masses. The labels 3 and γ are reserved for the unlike pion in τ, τ' (i.e., π^- in τ, and π^+ in τ'), and for π^0 in $K_2^0\to\pi^+\pi^-\pi^0$. Note that we are visualizing the decay amplitudes *consistently* in the (linear) momentum representation. For purposes like the one in hand it is much simpler than any angular momentum representation; moreover, relativistic invariance is secured automatically.

To relate the constants $E(\alpha\beta\gamma)$ and $\sigma(\alpha\beta\gamma)$ (which is presumed small compared with unity) to experimental quantities, we write the rate, in suitable units, as

$$\int |M(\alpha\beta\gamma)|^2 dy \approx |E(\alpha\beta\gamma)|^2 \int [1-2M_K T_{\max}\sigma(\alpha\beta\gamma)y]dy,$$

where y is the usual Dalitz variable $(2t_3-1)$, $-1\leqslant y\leqslant +1$, and $T_{\max}$ is the maximum kinetic energy of π_3, i.e.,

$$2M_K T_{\max}=(M_K-m_3)^2-(m_1+m_2)^2.$$

Thus

$$|E(\alpha\beta\gamma)|^2=\frac{\text{observed rate}}{\text{phase space available}}=R(\alpha\beta\gamma),$$

$$\sigma(\alpha\beta\gamma)=-\frac{\text{observed slope}}{2M_K T_{\max}}. \qquad (3)$$

The experimental values[3] of $R(\alpha\beta\gamma)$, in consistent but arbitrary units, viz.,

$$\begin{aligned} R(+\,+\,-)&=4.65\pm0.15,\\ R(0\;\;0\;+)&=1.12\pm0.09,\\ R(+\,-\;\;0\,)&=1.12\pm0.33, \end{aligned} \qquad (4)$$

and of $\sigma(\alpha\beta\gamma)$,[5–7]

$$\begin{aligned} \sigma(+\,+\,-)&=-(5.2\pm0.9)\times10^{-6}\ (\text{MeV})^{-2},\\ \sigma(0\;\;0\;+)&=+(11\pm6)\times10^{-6}\ (\text{MeV})^{-2},\\ \sigma(+\,-\;\;0\,)&=+(13\pm8)\times10^{-6}\ (\text{MeV})^{-2}, \end{aligned} \qquad (5)$$

are to be compared with the predictions of the $\Delta T=\frac{1}{2}$ rule:

$$R(+\,+\,-)=4R(0\,0\,+)=2R(+\,-\,0), \qquad (6)$$

$$2\sigma(+\,+\,-)=-\sigma(0\,0\,+)=-\sigma(+\,-\,0). \qquad (7)$$

While the observed value of $\sigma(+\,-\,0)$ is equal to that of $-2\sigma(+\,+\,-)$ within the rather large experimental errors, the value of $R(+\,-\,0)$ definitely does not fit Eq. (6).

For our analysis, it is necessary to classify the possible 3π final states by their total isotopic spin. In general there are seven such states: one with $T=3$, two with $T=2$, three with $T=1$, and one with $T=0$. From the requirements (i) that the states be symmetric under the interchange of all coordinates (i.e., spatial and isotopic) of any pair of pions; and (ii) that their dependence on the variables s_i be at most linear, it follows that only four of the seven states can contribute to $K\to3\pi$. They are

$$\begin{aligned} |1,T_z(S)\rangle&=(5^{1/2}/3)|((j_1,j_2)0,j_3)1,T_z\rangle\\ &\qquad+\tfrac{2}{3}|((j_1,j_2)2,j_3)1,T_z\rangle, \end{aligned} \qquad (8)$$

$$|3,T_z(S)\rangle=|((j_1j_2)2,j_3),T_z\rangle,$$

and

$$\begin{aligned} |1,T_z(L)\rangle&=\{\tfrac{2}{3}|((j_1,j_2)0,j_3)1,T_z\rangle\\ &\qquad-(5^{1/2}/3)|((j_1,j_2)2,j_3)1,T_z\rangle\}(s_3-s_0)\\ &\qquad+(1/\sqrt{3})|((j_1,j_2)1,j_3)1,T_z\rangle(s_2-s_1), \end{aligned} \qquad (9)$$

$$\begin{aligned} |2,T_z(L)\rangle&=|((j_1,j_2)2,j_3)2,T_z\rangle(s_3-s_0)\\ &\qquad+(1/\sqrt{3})|((j_1,j_2)1,j_3)2,T_z\rangle(s_2-s_1), \end{aligned}$$

where[8]

$$\begin{aligned} |((j_1,j_2)T_a,j_3)T,T_z\rangle &=\sum_{m,\sigma}C_{T_z\sigma,T_z-\sigma}{}^{T,j_3,T_a}\,C_{T_z-\sigma,T_z-\sigma-m,m}{}^{T_a,j_2,j_1}\\ &\qquad\times|\pi_1{}^m\pi_2{}^{T_z-m-\sigma}\pi_3{}^\sigma\rangle. \end{aligned} \qquad (10)$$

The notation $|T,T_z(X)\rangle$ indicates states of total isotopic spin T, z-component T_z that are either independent of the variables s_i ($X\equiv S$) or linearly dependent on them ($X\equiv L$). When using (8), (9), and (10) to compute matrix elements, we adopt the convention that j_1 and j_2 represent the isotopic spins of the like pions in K^+ decay and the charged pions in $K_2^0\to\pi^+\pi^-\pi^0$; j_3 represents the isotopic spin of the unlike pion in K^+ decay and π^0 in $K_2^0\to\pi^+\pi^-\pi^0$.

MATRIX ELEMENTS, RATES, AND SPECTRA

The interaction Hamiltonian that gives rise to $K\to3\pi$ will, in general, be an admixture of $\Delta T=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$, and can be written as

$$H=H_{1/2}+H_{3/2}+H_{5/2}+H_{7/2}, \qquad (11)$$

where $H_{n/2}$ behaves like a $T=n/2$ quantity under rotations in isotopic spin space. We define a set of reduced matrix elements of the $H_{n/2}$ between the states (8), (9) and the $T=\frac{1}{2}$ K-meson doublet:

$$\begin{aligned} \lambda_n&=\langle1(S)\|H_{n/2}\|\tfrac{1}{2}\rangle, &&(n=1,3)\\ \mu_n&=\langle1(L)\|H_{n/2}\|\tfrac{1}{2}\rangle, &&(n=1,3)\\ \nu_n&=\langle2(L)\|H_{n/2}\|\tfrac{1}{2}\rangle, &&(n=3,5)\\ \eta_n&=\langle3(S)\|H_{n/2}\|\tfrac{1}{2}\rangle, &&(n=5,7) \end{aligned} \qquad (12)$$

[7] We compute $\sigma(+\,-\,0)$ directly from the Dalitz plot of the 58 K_2^0 events in reference 1, and thank Dr. Luers for sending us this information. This is preferable to working directly from the spectrum quoted in reference 1, viz., $W(T_3)\propto(1+aT_3)$ with $a=(-0.0171\pm0.0065)$ $(\text{MeV})^{-1}$. This is because only an analysis in the form $W(T_3)=A+B(T_3-\frac{1}{2}T_{\max})$ leads to independent probable errors in A and B. Our value of $\sigma(+\,-\,0)$ is equivalent to an $a=-(0.016_{-0.007}^{+0.004})$ $(\text{MeV})^{-1}$ which more nearly ensures a positive-definite $W(T_3)$ (note that $T_{\max}=53.8$ MeV).

[8] See, for example, M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

where

$$\langle T,T_z(X)|H_{n/2}|\tfrac{1}{2},\rho\rangle = C_{T_z,1/2,\rho}{}^{T,n/2,1/2}\langle T(X)\|H_{n/2}\|\tfrac{1}{2}\rangle \quad (13)$$

for $X\equiv S,L$; $\rho=+\frac{1}{2},-\frac{1}{2}$ correspond to K^+, K^0, respectively. Using these reduced matrix elements we can compute the final state for $K\to 3\pi$ and hence the matrix elements $M(\alpha\beta\gamma)$ of Eq. (1).

For K^+ decay, the final state is

$$a^+(1,S)|1,1(S)\rangle+a^+(1,L)|1,1(L)\rangle +a^+(2,L)|2,1(L)\rangle+a^+(3,S)|3,1(S)\rangle, \quad (14)$$

where $a^+(T,X)$ is the amplitude of the state $|T,1(X)\rangle$, and

$$\begin{aligned} a^+(1,S)&=(\lambda_1-\tfrac{1}{2}\lambda_3);\\ a^+(1,L)&=\mu_1-\tfrac{1}{2}\mu_3;\\ a^+(2,L)&=(\tfrac{3}{4})^{1/2}\nu_3-(1/\sqrt{3})\nu_5;\\ a^+(3,S)&=(\tfrac{2}{3})^{1/2}\eta_5-(\tfrac{3}{8})^{1/2}\eta_7.\end{aligned} \quad (15)$$

The matrix elements for the τ and τ' decay modes are

$$\begin{aligned} M(+\,+\,-)&=[2/(15)^{1/2}]a^+(1,S)+[1/(15)^{1/2}]a^+(3,S)\\ &\quad-(1/\sqrt{3})[a^+(1,L)+a^+(2,L)](s_3-s_0), \quad (16)\\ M(0\ \ 0\ +)&=-[1/(15)^{1/2}]a^+(1,S)\\ &\quad+[2/(15)^{1/2}]a^+(3,S)\\ &\quad-(1/\sqrt{3})[a^+(1,L)-a^+(2,L)](s_3-s_0),\end{aligned}$$

and the rates divided by phase space, and slopes divided by $2M_KT_{\max}$ [see (1) and (3)] are given by

$$\begin{aligned} R(+\,+\,-)&=(1/2!\times 15)|2a^+(1,S)+a^+(3,S)|^2,\\ R(0\ \ 0\ +)&=(1/2!\times 15)|-a^+(1,S)+2a^+(3,S)|^2,\end{aligned} \quad (17)$$

$$\begin{aligned} \sigma(+\,+\,-)&=-\frac{5^{1/2}[a^+(1,L)+a^+(2,L)]}{[2a^+(1,S)+a^+(3,S)]},\\ \sigma(0\ \ 0\ +)&=+\frac{5^{1/2}[a^+(1,L)-a^+(2,L)]}{[a^+(1,S)-2a^+(3,S)]}.\end{aligned} \quad (18)$$

The 2! in (17) is the Bose-Einstein statistical factor for two like pions. Notice that the terms containing (s_2-s_1) in the states $|1,1(L)\rangle$ and $|2,1(L)\rangle$ [see Eq. (9)] do not contribute to the matrix elements for τ, τ'; the reason for this is that in the relevant terms of (9) the like pions (π_1 and π_2) are coupled to a resultant $T=1$, and this state contains neither $\pi_1{}^+\pi_2{}^+$ nor $\pi_1{}^0\pi_2{}^0$.

If the state (14) is pure $T=1$, i.e.,

$$a^+(2,L)=a^+(3,S)=0, \quad (19)$$

then the rates and slopes in (17), (18) satisfy the appropriate relations in (6), (7) for all values of $a^+(1,S)$, $a^+(1,L)$, and hence for all λ_1, λ_3, μ_1, μ_3. In other words, if the interaction Hamiltonian contains no admixtures of $\Delta T=\frac{5}{2}$, $\frac{7}{2}$, the relations between τ and τ' in (6), (7) will be satisfied whatever the admixture of $\Delta T=\frac{1}{2}$, $\frac{3}{2}$ may be. We have thus rederived Dalitz's[4] result for the τ to τ' branching ratio, and have shown that Weinberg's relation[2] between the slopes of the spectra depends only on the 3π final state being $T=1$.

Let us now consider $K_2{}^0$ decay. CP invariance implies that only the $K_2{}^0$ component of K^0 can decay into three pions, and also that the 3π final state cannot contain admixtures of even isotopic spin. Therefore,

$$\begin{aligned}\langle K^0|3\pi\rangle&=(1/\sqrt{2})\langle K_1{}^0+K_2{}^0|3\pi\rangle=(1/\sqrt{2})\langle K_2{}^0|3\pi\rangle, \quad (20)\\ \langle K_2{}^0|2,0(L)\rangle&=0,\end{aligned}$$

and it follows that the final state for $K_2{}^0$ decay is

$$a^0(1,S)|1,0(S)\rangle+a^0(1,L)|1,0(L)\rangle +a^0(3,S)|3,0(S)\rangle, \quad (21)$$

where

$$a^0(1,S)=(\lambda_1+\lambda_3);\quad a^0(1,L)=\mu_1+\mu_3;\quad a^0(3,S)=(\eta_5+\eta_7). \quad (22)$$

The matrix elements for

$$K_2{}^0\to\pi^++\pi^-+\pi^0,$$
$$K_2{}^0\to\pi^0+\pi^0+\pi^0,$$

are then

$$\begin{aligned} M(+\,-\,0)&=[1/(15)^{1/2}]a^0(1,S)+[1/(10)^{1/2}]a^0(3,S)\\ &\quad+(1/\sqrt{3})a^0(1,L)(s_3-s_0), \quad (23)\\ M(0\ \ 0\ \ 0)&=-(\tfrac{3}{5})^{1/2}a^0(1,S)+(\tfrac{2}{5})^{1/2}a^0(3,S),\end{aligned}$$

and the corresponding rates and slopes are

$$\begin{aligned} R(+\,-\,0)&=(1/30)|\sqrt{2}a^0(1,S)+\sqrt{3}a^{(0)}(3,S)|^2,\\ R(0\ \ 0\ \ 0)&=(1/3!\times 5)|-\sqrt{3}a^0(1,S)+\sqrt{2}a^0(3,S)|^2,\end{aligned} \quad (24)$$

$$\begin{aligned} \sigma(+\,-\,0)&=\frac{(10)^{1/2}a^0(1,L)}{[\sqrt{2}a^0(1,S)+\sqrt{3}a^0(3,S)]},\\ \sigma(0\ \ 0\ \ 0)&=0.\end{aligned} \quad (25)$$

The 3! in $R(0\,0\,0)$ is the Bose-Einstein statistical factor for three like pions.

If the 3π final state in $K_2{}^0$ decay is also pure $T=1$, then in addition to (19) we have

$$a^0(3,S)=0, \quad (26)$$

and hence

$$R(0\,0\,0)=\tfrac{3}{2}R(+\,-\,0). \quad (27)$$

Let us now compare the rate and slope for $K_2{}^0\to\pi^+\pi^-\pi^0$ [see (24) and (25)] with those for $K^+\to\pi^0\pi^0\pi^+$ [see (17) and (18)]. Since $a^+(1,S)$ and $a^0(1,S)$ are two different combinations of λ_1, λ_3, and $a^+(1,L)$, $a^0(1,L)$ are two different combinations of μ_1, μ_3 [(12), (15), and (22)], the fact that the final state is pure $T=1$ does not imply anything about the ratio of the rates, or the ratio of the slopes, for these two decays. In order to make a prediction, we must make an assumption about the interaction Hamiltonian itself: If, for example, we assume H to be pure $\Delta T=\frac{1}{2}$, then λ_3, μ_3 will be zero and we would predict

$$\begin{aligned} 2R(0\,0\,+)&=R(+\,-\,0),\\ \sigma(0\,0\,+)&=\sigma(+\,-\,0),\end{aligned} \quad (28)$$

similarly if H is assumed to be pure $\Delta T=\frac{3}{2}$ we would predict

$$8R(0\,0\,+)=R(+\,-\,0),\quad \sigma(0\,0\,+)=\sigma(+\,-\,0). \tag{29}$$

CONCLUSION

We now see that the situation in $K\to 3\pi$ is as follows: By comparing the data on τ decay with the data on τ' we may reasonably conclude that the final state in $K^+\to 3\pi$ is pure $T=1$; similarly for the two modes of $K_2{}^0$ decay [assuming, of course, that the $K_2{}^0\to\pi^0\pi^0\pi^0$ branching ratio is $\frac{3}{2}$; see (27)]. In this way, we can rule out admixtures of $\Delta T=\frac{5}{2}$ and $\frac{7}{2}$ in the interaction Hamiltonian; but in order to establish whether or not H contains $\Delta T=\frac{3}{2}$, we must compare $K^+\to 3\pi$ with $K_2{}^0\to 3\pi$. From such a comparison of the data [(4) and (5)] with the predictions of the $\Delta T=\frac{1}{2}$ rule [(5) and (6)], we see that in fact the admixture of $\Delta T=\frac{3}{2}$ must be nonzero. The appropriate values of the two reduced matrix elements λ_1, λ_3 can be calculated from the known rates of $K^+\to 3\pi$ and $K_2{}^0\to 3\pi$, and the values of μ_1, μ_3 from the known slopes of the spectra.

Our main conclusion, then, is that the present data on $K\to 3\pi$ are consistent with a $T=1$ final state and an interaction Hamiltonian containing only $\Delta T=\frac{1}{2}$ and $\Delta T=\frac{3}{2}$. One important test remaining is the branching ratio of $K_2{}^0\to\pi^0\pi^0\pi^0$ to $K_2{}^0\to\pi^+\pi^-\pi^0$; if, in future experiments, it is shown to be $\frac{3}{2}$ [see (27)], then we can reasonably conclude that $a^0(3,S)$ is zero [see (26)]. If it should differ significantly from $\frac{3}{2}$, then the interaction must involve at least $\Delta T=\frac{5}{2}$, and possibly $\frac{7}{2}$; we would then have to consider seriously the possibility that (4), (5) can be fitted by nonzero values of $a^+(2,L)$, $a^+(3,S)$, $a^0(3,S)$, i.e., that all possible final states and all possible ΔT are realized.

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Strange Particle Production by 4.65-BeV/c π^- Mesons*

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Hydrogen bubble chamber photographs taken in an unseparated 4.65-BeV/c π^- beam at the Brookhaven alternating gradient synchrotron give partial cross sections for many channels involving associated production of $Y+K$ and production of K pairs. Associated production channels total 1.11 mb, K pair 0.57 mb. Most channels involve one or more pions in the final state. Peripheral collisions appear important for such processes. The only resonance clearly observed is K^* with the mass of 895 MeV.

INTRODUCTION

THE production of hyperons and K mesons by high-energy pions has been observed at energies up to 18 BeV,[1] in addition to the more complete data obtained at energies near the associated production threshold. The high-energy experiments have indicated that production of K-meson pairs becomes more com-

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