similarly if H is assumed to be pure $\Delta T = \frac{3}{2}$ we would predict

$$8R(0\ 0\ +) = R(+\ -\ 0),$$
$$\sigma(0\ 0\ +) = \sigma(+\ -\ 0). \tag{29}$$

CONCLUSION

We now see that the situation in $K \to 3\pi$ is as follows: By comparing the data on τ decay with the data on τ' we may reasonably conclude that the final state in $K^+ \to 3\pi$ is pure $T = 1$; similarly for the two modes of K_2^0 decay [assuming, of course, that the $K_2^0 \to \pi^0\pi^0\pi^0$ branching ratio is $\frac{3}{2}$; see (27)]. In this way, we can rule out admixtures of $\Delta T = \frac{5}{2}$ and $\frac{7}{2}$ in the interaction Hamiltonian; but in order to establish whether or not H contains $\Delta T = \frac{3}{2}$, we must compare $K^+ \to 3\pi$ with $K_2^0 \to 3\pi$. From such a comparison of the data [(4) and (5)] with the predictions of the $\Delta T = \frac{1}{2}$ rule [(5) and (6)], we see that in fact the admixture of $\Delta T = \frac{3}{2}$ must be nonzero. The appropriate values of the two reduced matrix elements λ_1, λ_3 can be calculated from the known

rates of $K^+ \to 3\pi$ and $K_2^0 \to 3\pi$, and the values of μ_1, μ_3 from the known slopes of the spectra.

Our main conclusion, then, is that the present data on $K \to 3\pi$ are consistent with a $T = 1$ final state and an interaction Hamiltonian containing only $\Delta T = \frac{1}{2}$ and $\Delta T = \frac{3}{2}$. One important test remaining is the branching ratio of $K_2^0 \to \pi^0\pi^0\pi^0$ to $K_2^0 \to \pi^+\pi^-\pi^0$; if, in future experiments, it is shown to be $\frac{3}{2}$ [see (27)], then we can reasonably conclude that $a^0(3,S)$ is zero [see (26)]. If it should differ significantly from $\frac{3}{2}$, then the interaction must involve at least $\Delta T = \frac{5}{2}$, and possibly $\frac{7}{2}$; we would then have to consider seriously the possibility that (4), (5) can be fitted by nonzero values of $a^+(2,L)$, $a^+(3,S)$, $a^0(3,S)$, i.e., that all possible final states and all possible ΔT are realized.

Strange Particle Production by 4.65-BeV/c π⁻ Mesons*

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Hydrogen bubble chamber photographs taken in an unseparated 4.65-BeV/c π⁻ beam at the Brookhaven alternating gradient synchrotron give partial cross sections for many channels involving associated production of $Y + K$ and production of K pairs. Associated production channels total 1.11 mb, K pair 0.57 mb. Most channels involve one or more pions in the final state. Peripheral collisions appear important for such processes. The only resonance clearly observed is K^* with the mass of 895 MeV.

INTRODUCTION

THE production of hyperons and K mesons by high-energy pions has been observed at energies up to 18 BeV,[1] in addition to the more complete data

obtained at energies near the associated production threshold. The high-energy experiments have indicated that production of K-meson pairs becomes more com-

[1] See, for example, G. Maenchen, W. B. Fowler, W. M. Powell, and R. W. Wright, Phys. Rev. 108, 850 (1957); W. B. Fowler, W. M. Powell, and J. I. Shonle, Nuovo Cimento, 11, 428 (1959); Wang Kang-Ch'ang, Wang Ts'u-Tseng, V. I. Veksler, J. Vrana, Ting Ta-Ts'ao, V. G. Ivanov, E. N. Kladnitskaya, A. A. Kuznetsov, Nguyen Dinh Tu, A. V. Nikitin, M. I. Solov'ev, and Ch'eng Ling-Yen, Soviet Phys.—JETP 13, 323 (1961); Wang Kang-Ch'ang, Wang Ts'u-Tseng, N. M. Virasov, Ting Ta-Ts'ao Kim Hi In, E. N. Kladnitskaya, A. A. Kuznetsov, A. Mikhul, Nguyen Dinh Tu, A. V. Nikitin, and M. I. Solov'ev, ibid. 13, 512 (1961); J. Bartke, R. Bock, R. Budde, W. A. Cooper, H. Filthuth,

Y. Goldschmidt-Clermont, F. Grard, G. R. MacLeod, A. Minguzzi-Ranzi, L. Montanet, W. G. Moorhead, D. R. O. Morrison, S. Nilsson, C. Peyrou, B. W. Powell, J. Trembley, D. Wiskott, I. Bertanza, C. Franzinetti, I. Manelli, V. Silvestrini, G. Brautti, M. Ceschia, and L. Chervosani, Phys. Rev. Letters 6, 303 (1961); CERN HBC and IEP groups, University of Pisa, University of Trieste, *Proceedings of the Aix-en-Provence International Conference on Elementary Particles*, 1961 (Centre d'Etudes Nucléaires de Saclay, Seine-et-Oise, 1961), p. 93; M. di Corato, E. S. Gelsema, A. Minguzzi-Ranzi, J. Belliere, H. H. Bingham, M. Bloch, D. Drijard, J. Hennessy, P. Mittner, A. Orkin-Lecourtois, M. I. Ferrero, C. M. Garelli, M. Vigone, A. Grigoletto, S. Limentani, A. Loria, F. Waldner, C. Baglin, A. Lagarrigue, P. Rancon, A. Rousset, B. de Raad, R. Salmeron, and R. Voss, in *Proceedings of the Aix-en-Provence International Conference on Elementary Particles, 1961* (Centre d'Etudes Nucléaires de Saclay, Seine-et-Oise, 1961), p. 101.

mon than associated production of hyperon and K meson at the highest energies. Information on angle and momentum distributions of the strange particles has been obtained which indicated an important role played by peripheral collisions, but individual events were not identified by kinematic analysis. In the present experiment an intermediate energy was selected—at a beam momentum of 4.65 BeV/c, and an effort was made to identify the final-state particles in each production event.

The data were obtained during the first operation of the BNL 20-in. liquid-hydrogen bubble chamber at the alternating gradient synchrotron (AGS). This first "survey experiment" was intended to serve as a technical test in two respects as well as to provide data concerning high-energy phenomena. The first respect was to investigate beam intensity, timing, background, and facilities needed for bubble chamber operation at the AGS. The second was to see how easily kinematic fitting programs such as KICK could be applied to the processes occurring at such energies.

In this paper we present some results on the production of hyperons and K mesons. Pion-production reactions have been analyzed independently and reported previously.[2]

PROCEDURE

The particles that passed through the chamber were emitted from an aluminum target at 4.7°, collimated, deflected 4° by a 72-in. magnet, and passed through a second collimator. This simplest possible secondary beam involved neither focusing nor mass separation. The beam momentum is 4.65 BeV/c as measured in the chamber, with an expected spread due to collimator apertures of about ±0.10 BeV/c. The measured distribution has a standard deviation of 0.17 BeV/c, which includes the measurement error. The beam consisted primarily of π^-. Čerenkov counter measurements indicated K^- and $\bar{p}$ components of about 1%. These are considered negligible for the purposes of this rough survey. From the total number of interactions and the known total cross section the noninteracting component (presumably μ^-) is estimated to be 14%.

A beam duration of about 0.2 msec was obtained by flipping a very light target through part of the circulating beam with a solenoid, since the electromagnetic rapid beam deflector was not yet available. Beam intensity, controlled by shifting target position, was rather variable, and out of 45 000 pictures about 20 000 provided useful data. The bubble chamber was operated at 25.7°K, with an expansion of somewhat under 1%, so as to give a close bubble spacing, but not continuous tracks, for particles of minimum ionization.

In scanning the film, pictures having many tracks were handled in the following way: Any events that were noticed were recorded, but detailed scanning was not required of pictures with more than 50 tracks. They were not used for cross-section determinations. The fraction of the pictures which were rescanned was 0.49. Measurement was made on projectors with digitized stage motion and automatic track centering.

Geometrical reconstruction of tracks was carried out by the BNL TRED program on the 7090 computer. Kinematic fitting of events was performed by the BNL version of KICK. For each event type, mass hypotheses were tried for each of the many possible final states. As an example, those tried for two-prong interaction with V^0 (type 112) are listed below:

$$
\begin{aligned}
\pi^- + p &\rightarrow \Lambda^0 + \pi^- + K^+, \\
&\rightarrow \Lambda^0 + \pi^- + K^+ + (\pi^0), \\
&\rightarrow \Lambda^0 + \pi^- + \pi^+ + (\theta^0), \\
&\rightarrow \Sigma^0 + \pi^- + K^+, \\
&\rightarrow \theta^0 + \pi^- + \pi^+ + (\Lambda^0), \\
&\rightarrow \theta^0 + \pi^- + \pi^+ + (\Sigma^0), \\
&\rightarrow \theta^0 + K^- + p, \\
&\rightarrow \theta^0 + K^- + p + (\pi^0), \\
&\rightarrow \theta^0 + \pi^- + p + (\theta^0), \\
&\rightarrow \theta^0 + K^- + \pi^+ + (n), \\
&\rightarrow \theta^0 + \pi^- + K^+ + (n).
\end{aligned}
\tag{1}
$$

In Eq. (1) the particles in parenthesis are inferred from the kinematic fit, but are not observed directly.

The primary criterion for the identification of events was the value of χ^2 obtained by KICK for that final state. The probability of obtaining an equal or greater value of χ^2 was calculated for both decay and production fits. The product of these two probabilities was used as a figure of merit, which was required to be ≥ 0.001 for an acceptable fit. Where two or more acceptable fits were found, density of ionization was used to distinguish between them whenever possible, either by visual estimate or gap length measurement. In the absence of other deciding information, the event was identified as having the final state giving the highest figure of merit. In many cases, however, another final state also gave an acceptable fit, so that the identification is not certain.

Both neutral and charged V's of relativistic velocities can be difficult to identify. V^0's that fit as either Λ^0's or θ^0's were considered to be Λ^0's because Λ^0's can often be fitted as θ^0's but the opposite is not often the case.[3] In the case of charged decays the track of the decaying particle is often so short that a meaningful momentum measurement is impossible. In such cases KICK is

[2] N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters **9**, 139 (1962); B. A. Munir and G. T. Zorn (private communication).

[3] If the ambiguous cases are fitted as θ^0's one finds that in the rest system of the θ^0 the π^+ always has to be emitted forwards and the π^- backwards. Adding these cases to the identified cases would give an asymmetrical distribution that is impossible for a spin 0 particle. The same effect shows up in laboratory momenta. For θ^0's the laboratory momentum distribution should be the same for π^+ and π^-, but if the ambiguous V^0's are called θ^0's the π^+ has more high momenta than the π^- (because the ambiguous cases are really Λ^0's, and there the proton usually has much higher laboratory momentum than the π^-).

programmed to ignore the momentum, and the decay fit becomes zero constraint, for which a $K^{\pm}$ solution is almost always possible if a $\Sigma^{\pm}$ solution is possible. In such cases, however, the distribution of path lengths to decay is not uniform, as would be the case for a long-lived particle, so they cannot, in general, be K's.[4] Consequently, any charged V that could be fitted as a Σ is assumed to be identified as Σ, even if a K fit is also possible.

PARTIAL CROSS SECTIONS

Events to be used for partial cross sections were required to be in a fiducial region with a length of 23.5 cm located so that secondary tracks would be long enough to give measurements of fair accuracy. A count of beam tracks using the biased-sample method of Crawford[5] gave a path length, corrected for beam contamination, of 710 kg/cm² of hydrogen.

A number of corrections must be made to the observed number of events of any given final state. Not all events are found in scanning, and the great variation in beam intensity made these photographs difficult to scan. Based on repeated scanning, our estimate of efficiency is 80%. Not all events found could be identified by the fitting procedure used.[6] This identification efficiency is estimated at 95% for events with neutral decays and 65% for events with charged decays. The effects of neutral decay modes, and decays too near the point of production, or too far away, or at too small an angle are combined in an average detection probability for a given type of particle, and the inverse of this probability used as a weighting factor. These probabilities were as given in Table I. In the case of Σ^{+} the efficiency of detection for the decay mode $\Sigma^{+} \rightarrow p+\pi^{0}$ is very sensitive to the losses of small angle decays, which are hard to evaluate. Consequently, only the decay mode $\Sigma^{+} \rightarrow \pi^{+}+n$ was used for cross-section determinations, and the 50% that are proton decays considered unobservable.

TABLE I. Average probability of observing decay.

Unstable particle	Probability
Λ^{0}	0.58
θ^{0}	0.27
Σ^{+} (π^{+} decay only)	0.42
Σ^{-}	0.82
K^{+}	0.045
K^{-}	0.040

[4] A plot of l_d/l_p, where l_d is the length to point of decay and l_p the potential path in the chamber should be approximately uniformly distributed from 0 to 1 for charged K's, but the actual distribution for all decays that fit best kinematically as K's shows some peaking for small values—though less pronounced than for Σ's.

[5] F. S. Crawford, Rev. Sci. Instr. **30**, 1096 (1959).

[6] We consider these to be genuine events in which the failure to fit is due to a track with distortion or undetected scattering, short track with no useful momentum measurements, failure to converge in calculation, and similar causes.

TABLE II. Partial cross sections for strange particle production channels.

Channel	Number of events used	σ (mb)
$(\Lambda^{0}$ or $\Sigma^{0})+\theta^{0}$	8	0.04
$(\Lambda^{0}$ or $\Sigma^{0})+\theta^{0}+\pi^{0}$	6	0.12
$(\Lambda^{0}$ or $\Sigma^{0})+\theta^{0}+$neutrals	11.8ᵃ	0.09
$(\Lambda^{0}$ or $\Sigma^{0})+K^{+}+\pi^{-}$	6	0.03
$(\Lambda^{0}$ or $\Sigma^{0})+K^{+}+\pi^{-}+\pi^{0}$	24	0.13
$(\Lambda^{0}$ or $\Sigma^{0})+\theta^{0}+\pi^{+}+\pi^{-}$	34	0.20
$(\Lambda^{0}$ or $\Sigma^{0})+$ch$+$neutrals	16	0.10
$(\Lambda^{0}$ or $\Sigma^{0})+\pi^{-}+\pi^{-}+\pi^{+}+K^{+}$	1	0.01
$(\Lambda^{0}$ or $\Sigma^{0})+\pi^{-}+\pi^{-}+\pi^{+}+K^{+}+\pi^{0}$	5	0.05
$(\Lambda^{0}$ or $\Sigma^{0})+\pi^{-}+\pi^{-}+\pi^{+}+\pi^{+}+\theta^{0}$	2	0.02
$(\Lambda^{0}$ or $\Sigma^{0})+\pi^{-}+K^{-}+\pi^{+}+K^{+}+\theta^{0}$	1	0.01
$\theta^{0}+\bar{\theta}^{0}+n$ $\theta^{0}+\bar{\theta}^{0}+n+(\pi^{0}$ or π^{0}'s$)$	28.6ᵃ	0.19
$\theta^{0}+K^{-}+p$	2	0.02
$\theta^{0}+K^{-}+p+\pi^{0}$	8	0.09
$\theta^{0}+\bar{\theta}^{0}+p+\pi^{-}$	2	0.02
$\theta^{0}+K^{-}+n+\pi^{+}$	8	0.09
$\bar{\theta}^{0}+K^{+}+n+\pi^{-}$	6	0.07
$\theta^{0}+K^{-}+p+\pi^{+}+\pi^{-}$	1	0.02
$\theta^{0}+\bar{\theta}^{0}+p+\pi^{+}+\pi^{-}+\pi^{-}$	2	0.03
$\theta^{0}+K^{-}+p+\pi^{-}+\pi^{-}+\pi^{0}$	2	0.04
$\Sigma^{+}+\theta^{0}+\pi^{-}$	3	0.03
$\Sigma^{-}+\theta^{0}+\pi^{+}$	7	0.04
$\Sigma^{-}+K^{+}+\pi^{0}$	10	0.06
$\Sigma^{+}+$ch$+$neutrals	3	0.03
$\Sigma^{-}+$ch$+$neutrals	4	0.02
$\Sigma^{-}+K^{+}+\pi^{+}+\pi^{-}$	4	0.02
$\Sigma^{+}+K^{+}+\pi^{-}+\pi^{-}+\pi^{0}$	1	0.01
$\Sigma^{+}+\theta^{0}+\pi^{+}+\pi^{-}+\pi^{-}$	2	0.02
$\Sigma^{-}+K^{+}+\pi^{+}+\pi^{-}+\pi^{0}$	7	0.04
$\Sigma^{-}+\theta^{0}+\pi^{+}+\pi^{+}+\pi^{-}$	3	0.02
$\Sigma^{+}+3$ ch$+$neutrals	1	0.01
$K^{+}+K^{-}+n$	2	0.11
$K^{+}+K^{-}+p+\pi^{-}$	1	0.05
$K^{+}+K^{-}+n+\pi^{+}+\pi^{-}$	1	0.05

ᵃ These fractional numbers are deduced by a subtraction process.

The probabilities were considered to be independent for the two particles in order to estimate the *a priori* probability of observing one or two decays from a particular final state. In the case of states involving two K mesons, correlations between the modes of decay presumably do exist,[7] but these data are not adequate to determine the correlations, so have assumed them independent as a simplifying approximation.[8]

We were not able to make a reliable distinction between Λ^{0} and Σ^{0}, since the energy and momentum carried by the γ ray from Σ^{0} decay are small compared with those of other secondary particles in 4.65-BeV/c collisions.

Estimates of the partial cross sections corrected for scanning efficiency, identification efficiency, and probability of observation are given in Table II.

Here the channels $\theta^{0}+\bar{\theta}^{0}+n$, $\theta^{0}+\bar{\theta}^{0}+n+\pi^{0}$, and $\theta^{0}+\bar{\theta}^{0}+n+(\geq 2\pi^{0})$ are simply grouped together because most of them are observed as zero-prong interactions with an associated θ^{0}, which are not fittable by KICK.

[7] M. Goldhaber, T. D. Lee, and C. N. Yang, Phys. Rev. **112**, 1796 (1958); A. R. Erwin, G. A. Hoyer, R. H. March, W. D. Walker, and T. P. Wangler, Phys. Rev. Letters **9**, 34 (1962).

[8] For example, in the process $\pi^{-}+p \rightarrow n+\theta^{0}+\bar{\theta}^{0}$, the probability of observing both decays is taken to be 0.27×0.27.

Three obvious features of Table II are the following: (1) There are so many channels that no one of them has enough cases to permit detailed study; (2) practically all strange particle production events involve one or more π mesons in the final state; and (3) production of K-meson pairs is comparable with that for hyperon+K meson—0.57 mb compared with 1.11 mb, when summed from Table II.

A general confirmation of this last observation can be obtained from the total numbers of Λ^0's and θ^0's observed (without restriction to fiducial region or beam intensity) as given in Table III. Those cases that fit as either Λ^0 or θ^0 were apportioned as being 11 θ^0 and 76 Λ^0 so as to make a plot of $(p^+-p^-)/(p^++p^-)$ symmetrical for θ^0's, when p^+ is the momentum of the positive prong
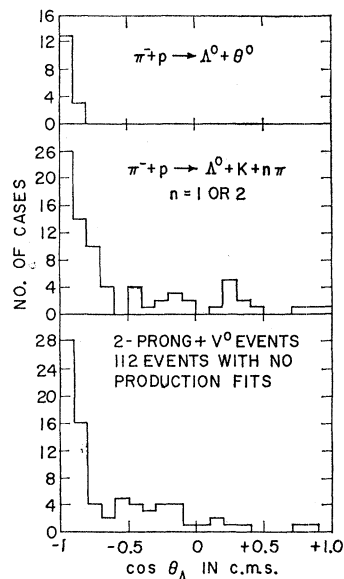

FIG. 1. Distribution of production angles (in the c.m. system) for Λ^0's. The top histogram is for $\Lambda^0+\theta^0$ events, the middle are for events with one or two π's determined by kinematic fits, and the bottom for events having no production fits, and hence additional π^0's.

and p^- that of the negative. The corrected numbers show an excess of θ^0's (585) over Λ^0's (343) that is a qualitative measure of the frequency of K-pair production.

In addition to the channels listed in Table II three examples of production of Ξ^- were observed, of which only one was in a picture used for the cross-section estimate, which was identified as $\Xi^-K^+\theta^0$. Corrections are hard to evaluate, but a rough estimate yields a partial cross section of 0.01–0.02 mb.

TABLE III. Total numbers of V^0 decays.

	Fit as Λ^0	Fit as θ^0	Fit as either
Observed	123	147	87
With ambiguous cases apportioned	199	158	$\cdots$
Corrected for average probability of detection	343	585	$\cdots$

FIG. 2. Distribution of production angles (in the c.m. system) for nucleons in events that involve production of a pair of K mesons. The 3-body states consist of nucleon plus K pair, while the 4-body states also have a π meson.
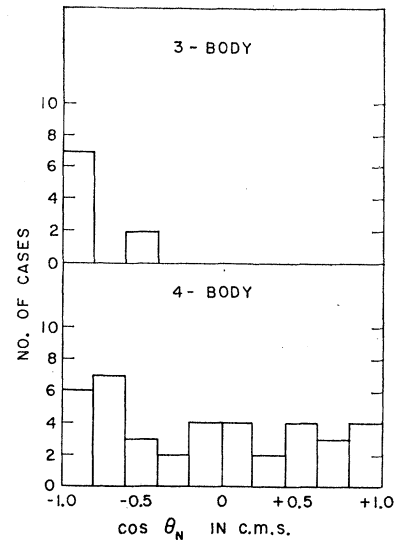
ANGULAR DISTRIBUTIONS

Previous experiments have shown that Λ^0's are emitted backwards in the center-of-mass system, corresponding to a low momentum transfer to the baryon. The effect is strongly evident in the Λ^0 angular distributions in Fig. 1. When one or more π mesons are produced, however, some of the events do have Λ^0's emitted at large angles, which would be understandable if collisions with large momentum transfer result in pion emission in addition to strange particles.

A similar comparison can be made for K-pair production, although the number of cases is very small. In Fig. 2, the nucleon angular distributions show the familiar backward peak for 3-body processes (KKN), but for 4-body processes ($KKN\pi$) the nucleon distribution shows no significant difference from isotropy. The corresponding forward peaking for K's from 3-body
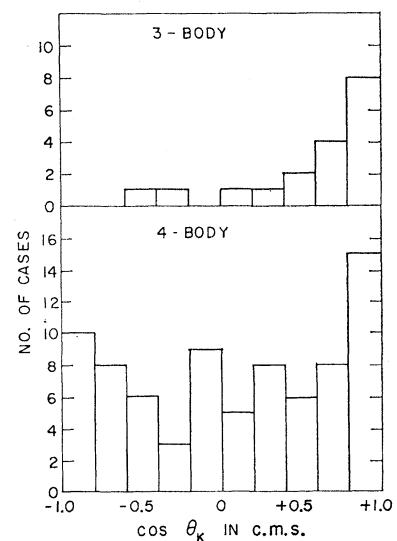

FIG. 3. Distribution of production angles (in the c.m. system) for K mesons in events that involve a pair of K mesons.

states, and isotropic distribution for those from 4-body states, are shown in Fig. 3. Apparently, peripheral collisions produce K pairs alone, while those that are more central have larger momentum transfer to the nucleon and also emit a π meson.

A number of effective mass distributions have been plotted for various two- and three-body combinations. Because of the small number of events available in any given reaction, most of such mass distributions cannot yield very meaningful information on resonant states. Whenever possible, however, the same mass combinations in the same isospin state were added to improve the statistics. Figure 4 shows the $K\pi$ mass distributions obtained by adding the $K\pi$ combinations from the following reactions:
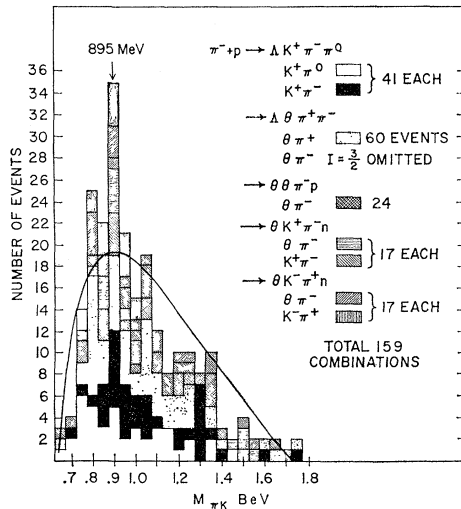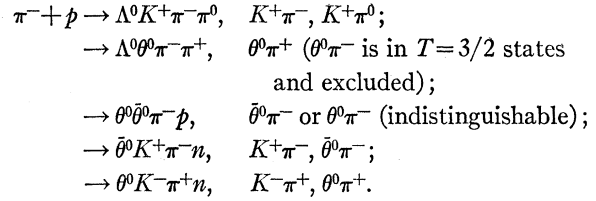
$$\pi^- + p \rightarrow \Lambda^0 K^+ \pi^- \pi^0, \quad K^+\pi^-, \ K^+\pi^0;$$
$$\rightarrow \Lambda^0 \theta^0 \pi^- \pi^+, \quad \theta^0\pi^+ \ (\theta^0\pi^- \text{ is in } T=3/2 \text{ states}$$
$$\text{and excluded});$$
$$\rightarrow \theta^0\bar{\theta}^0\pi^- p, \quad \bar{\theta}^0\pi^- \text{ or } \theta^0\pi^- \text{ (indistinguishable)};$$
$$\rightarrow \bar{\theta}^0 K^+ \pi^- n, \quad K^+\pi^-, \ \bar{\theta}^0\pi^-;$$
$$\rightarrow \theta^0 K^- \pi^+ n, \quad K^-\pi^+, \ \theta^0\pi^+.$$

The phase space drawn is a composite of phase spaces for the above reactions normalized to the total number of events. The formation of K^* at a mass of 895 MeV is indicated. The width of this resonance is probably less than 50 MeV.

The similar plots were made for $\Lambda^0\pi^\pm$, $\theta^0 K^\pm$, $\theta^0\theta^0$, and K^+K^-, but no marked deviations from phase space were observed.

CONCLUSIONS

At this energy the production of K-meson pairs has a cross section of about $\frac{1}{2}$ mb (not including K^+K^- pairs), while the associated production of hyperon and K meson has a cross section of about one millibarn. A great many individual channels participate, most of which include one or more π mesons in the final state. In general the baryons show strong backward peaking in the c.m. system indicative of a peripheral collision. This effect is less pronounced for $K+K$ production than for $Y+K$, and becomes less as pion multiplicity increases, showing that there is a higher pion multiplicity in the more central collisions. K^* in $T=1/2$ state seems to be formed. There are indications for Y_1^* and Y_0^*, but statistics are not good enough to confirm their formation.

FIG. 4. Distribution of effective masses for $K\pi$ combinations from $\Lambda^0\theta^0\pi^+\pi^-$, $\Lambda^0 K^+\pi^-\pi^0$, $\theta^0\bar{\theta}^0\pi^- p$, $\bar{\theta}^0 K^+\pi^- n$, and $\theta^0 K^-\pi^+ n$ final states.