Calculations of Energy Spectra of Nuclei in the 2s, 1d Shell*

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Using the shell-model classification scheme based on the group $SU3$, the energy spectrum of Mg^{24} is studied in detail. Using only the lowest $SU3$ state it is not possible to obtain a good fit to the experimental data. A variational method is introduced within the framework of the $SU3$ scheme which admixes higher states. This yields good agreement between theory and experiment.

INTRODUCTION

THE group $SU3$ has recently been introduced as a basis for shell-model calculations by Elliott.[1] He has shown how shell-model states which have the properties of rotational motion can be constructed. A detailed explanation of the operator techniques required for the use of $SU3$, with particular emphasis on the $2s$, $1d$ shell, has been given by Banerjee and Levinson[2] (hereafter referred to as I). In this paper we shall adopt the notation and terminology of I. Using the same kind of techniques as those described in I, Koltun has examined the $1p$ shell in detail.[3]

The present work deals with the calculation of the energy levels of some nuclei in the $2s$, $1d$ shell, and the interpretation of the level spectra which result. The nuclei Ne^{20} and Mg^{24} are studied in detail. One important conclusion is that calculations of low-lying spectra using zero-order $SU3$ wave functions alone, neglecting mixing of higher $SU3$ states, yield moments of inertia which are larger than observed from experiment. Our physical interpretation of this result is that the zero-order states are *too* deformed and that some less deformed states must be admixed into the wave functions in order to produce agreement with experiment. The means chosen to produce this mixing is a variational procedure similar to that used by Feingold.[4]

In Sec. I we review briefly the techniques required to calculate the matrix elements of the Hamiltonian in the $SU3$ representation. In Sec. II we carry out the explicit evaluation of the matrix elements for Mg^{24}, set up the energy matrix, and calculate the energy levels. In Sec. III we discuss the variational procedure. We treat the case of Ne^{20} in detail in order to illustrate the validity of this approach. Section IV deals with the detailed application of the variational approach to Mg^{24}. Our conclusions are given in Sec. V.

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I. CALCULATION TECHNIQUES

As discussed in I, the wave functions used in the evaluation of the matrix elements are the projected functions:

$$\Psi_M{}^{J,L,S}=P_M{}^J\Phi([f](\lambda\mu)K\epsilon_{\max}:S\sigma). \quad (1)$$

$P_M{}^J$ is a projection operator which projects out that part of the totally antisymmetric intrinsic state $\Phi([f](\lambda\mu)K\epsilon_{\max}:S\sigma)$ with total angular momentum J and which changes the eigenvalue of J_z to M. The intrinsic state Φ describes an axially symmetric rotator composed of independent particles in a deformed well. It is specified by the space symmetry $[f]$, and the $SU3$ representation $(\lambda\mu)$ to which it belongs. K is the projection of the orbital angular momentum on the symmetry axis and $\epsilon_{\max}$ is the quadrupole moment of the intrinsic state. σ is the projection of the spin angular momentum on the symmetry axis. In this representation, $K+\sigma=\tau$ is the band quantum number (ordinarily called K in most discussions) corresponding to the projection of J on the body fixed symmetry axis. τ takes on all values consistent with $K+\sigma\geq 0$, where

$$\pm K=\min(\lambda\mu),\quad \min(\lambda\mu)-2,\ \cdots 0 \text{ or } 1,$$
$$\pm\sigma=S,\quad S-1,\ \cdots 0 \text{ or } \tfrac{1}{2}.$$

Since we consider only $S=0$ systems in the cases of Ne^{20} and Mg^{24}, only the spin-independent wave functions

$$\Psi_M{}^L=P_M{}^L\Phi([f](\lambda\mu)K\epsilon_{\max}) \quad (2)$$

need be considered.

For the case $S=0$, the secular equation for the eigenvalues E assumes the form of Eq. (C32) of I.

$$\mathrm{Det}\{[(\lambda'\mu')K'|L|(\lambda\mu)K]-E\delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{KK'}\}=0. \quad (3)$$

As discussed in I, the coefficients $[(\lambda\mu)|L|(\lambda'\mu')K']$ are defined in the equation

$$HP_M{}^L\Phi_{\epsilon K}{}^{\lambda\mu}=\sum_{\lambda'\mu'K'}[(\lambda\mu)K|L|(\lambda'\mu')K']P_M{}^L\Phi_{\epsilon K'}{}^{\lambda'\mu'}, \quad (4)$$

where we have introduced a shorter notation for $\Phi([f](\lambda\mu)K\epsilon_{\max})$. Since the representation spanned by the $P_M{}^L\Phi_{\epsilon K}{}^{\lambda\mu}$ is not orthogonal, the matrix elements so defined are not equal to the usual integrals $\langle P_M{}^L\Phi_{\epsilon K}{}^{\lambda\mu}|H|P_M{}^L\Phi_{\epsilon' K'}{}^{\lambda'\mu'}\rangle$ used with orthogonal sets

TABLE I. $[f]$ and $(\lambda\mu)$ symmetry for Mg^{24}. The value of each $9C$ unit is -0.21121 MeV; the value of each P^x unit is -1.05288 MeV. ΔP^x is the energy difference from the lowest state due to the P^x term, $9\Delta C$ is the energy difference from the lowest state due to the $9C$ term, and ΔE is the total energy difference.

$[f]$	$(\lambda\mu)$	ϵ_{max}	P^x (eigenvalue)	$9C(\lambda\mu)$	ΔP^x (MeV)	$9\Delta C$ (MeV)	ΔE (MeV)
[44]	(84)	20	8	148	0	0	0
	(73)	17	8	109	0	8.24	8.24
	(81)	17	8	100	0	10.14	10.14
[431]	(92)	20	4	136	4.21	2.53	6.74
	(65)	17	4	124	4.21	5.07	9.28
	(73)	17	4	109	4.21	8.24	12.45
	(81)	17	4	100	4.21	10.14	14.35

of functions. If we write an eigenvector of H as

$$\Psi^{L,M}=\sum_{\lambda'\mu'K'} A^L(\lambda'\mu'K')P_M{}^L\Phi_{\epsilon'K'}{}^{\lambda'\mu'}, \quad (5)$$

where the $P_M{}^L\Phi_{\epsilon K}{}^{\lambda\mu}$ form a complete set, then the amplitudes $A^L(\lambda'\mu',K')$ must satisfy the equation

$$\sum_{\lambda'\mu'K'} A^L(\lambda'\mu',K')[(\lambda'\mu')|L|(\lambda\mu)K]=EA^L(\lambda\mu,K) \quad (6)$$

and (3) follows immediately.

As may be recalled from Sec. C of I, the evaluation of the elements $[(\lambda'\mu')K'|L|(\lambda\mu)K]$ involves a calculation of the quantities $C_\epsilon{}^{\lambda\mu K}$, $a_i{}^{\lambda\mu K}$, and $b_i{}^{\lambda\mu K}$ which appear in Eq. (C15) of I. These must be evaluated separately for each nucleus considered, with the help of the table of inhomogeneous matrix elements (Table IV-1, of I).

If we assume that the shell-model wave functions can be well approximated by one $(\lambda\mu)$ symmetry, then we can restrict our attention to the submatrix:

$$[(\lambda\mu)K'|L|(\lambda\mu)K], \quad (7)$$

where $(\lambda\mu)$ is fixed and only K varies. The choice of the dominant $(\lambda\mu)$ symmetry is determined by an examination of the matrix elements:

$$\langle\Phi_{\epsilon K}{}^{\lambda\mu}|H|\Phi_{\epsilon K}{}^{\lambda\mu}\rangle.$$

In order to evaluate these approximately for the purpose of finding the lowest $(\lambda\mu)$, it is sufficient to consider only the contribution of the operators P^x and C in the expansion [cf., (A13) in I] of H. Once a lowest symmetry is found, the problem reduces to diagonalization of the submatrix shown above in (7). The matrix elements depend explicitly on the parameters of the original Hamiltonian. In general, the dimension of the submatrix s quite small. Explicit calculations in the lowest $(\lambda\mu)$ symmetry yield very small off-diagonal elements. This means that to a good approximation there is very little band mixing in the *sd* shell and that the eigenvalues are approximately given by the form:

$$a(K)+b(K)L(L+1)+c(K)L^2(L+1)^2\simeq E_{KL}.$$

II. Mg^{24} IN THE SINGLE $(\lambda\mu)$ SYMMETRY LIMIT

Let us consider the detailed calculation of the energy levels of Mg^{24}. We will use the Serber exchange two-body potential discussed in I. It has the form:

$$V=V_0\frac{e^{-r/a}}{r/a}\frac{(1+P^x)}{2}, \quad (8)$$

where we take $V_0=-45$ MeV and $a=1.37\times10^{-13}$ cm. A harmonic oscillator length parameter $b=1.64\times10^{-13}$ cm is used to define the shell-model radial wave functions. We wish to restrict our lowest order calculation to one $(\lambda\mu)$ representation. Our criterion for choosing this configuration is that it be the one with the lowest energy expectation value obtained by considering the matrix element:

$$\langle\Phi_{\epsilon K}{}^{\lambda\mu}|\sum_{i<j} V(r_{ij})|\Phi_{\epsilon K}{}^{\lambda\mu}\rangle.$$

The two-body interaction $V(r_{ij})$ can be expanded exactly in terms of ten operators. This expansion is carried out in I and the results appear in (A13) of I. For our purposes it is sufficiently accurate to consider only the contributions of the Casimir operator C and the Majorana operator P^x to the matrix element in (3) above. The lowest states naturally have the highest space symmetry, so we only consider the two highest space symmetries [44] and [431], with P^x eigenvalues of 8 and 4, respectively. In Table I are listed the low lying symmetries $(\lambda\mu)$ within each of the two lowest space symmetries, along with the eigenvalues of P^x and C. We are approximating our Hamiltonian by

$$H\simeq-0.21121(9C)-1.05288P^x. \quad (9)$$

The numbers in the expression above come from (A13) of I. The final approximate energies for each band labeled by $[f](\lambda\mu)$ appear in Table I.

We see that the $(\lambda\mu)$ symmetry (84) with space symmetry [44] is the lowest configuration by 6.74 MeV, giving a clear choice for the lowest state. The situation is not always so straightforward. In Al^{25}, for example, there are three competing $(\lambda\mu)$'s arising from two space symmetries, which are very closely spaced.

In addition to the two-body force, one must, of course, consider the single-particle spin-orbit force of the form $a\sum_i \mathbf{l}^i\cdot\mathbf{s}^i$. If one considers only the [44] space symmetry which is a pure spin singlet then this force has no effect. When we consider admixtures of higher lying

TABLE II. Energy matrix for the [44], (84) symmetry in Mg^{24}. All entries must be multiplied by 10^{-2} MeV.

	$K=0$	$K=2$	$K=4$
$K=0$	$-15.78-13.17L(L+1)$ $+0.0779[L(L+1)]^2$	$0.1087[1+(-)^L][6(L-1)L(L+1)(L+2)]^{1/2}$	0
$K=2$	$0.3187[6(L-1)L(L+1)(L+2)]^{1/2}$	$-137.27-12.17L(L+1)+0.2300[L(L+1)]^2$	$-0.3756[(L-3)(L-2)(L+3)(L+4)]^{1/2}$
$K=4$	0	$0.8325[(L-3)(L-2)(L+3)(L+4)]^{1/2}$	$-529.90-9.246L(L+1)$ $+0.0005[L(L+1)]^2$

symmetries, however, we must not neglect this force.

Using the potential (8) it is possible, using the techniques described in I, to compute the submatrix (7) where $(\lambda\mu)$ is fixed at (84) and $[f]$ is fixed at [44]. The effect of a Hamiltonian H operating on a state has been shown in I to be characterized by parameters, $C_\epsilon^{\lambda\mu K}$, $a_i^{\lambda\mu K}$, and $b_i^{\lambda\mu K}$, [c.f., Eq. (C15) of I]. The contributions from the $b_i^{\lambda\mu K}$ coefficients (which come from the $\Delta\epsilon=12$ part of the force) are so small relative to the contributions from the $a_i^{\lambda\mu K}$ coefficients that they can be neglected except for the $L^2(L+1)^2$ term. The final submatrix is given in Table II where the L dependence of the matrix elements is given explicitly. We note that:

(1) The L dependence of the diagonal elements is of the form

$$A+BL(L+1)+CL^2(L+1)^2.$$

(2) The off-diagonal matrix elements are quite small and give rise to small K-band mixing.

This matrix was diagonalized for each L and the eigenspectrum is shown in Fig. 1. A comparison with the experimental results which are also shown in Fig. 1 indicates that the lowest order calculation neglecting $(\lambda\mu)$ and $[f]$ mixing is insufficient. The theoretical spectrum is much too densely packed, showing that the moment of inertia is too large. The (84) state we have used has the largest intrinsic quadrupole moment. Our results indicate that the corresponding deformation is too big and that we must admix states of other $(\lambda\mu)$ and $[f]$ which are not so deformed. The technique we have used to compute the effects of these admixtures is based on the variational principle used by Feingold.

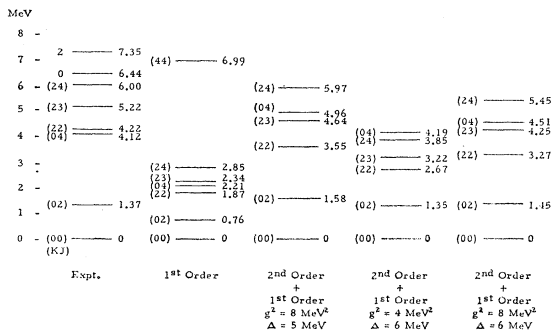

FIG. 1. Energies of Mg^{24}.

III. VARIATIONAL TREATMENT

We wish to use the same type of variational approach that Feingold[4] utilized in his treatment of the tensor force in nuclear structure calculations. We are given a shell-model Hamiltonian matrix H and an $SU3$ wave function Φ_L. Introducing the renormalized Hamiltonian:

$$\tilde{H}\equiv H-\langle\Phi_L|H|\Phi_L\rangle, \tag{10}$$

we consider the variation wave function:

$$\Psi_L(\lambda)\equiv\Phi_L+\lambda\tilde{H}\Phi_L \tag{11}$$

and variational energy:

$$E_L(\lambda)=\frac{\langle\Phi_L+\lambda\tilde{H}\Phi_L|\tilde{H}|\Phi_L+\lambda\tilde{H}\Phi_L\rangle}{\langle\Phi_L+\lambda\tilde{H}\Phi_L|\Phi_L+\lambda\tilde{H}\Phi_L\rangle}+\langle\Phi_L|H|\Phi_L\rangle. \tag{12}$$

Setting $dE_L(\lambda)/d\lambda=0$ and remembering that $\langle\Phi_L|\tilde{H}|\Phi_L\rangle=0$ we find the equation:

$$\lambda^2\langle\tilde{H}^2\rangle\langle\tilde{H}^2\rangle-\lambda\langle\tilde{H}^3\rangle-\langle\tilde{H}^2\rangle=0, \tag{13}$$

where $\langle\tilde{H}^2\rangle=\langle\Phi_L|\tilde{H}^2|\Phi_L\rangle$ notation is introduced. Choosing the smaller root, [in practice, this choice corresponds to the lowest value, $E_L(\lambda_0)$], we obtain [cf. Eq. (18) in Feingold]

$$\lambda_0=\frac{\langle\tilde{H}^3\rangle}{2\langle\tilde{H}^2\rangle\langle\tilde{H}^2\rangle}[1-(1+4k)^{1/2}];\quad k=\frac{\langle\tilde{H}^2\rangle^3}{\langle\tilde{H}^3\rangle^2}. \tag{14}$$

In general, k is expected to be small and one can approximate

$$1-(1+4k)^{1/2}\simeq-2k. \tag{15}$$

We shall define a correction factor $\tau\sim1$ by the equation:

$$1-(1+4k)^{1/2}=-2k\tau. \tag{16}$$

Then we have

$$\lambda_0=-\langle\tilde{H}^2\rangle\tau/\langle\tilde{H}^3\rangle, \tag{17}$$

$$E_L(\lambda_0)=\langle H\rangle-\tau\langle\tilde{H}^2\rangle/\langle\tilde{H}^3\rangle\langle\tilde{H}^2\rangle. \tag{18}$$

For small k the factor τ is approximately unity and the coefficient of τ in the energy expression has the form of a second-order perturbation correction with an average energy denominator Δ_L,

$$\Delta_L=\langle\tilde{H}^3\rangle/\langle\tilde{H}^2\rangle. \tag{19}$$

Δ_L is the average energy of excitation of the admixture $\tilde{H}|\Phi_L\rangle$. The average energy denominator given by the variational method is not Δ_L, but (Δ_L/τ).

TABLE III. Variational energies for Ne^{20}.

L	$-E_L$ (s.m.)	$-E_L(\lambda_0)$	$-\langle H\rangle$	$\langle\tilde{H}^2\rangle$	Δ_L (exact)	$E_L'-E_L$	Δ_L/τ	P
0	34.82	34.67	33.59	8.22	6.68	6.1	7.61	0.10
2	32.72	32.70	32.11	4.90	7.90	6.41	8.37	0.06
4	30.57	30.53	29.10	8.22	5.59	6.79	5.75	0.20
6	25.70	25.69	25.48	1.60	7.27	7.67	7.49	0.03

In order to test the validity of this variational form and, in particular, to test the L dependence of Δ_L, the four-nucleon problem in the $2s$, $1d$ shell was considered using the same two-body Hamiltonian described in the previous section. H was diagonalized in a standard manner to yield the shell-model energies. Diagonalizing the shell-model matrices in the [4] symmetry is equivalent to including all of the contributions from the (80), (42), (04), and (20) $(\lambda\mu)$ symmetries. We can, therefore, see how well our approximation scheme admixes states to the (80). The lowest energies correspond to the completely symmetric representations of symmetry [4]. The shell-model energies E_L of the lowest states of each L are given in Table III along with $\langle H\rangle$, $\langle\tilde{H}^2\rangle$, and $E_L(\lambda_0)$. A Δ_L(exact) is defined by

$$E_L=\langle H\rangle-\frac{\langle\tilde{H}^2\rangle}{\Delta_L(\text{exact})}. \tag{20}$$

This is compared with the variational "average energy" Δ_L/τ. The percentage admixture defined by

$$\frac{\lambda_0^2\langle\tilde{H}\Phi_L|\tilde{H}\Phi_L\rangle}{1+\lambda_0^2\langle\tilde{H}\Phi_L|\tilde{H}\Phi_L\rangle}\equiv P \tag{21}$$

is also listed.

As is seen in Table III, the variational method as applied to the case of Ne^{20} is quite good even with as much as a 20% admixture. We note also that the energy denominator Δ_L(exact) is fairly independent of L and is close to the energy separation between the lowest state at energy E_L and the next state of the same L at E_L'. That is,

$$\Delta_L(\text{exact})\sim E_L'-E_L. \tag{22}$$

These results lend support to the prescription we wish to propose for improving the lowest order Elliott calculations. Namely, we wish to set

$$E_L=\langle\Phi_L|H|\Phi_L\rangle-\frac{\langle\Phi_L|\tilde{H}^2|\Phi_L\rangle}{\Delta}, \tag{23}$$

where Δ is a parameter of the order of magnitude of the spacing between the lowest and first excited state of a given L. In general, these spacings for various L's are about the same so we shall simply take some average value. Alternatively we can consider Δ as an L-independent parameter to be fitted to the data. The form [Eq. (23)] would be accurate if the variational procedure outlined above is valid, and if the average excitation energy Δ is independent of L. Both of these assumptions are certainly valid for the problem discussed above.

IV. VARIATIONAL TREATMENT OF Mg^{24}

We apply the method discussed in the previous section to Mg^{24}. The correction to our lowest order calculation can be obtained by calculating the numerator in the term:

$$\text{"correction"}=\langle\Phi_L|\tilde{H}^2|\Phi_L\rangle/\Delta, \tag{24}$$

which appears in formula (23). We then adjust Δ to fit the data, noting that it should have a value which is of the order of the excitation of the first state which can be admixed. Actually the wave function Φ_L, which appears in Eq. (23), should be the solution of our lowest order calculation, and should involve small admixtures of different K bands. However, we use only the dominant K value in evaluating the correction term in Eq. (23). In evaluating $\langle\Phi_L|\tilde{H}^2|\Phi_L\rangle$ we must remember to add the one-body spin orbit force which now makes a contribution, even though we are in $S=0$ states. Our Hamiltonian becomes

$$H=h+g\sum\mathbf{l}^i\cdot\mathbf{s}^i,$$

where

$$h=\sum_{i<j}V(r_{ij}).$$

Since our wave functions Φ_L are spin singlets we get no contribution from the cross terms in $\tilde{H}^2$; thus, Eq. (24) becomes

$$\text{"correction"}=\frac{\langle\Phi_L|\tilde{h}^2|\Phi_L\rangle+g^2\langle\Phi_L|(\sum_i\mathbf{l}^i\cdot\mathbf{s}^i)^2|\Phi_L\rangle}{\Delta}. \tag{25}$$

The first term in (25) was evaluated, using the same method as in the lowest order calculation. To good approximation this matrix element has the form: $\alpha(K)L(L+1)+\beta(K)$. Normalizing to $\beta(0)=0$ we find [units of $(\text{MeV})^2$]

$$\begin{aligned}\langle\Phi_L|\tilde{h}^2|\Phi_L\rangle&=0.503L(L+1), && K=0\\ &=0.346L(L+1)+3.284, && K=2\\ &=0.0597L(L+1)+19.815, && K=4.\end{aligned} \tag{26}$$

Similarly the second term in Eq. (25) gives

$$\begin{aligned}g^2\langle\Phi_L|(\textstyle\sum_i\mathbf{l}^i\cdot\mathbf{s}^i)^2|\Phi_L\rangle&\\ =g^2\times0.023L(L+1), && K=0\\ =g^2\times[0.021L(L+1)+0.253], && K=2\\ =g^2\times[0.016L(L+1)+1.085], && K=4.\end{aligned} \tag{27}$$

The total correction is

$$\begin{aligned}\langle\tilde{H}^2\rangle &= (0.503+0.023g^2)L(L+1), && K=0\\ &= (0.346+0.021g^2)L(L+1)\\ &\quad +(3.284+0.253g^2), && K=2\\ &= (0.0597+0.0169g^2)L(L+1)\\ &\quad +(19.815+1.0865g^2), && K=4. \quad (28)\end{aligned}$$

For the parameters listed below, these corrections reduce to,

for $g^2=8$, $\Delta=6$:

$$\begin{aligned}&0.115L(L+1), && K=0\\ &0.0856L(L+1)+0.885, && K=2;\end{aligned}$$

for $g^2=8$, $\Delta=5$:

$$\begin{aligned}&0.137L(L+1), && K=0\\ &0.1028L(L+1)+1.062, && K=2;\end{aligned}$$

for $g^2=4$, $\Delta=6$:

$$\begin{aligned}&0.0992L(L+1), && K=0\\ &0.0717L(L+1)+0.716, && K=2.\end{aligned}$$

From the 5.08-MeV spin-orbit splitting in O^{17}, we conclude that $g\sim 2$ MeV. It is possible that g is larger for Mg^{24} if the trend in the $2s$, $1d$ shell is the same as in the p shell where g probably doubles[5] as one progresses from one end of the p shell to the other.

The value of Δ should be of the order of the energy separation of the first state which can be admixed to a given state. For $J=0$ we find another $J=0$ state at 6.44 MeV. In the case of $J=2$ we find two states experimentally separated by only 2.85 MeV. However, one is $K=0$ and the other is $K=2$ and as a result are not mixed as we have seen. The next $J=2$, $K=2$ state should come from the (92) symmetry and is probably at 7.35 MeV. We thus assume $\Delta\sim 6$ MeV and take $g\sim 2$ MeV. With these values we can compare our theory with the data. We see in Fig. 1 that for these values of g and Δ the $K=0$ band fits quite well, but the $K=2$ band lies too low by about 2 MeV.

If the spin-orbit force is indeed increasing in the $2s$, $1d$ shell as more particles are added then g must be increased. As an example, we have also computed the result for $g^2=8$ MeV^2. This value of g would give rise to a $d_{5/2}-d_{3/2}$ splitting of 7 MeV instead of 5 MeV. The resulting spectrum is shown also in Fig. 1. This certainly improves the fit to the $K=2$ band without sacrificing the $K=0$ fit. Since we have neglected the spin independence of the central force, it is not worthwhile attempting a better fit at this point.

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CONCLUSIONS

It is clear from these calculations that the lowest order $SU3$ results are inadequate. The correction terms from the variational approach are about as large as the zero-order terms in their contribution to the splittings. It is certainly encouraging that the ground-state band can be fitted well by this method. The fact that the $K=2$ band can be better fitted with a spin-orbit force which is larger than that in O^{17} indicates that this force is increasing as more particles are added.

The weakest point in this calculation concerns the empirical evaluation of Δ in the variational method. If $\langle\Phi_L|\tilde{H}^3|\Phi_L\rangle$ were evaluated, this ambiguity would be removed.

The effect of admixtures was seen to increase the moment of inertia of the excited band relative to the ground-state band. This effect also occurred in the zero-order calculation and seems to be a general characteristic of the observed energy spectra of nuclei.

The quantum number K emerges as being quite good in Mg^{24} and will probably remain good throughout the $2s$, $1d$ shell. This implies that these nuclei are axially symmetric.