Electron Velocity Distribution in a Slightly Ionized Gas with Crossed Electric and Strong Magnetic Fields*

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Assuming $(\omega_b\tau)^2\gg1$, equations that determine the electron energy distribution and drift velocity in the $\mathbf{E}\times\mathbf{B}$ drift frame of reference are derived, including a detailed treatment of inelastic collisions. These equations are valid when $\mathbf{E}$, $\mathbf{B}$, and the distribution function of the gas molecules are spacially uniform and constant in time, the component of $\mathbf{E}$ along $\mathbf{B}$ is negligible, and $E\ll B$ (in Gaussian units). The equations are suitable for numerical computations if the electron velocity distribution is assumed to be isotropic in the $\mathbf{E}\times\mathbf{B}$ drift frame. When the rms molecular speed in this frame is much smaller than the rms electron speed, the equations can be greatly simplified. Although the resulting equations are not new, this derivation clarifies their physical interpretation and limitations. The treatment is, of course, also applicable to cases where slightly ionized gas is moving across a magnetic field.

I. INTRODUCTION

THE distribution function of the electrons in a slightly ionized gas under the influence of external fields is determined by the Boltzmann equation. The electron distribution function in the gas frame, i.e., the reference frame in which the mean velocity of the gas molecules vanishes, is usually found by expanding the electron distribution function in spherical harmonics in velocity space.[1] When the rms electron speed is large in comparison with the electron drift speed and the rms speed of the gas molecules, the expansion converges rapidly and this "usual" or "conventional" method of solution works well. Under such conditions, elastic collisions are often much more probable than any other; this leads to further simplification of the calculations.

When in the gas frame the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ are perpendicular with $E<B$ (in Gaussian units) and the electron cyclotron frequency ω_b is much larger than the electron collision frequency $1/\tau$, it is advantageous to use a reference frame moving at velocity $\mathbf{v}_d=c(\mathbf{E}\times\mathbf{B})/B^2$ with respect to the gas frame. In this frame, which we will call the drift frame, the electric field vanishes and the gas flows with velocity $-\mathbf{v}_d$ perpendicular to the magnetic field; therefore, the (kinetic) energy of an electron is constant between its relatively infrequent collisions. The basic problem is to solve the Boltzmann equation in the presence of a gas wind instead of an electric field. This procedure is preferable because when v_d is comparable with the rms electron speed the electron velocity distribution is quite anisotropic in the gas frame but may be nearly isotropic in the drift frame. Notice that the phenomenon of runaway electrons[2] cannot occur as it does when $\mathbf{E}$ is not perpendicular to $\mathbf{B}$ or when B is smaller than E.

In this paper we restrict ourselves to the nonrelativistic limit, which requires $E\ll B$. The derivation of the equation governing the electron energy distribution in the drift frame is outlined. We discuss the evaluation of quantities of physical interest, including the electron drift. The physical interpretation of the results is made clearer by considering the limit of "small" v_d, in which the usual approach would also be adequate. Further discussion and details of the derivations as well as some numerical results of calculations applied to molecular hydrogen are given elsewhere.[3]

II. COLLISION PROCESSES, CROSS SECTIONS, AND COLLISION FREQUENCIES

The electron-molecule collision process j is distinguished by the energy $\epsilon_j=\frac{1}{2}m\alpha_j$ inelastically transferred to the molecule and by the number n_j of electrons leaving the collision. Considering a molecule initially at rest, we denote the velocity of the incident electron by $\mathbf{v}'$ and the velocities of the outgoing electrons by $\mathbf{v}_i$, where $i=1, 2\cdots n_j$, and we define $\cos\theta_i=(\mathbf{v}'\cdot\mathbf{v}_i)/v'v_i$. For each process, the collision frequency is $\nu^j(v')=n_g v'\sigma^j(v')$, where $\sigma^j(v')$ is the cross section for the process and n_g is the number of molecules per unit volume.

The momentum-transfer collision frequency $\nu_m{}^j(v')$ for process j is defined by

$$v'\nu_m{}^j(v')=\nu^j(v')\langle v'-\sum_{i=1}^{n_j} v_i\cos\theta_i\rangle_j, \tag{1}$$

where $\langle\ \rangle_j$ denotes the average effect of a collision process j for specified incident speed v'. The total momentum-transfer collision frequency is

$$\nu_m(v')=\textstyle\sum_j \nu_m{}^j(v'). \tag{2}$$

In this paper we may consider τ to be defined as $1/\nu_m$.

Processes with $n_j=0$ are electron attachment processes. From Eq. (1) we see that

$$\nu_m{}^j(v')=\nu^j(v'). \tag{3}$$

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[1] W. P. Allis, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21, p. 404.

[2] G. Ecker and K. G. Müller, Z. Naturforsch. **16a**, 246 (1961).

[3] G. A. Pearson and W. B. Kunkel, Lawrence Radiation Laboratory Report UCRL-10366, 1962 (unpublished). Recently A. G. Engelhardt and A. V. Phelps have reported numerical results for hydrogen and deuterium; see Bull. Am. Phys. Soc. 8, 161 (1963). These results, however, were based upon the "usual" method of solution.

Processes with $n_j=1$ are elastic if $\epsilon_j=0$, and are inelastic otherwise. They are characterized by a differential cross section $\sigma_\theta{}^j(\theta; v')$ in terms of which

$$\sigma^j(v')=2\pi\int_0^\pi \sigma_\theta{}^j(\theta; v')\sin\theta d\theta \tag{4}$$

and

$$\nu_m{}^j(v')=2\pi n_g v'\int_0^\pi \sigma_\theta{}^j(\theta; v') \times\sin\theta\left[1-\left(1-\frac{\alpha_j}{v'^2}\right)^{1/2}\cos\theta\right]d\theta, \tag{5}$$

if terms involving m/M are neglected.

Processes with $n_j\geqslant 2$ are ionization processes characterized by a differential cross section $\sigma_{\theta,v}{}^j(\theta,v; v')$ in terms of which

$$n_j\sigma^j(v')=2\pi\int_0^\infty dv\int_0^\pi \sigma_{\theta,v}{}^j(\theta,v; v')\sin\theta d\theta \tag{6}$$

and

$$\nu_m{}^j(v')=2\pi n_g v'\int_0^\infty dv\int_0^\pi \sigma_{\theta,v}{}^j(\theta,v; v') \times\sin\theta\left(\frac{1}{n_j}-\frac{v\cos\theta}{v'}\right)d\theta, \tag{7}$$

if terms involving m/M are neglected.

III. ELECTRON ENERGY DISTRIBUTION IN THE DRIFT FRAME

We limit ourselves to cases in which the applied fields and the distribution function of the gas molecules are spacially uniform and constant in time. We may avoid considering the spacial dependence of the electron distribution function either (a) when it has no spacial dependence or (b) when a localized swarm of electrons is being considered. In case (b) we integrate the Boltzmann equation spacially over the entire swarm. Although the integrated Boltzmann function in case (b) and the Boltzmann function in case (a) have different dimensions, they are governed by the same equation.

In the drift frame the energy ϵ of a free electron is constant between collisions; therefore, the electron energy distribution $f(\epsilon,t)$ changes with time only because of collisions. Because we have assumed $\omega_b\tau\gg 1$, the effects of each collision process are independent of the other collision processes, and we have

$$\frac{\partial f(\epsilon,t)}{\partial t}=\sum_j\left(\frac{\partial f}{\partial t}\right)_j, \tag{8}$$

where $(\partial f/\partial t)_j$ is the effect of collisions of process j.[4]

In evaluating the collision terms in Eq. (8), we will simplify the algebra by neglecting molecular recoil and by assuming that the gas molecules are stationary in the gas frame. These assumptions are not necessary; they will be relaxed and discussed in Sec. V. We also assume that even with the strong fields, the scattering is independent of the azimuthal angle about the direction of the incident velocity.

We denote the probability distribution of a real quantity α for specified values of parameters $\beta, \gamma\cdots$ by $P(\alpha; \beta,\gamma\cdots)$, where

$$P(\alpha; \beta,\gamma\cdots)\geqslant 0 \quad\text{and}\quad \int P(\alpha; \beta,\gamma\cdots)d\alpha=1.$$

The function $P(v; \epsilon)$, which is the distribution of speed v *in the gas frame* of the electrons with energy $\epsilon=\frac{1}{2}mV^2$ *in the drift frame*, will appear throughout our formulas. Notice that $P(v; \epsilon)=0$ unless $|V-v_d|\leqslant v\leqslant V+v_d$. We will discuss the evaluation of $P(v; \epsilon)$ later.

The collision terms for each process j can be written in the form

$$(\partial f/\partial t)_j=\int_0^\infty G^j(\epsilon; \epsilon')f(\epsilon',t)d\epsilon'-N^j(\epsilon)f(\epsilon,t). \tag{9}$$

Equation (9) states that electrons are removed from the distribution at a rate proportional to $N^j(\epsilon)$, and electrons are inserted into the distribution by the first term in a manner determined by $G^j(\epsilon; \epsilon')$. Clearly, we have

$$N^j(\epsilon)=\int_0^\infty \nu^j(v)P(v; \epsilon)dv. \tag{10}$$

The general properties of $G^j(\epsilon; \epsilon')$ are that

$$G^j(\epsilon; \epsilon')\geqslant 0 \quad\text{and}\quad \int_0^\infty G^j(\epsilon; \epsilon')d\epsilon=n_jN^j(\epsilon'). \tag{11}$$

For electron-attachment processes, $n_j=0$, so we have

$$G^j(\epsilon; \epsilon')=0. \tag{12}$$

For ionization processes, we may write

$$G^j(\epsilon; \epsilon')=\int_{|V'-v_d|}^{v_d+V'}dv'\int_0^\pi d\theta\int_{|V-v_d|}^{V+v_d}dv\ P(v'; \epsilon') \times[2\pi\sin\theta\, n_g v'\sigma_{\theta,v}{}^j(\theta,v; v')]P(\epsilon; v',v,\theta,\epsilon'), \tag{13}$$

where $P(\epsilon; v',v,\theta,\epsilon')$ is the distribution of energy ϵ of the electrons ejected at speed v and angle θ from collisions in which the incident electron speed is v'. The function $P(\epsilon; v',v,\theta,\epsilon')$ can be derived from the collision kinematics with the result[3]

$$P(\epsilon; v',v,\theta,\epsilon')=\frac{1}{\pi m v_d v}[-\cos^2\theta+2b\cos\theta+c]^{-1/2}, \tag{14}$$

where

$$b=\left(\frac{v'^2+v_d{}^2-V'^2}{2v_dv'}\right)\left(\frac{v^2+v_d{}^2-V^2}{2v_dv}\right)$$

[4] If free electrons are produced by processes other than ionization of gas molecules by electron impact, additional terms must be added to Eq. (8); these terms will not be discussed here.

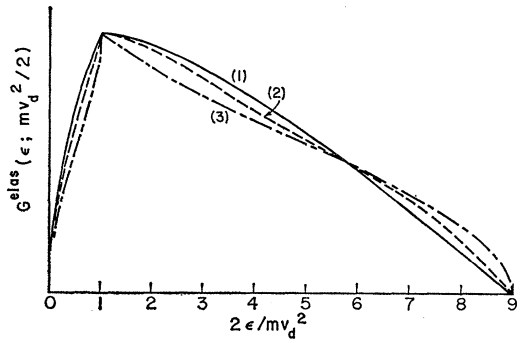

FIG. 1. The energy-scatter function for elastic scattering when $\epsilon'=\frac{1}{2}mv_d^2$ and when the scattering is isotropic and the cross section is constant. The electron velocity distribution in the drift frame is assumed to (1) be isotropic, (2) be weighted by $\sin^2\xi$, or (3) contain only $\xi=\pi/2$, where ξ is the angle between $\mathbf{B}$ and the electron velocity in the drift frame.

and

$$c=1-\left(\frac{v'^2+v_d^2-V'^2}{2v_dv'}\right)^2-\left(\frac{v^2+v_d^2-V^2}{2v_dv}\right)^2.$$

This function is defined to be zero except when

$$-1\leqslant b-(b^2+c)^{1/2}\leqslant\cos\theta\leqslant b+(b^2+c)^{1/2}\leqslant 1,$$

and when $b^2+c\geqslant 0$. The latter condition simply implies that $|V-v_d|\leqslant v\leqslant V+v_d$ must be satisfied whenever $|V'-v_d|\leqslant v'\leqslant V'+v_d$ is satisfied.

If the scattering is isotropic, the angular integration in Eq. (13) is trivial and yields

$$G^j(\epsilon;\epsilon')=\frac{1}{2mv_d}\int_{|V-v_d|}^{V+v_d}\frac{dv}{v}\times\int_{|V'-v_d|}^{V'+v_d}P(v';\epsilon')n_g v'\sigma_v{}^j(v;v')dv', \quad (15)$$

where

$$\sigma_v{}^j(v;v')\equiv 2\pi\int_0^\pi\sin\theta\,\sigma_{\theta,v}{}^j(\theta,v;v')d\theta.$$

For ionization process j, $G^j(\epsilon;\epsilon')$ is nonzero for values of ϵ between zero and approximately $\epsilon'-\epsilon_j$.

For elastic or inelastic processes, we may write

$$G^j(\epsilon;\epsilon')=\int_{|V'-v_d|}^{V'+v_d}dv'\int_0^\pi d\theta\,P(v';\epsilon')\times[2\pi\sin\theta\,n_g v'\sigma_\theta{}^j(\theta;v')]P^j(\epsilon;v',\theta,\epsilon'), \quad (16)$$

where $P^j(\epsilon;v',\theta,\epsilon')$ is just $P(\epsilon;v',v,\theta,\epsilon')$ from above evaluated at $v^2=v'^2-\alpha_j$. If the scattering is isotropic, the angular integration in Eq. (16) yields

$$G^j(\epsilon;\epsilon')=\frac{1}{2mv_d}\int\frac{dv'}{v}P(v';\epsilon')\nu^j(v'), \quad (17)$$

where the limits are given by $|V-v_d|\leqslant(v'^2-\alpha_j)^{1/2}\leqslant V+v_d$. For elastic or inelastic process j, $G^j(\epsilon;\epsilon')$ is nonzero only for ϵ within a definite range about $\epsilon'-\epsilon_j$.

The function $P(v;\epsilon)$ that appears throughout the equations depends only upon the angular dependence of the electron velocity distribution in the drift frame. With large $\omega_b\tau$, the most general angular distribution of interest is independent of the azimuthal angle about $\mathbf{B}$, and is an even function of $\cos\xi$, where ξ is the polar angle measured from $\mathbf{B}$. Such a distribution can be expanded in the even Legendre polynomials of $\cos\xi$, and $P(v;\epsilon)$ can be written in terms of coefficients of the expansion. Since our procedure gives no way of calculating these coefficients we must make an assumption. We assume that the use of

$$P(v;\epsilon)=v/2v_dV \quad \text{for} \quad |V-v_d|\leqslant v\leqslant V+v_d, \quad (18)$$

which is correct for an isotropic distribution, will give a good approximation to the correct physical results.

From physical arguments we expect the anisotropy of the velocity distribution to be largest when $\epsilon\approx\frac{1}{2}mv_d^2$ and to decrease rapidly as ϵ increases or decreases from this value. We also expect that anisotropy to be such that large values of $|\cos\xi|$ are less likely than with an isotropic distribution. To illustrate that the assumption of using Eq. (18) can be rather good even when the anisotropy is large, we calculate $G^j(\epsilon;\epsilon')$ for elastic collisions from Eq. (17) for the case $\epsilon'=\frac{1}{2}mv_d^2$—assuming that $\nu^j(v')$ is proportional to v'. Figure 1 shows the results for (1) an isotropic distribution (zero-order Legendre polynomial only), (2) one weighted by $\sin^2\xi$ (the largest anisotropy using only the zero- and second-order Legendre polynomials), and (3) one containing only $\xi=\pi/2$ (the largest possible anisotropy).

Often the angular dependence of a scattering process is not known well enough for use in Eq. (13) or (16). In this case one can assume the scattering is isotropic and use Eq. (15) or (17). For elastic collisions, the elastic momentum-transfer collision frequency should then be used in evaluating Eqs. (10) and (17).

With the assumptions we have made, the electron energy distribution in the drift frame is determined by Eq. (8), which may be written symbolically as

$$\partial f(\epsilon,t)/\partial t=n_g\int\chi(\epsilon,\epsilon';v_d)f(\epsilon',t)d\epsilon', \quad (19)$$

where for a particular gas, $\chi(\epsilon,\epsilon';v_d)$ is a kernel depending only upon v_d. Physically we expect that any initial distribution will quickly approach the separable form

$$f(\epsilon,t)=Cf_0(\epsilon)e^{\beta t} \quad \text{with} \quad \int f_0(\epsilon)d\epsilon=1, \quad (20)$$

where C and β are constants, and $f_0(\epsilon)\geqslant 0$. Substitution of this important form of the solution into Eq. (19) shows that for a particular gas $f_0(\epsilon)$ and β/n_g depend only upon $v_d=cE/B$.

Once $f_0(\epsilon)$ is known, many quantities of interest can be calculated. The mean electron energy in the drift frame is $\bar{\epsilon}=\int f_0(\epsilon)\epsilon d\epsilon$. The distribution of electron speed in the gas frame is $\int P(v;\epsilon)f_0(\epsilon)d\epsilon$, and the mean electron energy in the gas frame is $\bar{\epsilon}+\frac{1}{2}mv_d^2$. The rate at which a collision process—such as ionization, dissociation, or excitation—proceeds is determined by

$$\int dv\nu^j(v)\int d\epsilon\, P(v;\epsilon)f_0(\epsilon)=\int d\epsilon\, N^j(\epsilon)f_0(\epsilon),$$

where $\nu^j(v)$ is the collision frequency for the process.

IV. DRIFT ALONG $-E$ IN THE GAS FRAME

Other quantities of interest are the diffusion tensor, which we will not discuss here, and the drift speed of the electrons along $-\mathbf{E}$ in the gas frame. To calculate the latter we assume $\mathbf{E}=E\hat{a}_y$ and $\mathbf{B}=B\hat{a}_z$ so that $\mathbf{v}_d=v_d\hat{a}_x$. From the electron equations of motion in the gas frame, the y position of an electron is $y=Y+(v_d-v_x)/\omega_b$, where Y is the y position of its guiding center. One can verify that the effect of a collision of process j at y upon the mean guiding-center position of the electrons is determined by

$$\sum_{i=1}^{n_j}(Y_i-Y')=\sum_{i=1}^{n_j}(v_{xi}-v_x')/\omega_b,$$

where the guiding center of the incident electron is at Y', and those of the outgoing electrons are at Y_i, and where the speeds are those at the collision. The contribution of collision process j to the drift speed of the electrons with energy $\epsilon'=\frac{1}{2}mV'^2$ and speed v' is

$$\nu^j(v')\langle\sum_{i=1}^{n_j}(Y'-Y_i)\rangle_j=[\nu_m{}^j(v')+(n_j-1)\nu^j(v')]v_x'/\omega_b,$$

where we have used the definition of the momentum-transfer collision frequency, and where

$$v_x'=(v'^2+v_d^2-V'^2)/2v_d.$$

Notice that electron attachment processes do not contribute, since for them $n_j=0$ and $\nu_m{}^j=\nu^j$. For elastic and inelastic processes only the momentum-transfer collision frequency is important, since for them $n_j=1$.

By averaging over ϵ' and v' and summing over the collision processes j, we find (dropping the primes)

$$v_E=\int_0^\infty d\epsilon\, f_0(\epsilon)\int_{|V-v_d|}^{V+v_d}dv\, P(v;\epsilon)\left(\frac{v^2+v_d^2-V^2}{2v_d\omega_b}\right)\times[\nu_m(v)+\sum_j(n_j-1)\nu^j(v)]. \quad (21)$$

In addition to the term involving ν_m, we have a term $\sum_j(n_j-1)\nu^j$ that accounts for the changing number of electrons. This result does not depend upon the neglect of molecular recoil.

V. SIMPLIFICATIONS WHEN v_d IS "SMALL"

In the conventional approach, the assumption of "large $\omega_b\tau$" consists of replacing $[1+(\omega_b\tau)^2]^{1/2}$ by $\omega_b\tau$ throughout. When this approximation is good, the two methods of solution should agree in the limit of small v_d. This agreement can be demonstrated.

To illustrate the comparison, we consider v_E as given by Eq. (21). By assuming that $v_d\ll V$ and that $P(v;\epsilon)$ is given by Eq. (18), we can carry out the integral over v by expanding the integrand in a Taylor series about V. The result is a power series in $(v_d/V)^2$ with the first term being

$$\frac{v_d}{3V^2\omega_b}\frac{\partial}{\partial V}\{V^3[\nu_m(V)+\sum_j(n_j-1)\nu^j(V)]\}.$$

If most of the electrons in the distribution satisfy the condition $(v_d/V)^2\ll 1$, we can use this term in Eq. (21) to find

$$v_E=v_d\int\frac{f_0(\epsilon)}{3V^2\omega_b}\frac{\partial}{\partial V}\times\{V^3[\nu_m(V)+\sum_j(n_j-1)\nu^j(V)]\}d\epsilon. \quad (22)$$

If we assume elastic collisions are much more probable than any other, this formula is identical to that found by the usual method.[1] We clearly see that the usual method is adequate when $\bar{\epsilon}\gg\frac{1}{2}mv_d^2$ and when elastic collisions are much more probable than any other. The conventional approach also shows that the drift speed along $\mathbf{E}\times\mathbf{B}$ in the gas frame is v_d, as expected.

In the same manner, we can demonstrate the equivalence of Eq. (8) to the equation in the usual approach that determines $F_0(v,t)$, the isotropic part of the electron velocity distribution in the gas frame. By using $P(v;\epsilon)$ as given by Eq. (18) and $G^j(\epsilon;\epsilon')$ as given by Eq. (15) or Eq. (17), we carry out the integrals in Eqs. (9) and (10) by assuming $V\gg v_d$ and proceeding as above. This yields $(\partial f/\partial t)_j$ in a power series in $(v_d/V)^2$. For inelastic, electron-attachment, and ionization processes the first term in this expansion is identical to that found by the usual approach.[5] For elastic collisions the first term vanishes, but because the usual approach assumes elastic collisions are much more probable than any other, we keep the second term. To facilitate comparison with the conventional method we introduce an isotropic distribution of velocity in the drift frame by defining $g(\mathbf{V},t)=mf(\epsilon,t)/4\pi V$. Then, after a considerable amount of algebra, we find

$$(\partial g/\partial t)_{\text{elas}}=\nabla_V\cdot D_V\nabla_V g(\mathbf{V},t), \quad (23)$$

where

$$D_V=v_d{}^2\nu_m{}^{\text{elas}}(V)/3.$$

Thus elastic collisions cause the isotropic velocity dis-

[5] T. Holstein, Phys. Rev. **70**, 367 (1946).

tribution $g(\mathbf{V},t)$ to diffuse in velocity space, the diffusion constant being that from kinetic theory with the mean free path replaced by v_d. This result can also be derived using the conventional approach.

In Eq. (19), v_d represents the molecular speed in the drift frame. If the molecules are not at rest in the gas frame, so that they have a distribution $P(u)$ of speed u in the drift frame, Eq. (19) is generalized to

$$\frac{\partial f(\epsilon,t)}{\partial t}=n_g\int f(\epsilon',t)d\epsilon'\int\chi(\epsilon,\epsilon';u)P(u)du. \quad (24)$$

When $\bar{\epsilon}\gg\frac{1}{2}mv_d^2$ and elastic collisions are much more probable than any other, we have shown above that $\chi(\epsilon,\epsilon';v_d)$ depends upon v_d only through D_V in Eq. (23). Thus the integral over u in Eq. (24) simply generalizes D_V from Eq. (23) to

$$D_V=\langle u^2\rangle\nu_m^{\text{elas}}(V)/3=(v_d^2+2\mathcal{E}/M)\nu_m^{\text{elas}}(V)/3, \quad (25)$$

where $\mathcal{E}$ is the mean kinetic energy of the molecules in the gas frame. This result generalizes the result of the usual approach, which assumes the molecules have a Maxwellian velocity distribution in the drift frame.[1]

The inclusion of the molecular recoil will lead to a term in $(\partial g/\partial t)_{\text{elas}}$ that is of zero order in (v_d^2/V^2). We can deduce this term from the following facts: (a) This term cannot alter the number of electrons, and it must vanish when $m/M=0$. (b) When $v_d=0$ and the gas molecules have a Maxwellian velocity distribution, $(\partial g/\partial t)_{\text{elas}}$ must vanish when $g(\mathbf{V})$ is a Maxwellian velocity distribution with the same temperature. The result is that when $\bar{\epsilon}\gg\frac{1}{2}mv_d^2$,

$$(\partial g/\partial t)_{\text{elas}}=\nabla_V\cdot[D_V\nabla_V g(\mathbf{V},t)+(m/M)\nu_m^{\text{elas}}(V)\mathbf{V}g(\mathbf{V},t)]. \quad (26)$$

The term in m/M agrees with that found by the conventional approach. By multiplying Eq. (26) by $\epsilon=\frac{1}{2}mV^2$ and integrating over velocity, we find

$$\left(\frac{\partial\bar{\epsilon}}{\partial t}\right)_{\text{elas}}=\left(mv_d^2+\frac{2m\mathcal{E}}{M}\right)\left\langle\nu_m^{\text{elas}}(V)+\frac{V}{3}\frac{\partial}{\partial V}\nu_m^{\text{elas}}(V)\right\rangle-\frac{2m}{M}\langle\epsilon\nu_m^{\text{elas}}(V)\rangle, \quad (27)$$

where the brackets denote averages over $g(\mathbf{V},t)$. In doing partial integrations leading to Eq. (27) we assumed that $V^3\nu_m^{\text{elas}}(V)g(\mathbf{V},t)$ vanishes at $V=0$, as is almost always the case. Equation (27) shows that the average energy gain per elastic collision (neglecting recoil) is approximately $mv_d^2+2m\mathcal{E}/M$. However, the recoil term prevents $\bar{\epsilon}$ from increasing beyond about $\frac{1}{2}Mv_d^2+\mathcal{E}$, which is clearly much greater than $\frac{1}{2}mv_d^2$. From Eq. (26) we actually conclude that elastic collisions always tend to make $g(\mathbf{V},t)$ a Maxwellian velocity distribution with $\bar{\epsilon}$ *equal to* $\frac{1}{2}Mv_d^2+\mathcal{E}$, the mean energy of a gas molecule in the drift frame, since for this distribution $(\partial g/\partial t)_{\text{elas}}$ vanishes. This result is correct for any $\nu_m^{\text{elas}}(V)$.

The conventional approach should be used whenever $\bar{\epsilon}\gg\frac{1}{2}mv_d^2$ because of its relative simplicity. From the above discussion we see that this condition is satisfied except when inelastic and ionization processes are very much more effective than molecular recoil in holding $\bar{\epsilon}$ to a low value. We expect $\frac{1}{2}mv_d^2$ to be comparable to $\bar{\epsilon}$ only when $\bar{\epsilon}$ becomes comparable to the ionization energy of the gas. Since $\mathcal{E}$ must be smaller than the ionization energy if the gas is to be only slightly ionized, we conclude from Eq. (27) that the gas "temperature" $\mathcal{E}$ and molecular recoil may be neglected whenever the usual approach is not adequate and our more general approach should be used.

VI. CONCLUSION

By assuming the electron velocity distribution is isotropic in the drift frame, we derive an equation that determines the electron energy distribution in the drift frame. We then showed how to calculate quantities of physical interest, such as ionization rates and electron drift speeds, from this energy distribution. We expect the results calculated with this assumption to be a good approximation to the correct results, as we discussed briefly.

Comparison of our results with those of the conventional method of solution in the "large $\omega_b\tau$" limit [when $(\omega_b\tau)^2\gg1$] showed that the usual method is adequate when the mean electron kinetic energy is large compared with $\frac{1}{2}mv_d^2$. When these conditions are satisfied, the effect of elastic collisions without recoil is a diffusion of the electron velocity distribution in velocity space, and the diffusion coefficient has a very simple form.