

## Ultrasonic Amplification in Extrinsic Semiconductors

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(Received 13 August 1962)

When a semiconductor, with only one type of carrier, is placed in crossed electric and magnetic fields, an amplification of sound waves similar to that discussed by Dumke and Haering for semimetals can take place. The sound wave is amplified instead of absorbed, when the drift velocity in the crossed fields,  $cE/H$ , is in the direction of propagation of the wave and exceeds the wave velocity  $V_s$ . When the carrier concentrations and the sound wave frequency are such that the screening of the ionic currents by the electronic currents breaks down, the amplification factor can become very large.

### I. INTRODUCTION

RECENTLY it has been shown that ultrasonic amplification is possible in semiconductors<sup>1,2</sup> when the drift velocity of the conduction electrons in an external electric field is greater than the velocity of sound. An alternate mechanism for the amplification of acoustic waves has been suggested by the nonlinearity observed by Esaki<sup>3</sup> in the magnetoresistance of bismuth. When we have crossed electric and magnetic fields, the electrons drift in the direction  $\mathbf{E} \times \mathbf{H}$  with velocity  $cE/H$ . When this drift velocity is in the direction of propagation of a sound wave and it exceeds the wave velocity  $V_s$ , the sound wave can be amplified.

Dumke and Haering<sup>4</sup> have shown that such an amplification can take place in a semimetal with equal concentrations of electrons and holes such that no appreciable Hall voltage is set up. The advantage of their method is that it can be used in materials with high carrier concentrations without the generation of large current densities in the direction of the external electric field. It has been shown that in semiconductors with only one kind of carrier, large absorption of ultrasound can occur even for low concentrations of carriers in high magnetic fields.<sup>5</sup> We show that we can obtain high amplification factors under similar conditions when the drift velocity exceeds the sound velocity.

### II. CALCULATION OF THE ABSORPTION COEFFICIENT

In calculating the absorption coefficient of the acoustic wave, we shall represent the effects of the wave on the conduction electrons by a self-consistent electromagnetic field derived from Maxwell's equations,  $\mathcal{E}$ , and a deformation potential,  $-C(u_x/V_s)$ , where  $u_x$  is the velocity field induced by the sound wave. We take the dc electric field  $\mathbf{E}$  to be applied along the  $y$  direction and a constant magnetic field  $\mathbf{H}$  to be applied along the  $z$  direction. We shall treat the case of a longitudinally

polarized sound wave propagating along the  $x$  direction with wave number  $q$  and frequency  $\omega$ .

When the sound wavelength is much greater than the cyclotron radius of the electrons and the cyclotron frequency is greater than the collision frequency, the electronic current density in the crossed electric and magnetic fields can be expressed as

$$J_x = -neV_H + ne\mu \left( \mathcal{E}_x - \omega_c \tau \mathcal{E}_y - iq \frac{Cu_x}{eV_s} - \frac{mu_x}{e\tau} \right) + eD \frac{\partial n}{\partial x}, \quad (2.1a)$$

$$J_y = \frac{neV_H}{\omega_c \tau} + ne\mu \left[ \mathcal{E}_y + \omega_c \tau \left( \mathcal{E}_x - iq \frac{Cu_x}{eV_s} - \frac{mu_x}{e\tau} \right) \right] + e\omega_c \tau D \frac{\partial n}{\partial x}, \quad (2.1b)$$

where  $V_H = cE/H$  is the electron drift velocity in the crossed fields and  $\tau$  is the electron relaxation time. In (2.1a-b),  $\mu$  and  $D$  are the dc electron mobility and diffusion coefficient, respectively, in the presence of the transverse magnetic field. We have then

$$\mu = e\tau/m(\omega_c \tau)^2, \quad eD = 2\mu E_F/3, \quad (2.2)$$

where  $\omega_c$  is the cyclotron frequency and  $E_F$  is the Fermi energy of the carriers. The equation of continuity is

$$-e(\partial n/\partial t) + \partial J_x/\partial x = 0. \quad (2.3)$$

From (2.1a) and (2.3), we obtain the following equation for the electron density:

$$\left[ \frac{\partial}{\partial t} + V_H \frac{\partial}{\partial x} - \frac{\partial}{\partial x} D \frac{\partial}{\partial x} \right] n = -ne\mu \left( \mathcal{E}_x - \omega_c \tau \mathcal{E}_y - iq \frac{Cu_x}{eV_s} - \frac{mu_x}{e\tau} \right). \quad (2.4)$$

When the effects of the sound wave can be treated as a perturbation on the motion of the electrons, we can write the electron density as  $n = n_0 + n_1$ , where  $n_0$  is the unperturbed electron density and  $n_1 \propto \exp[i(qx - \omega t)]$

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<sup>1</sup> A. R. Hutson, J. H. McFee, and D. L. White, Phys. Rev. Letters **7**, 237 (1961).

<sup>2</sup> H. N. Spector, Phys. Rev. **127**, 1084 (1962).

<sup>3</sup> L. Esaki, Phys. Rev. Letters **8**, 4 (1962).

<sup>4</sup> W. P. Dumke and R. R. Haering, Phys. Rev. **126**, 1974 (1962).

<sup>5</sup> H. N. Spector, Phys. Rev. **125**, 1880 (1962).

is a small perturbation in the density caused by the sound wave. Keeping only terms that are of first order in (2.4), we find that

$$n_1 = \frac{n_0 \mu i q}{q^2 D - i\omega(1 - V_H/V_s)} \times \left( \mathcal{E}_x - \omega_c \tau \mathcal{E}_y - i q \frac{C u_x}{e V_s} - \frac{m u_x}{e \tau} \right). \quad (2.5)$$

Using (2.5) in (2.1a-b) we find that the electronic current induced by the sound wave is

$$J_x = \sigma_{11} \left( \mathcal{E}_x - i q \frac{C u_x}{e V_s} - \frac{m u_x}{e \tau} \right) + \sigma_{12} \mathcal{E}_y, \quad (2.6a)$$

$$J_y = \sigma_{21} \left( \mathcal{E}_x - i q \frac{C u_x}{e V_s} - \frac{m u_x}{e \tau} \right) + \sigma_{22} \mathcal{E}_y, \quad (2.6b)$$

where

$$\begin{aligned} \sigma_{11} &= \frac{-n_0 e \mu i \omega}{q^2 D - i\omega(1 - V_H/V_s)}, \\ \sigma_{12} &= \frac{n_0 e \mu i \omega c \tau}{q^2 D - i\omega(1 - V_H/V_s)}, \\ \sigma_{22} &= \frac{-n_0 e \mu [i\omega - (\omega_c \tau)^2 q^2 D]}{q^2 D - i\omega(1 - V_H/V_s)}, \\ \sigma_{21} &= \frac{n_0 e \mu i \omega (1 - V_H/V_s) \omega_c \tau}{q^2 D - i\omega(1 - V_H/V_s)}. \end{aligned} \quad (2.7)$$

From Maxwell's equation we obtain the following relations between the self-consistent electromagnetic field and the electronic and ionic currents:

$$\mathcal{E}_x = \frac{4\pi}{i\omega} (J_x + n_0 e u_x), \quad \mathcal{E}_y = \frac{-4\pi}{i\omega} \left( \frac{V_s}{c} \right)^2 J_y. \quad (2.8)$$

In (2.8),  $n_0 e u_x$  is the ionic current induced by the passage of the sound wave.

The energy dissipated per unit time and per unit volume from the sound wave is<sup>5</sup>

$$Q = \frac{1}{2} \operatorname{Re} \left[ \mathbf{J}^* \left( \boldsymbol{\mathcal{E}} - i q \frac{C u_x}{e V_s} \right) - \frac{m u_x^*}{e \tau} (J_x + n_0 e u_x) \right], \quad (2.9)$$

and the absorption coefficient can be obtained from  $Q$  by dividing it by the incident energy flux

$$\alpha = Q / \left( \frac{1}{2} \rho |u_x|^2 v_s \right). \quad (2.10)$$

In (2.10),  $\alpha$  is the absorption coefficient and  $\rho$  is the density of the material. From (2.6 a-b), (2.8), (2.9), and (2.10) we obtain the following expression for the absorption coefficient:

$$\alpha = \frac{n_0 m}{\rho V_s \tau} \operatorname{Re} \left\{ \left[ 1 + \left( \frac{\omega}{\omega_p} \right)^2 \frac{C}{m V_s^2} \right]^2 / \left[ \frac{\sigma_{11}}{\sigma_0} \frac{i\omega}{\omega_p^2 \tau} - (\sigma_{12} \sigma_{21} / \sigma_0^2) / \left( \frac{\sigma_{22}}{\sigma_0} + \frac{i\omega}{\omega_p^2 \tau} \left( \frac{c}{V_s} \right)^2 \right) \right] \right\}. \quad (2.11)$$

In (2.11),  $\omega_p = (4\pi n_0 e^2 / m)^{1/2}$  is the plasma frequency of the carriers. When the carrier density  $n_0$  is less than  $10^{18}$  electrons/cm<sup>3</sup>, the inequality  $(\omega / \omega_p^2 \tau) (c / V_s)^2 > |\sigma_{22} / \sigma_0|$  holds for sound frequencies greater than a megacycle, i.e., the screening of the transverse currents breaks down, and the absorption coefficient becomes

$$\alpha = \frac{n_0 m}{\rho V_s \tau} \left( 1 - \frac{V_H}{V_s} \right) (\omega_c \tau)^2 \left[ 1 + \left( \frac{\omega}{\omega_p} \right)^2 \frac{C}{m V_s^2} \right]^2 / \left\{ \left[ 1 + \frac{1}{3} \left( \frac{V_F}{V_s} \right)^2 \left( \frac{\omega}{\omega_p} \right)^2 \right]^2 + (\omega_c \tau)^4 \left( 1 - \frac{V_H}{V_s} \right)^2 \left( \frac{\omega}{\omega_p} \right)^2 / (\omega_p \tau)^2 \right\}. \quad (2.12)$$

As can be seen, (2.12) becomes negative and we have amplification instead of attenuation when  $V_H > V_s$ . The maximum amplification and attenuation occur, respectively, when

$$\frac{V_H}{V_s} = 1 \pm \frac{\omega_p^2 \tau}{\omega (\omega_c \tau)^2} \left[ 1 + \frac{1}{3} \left( \frac{V_F}{V_s} \right)^2 \left( \frac{\omega}{\omega_p} \right)^2 \right], \quad (2.13)$$

and the maximum value of the absorption coefficient is

$$\alpha_{\max} = \pm \frac{n_0 m \omega (\omega_p / \omega)^2 [1 + (\omega / \omega_p)^2 C / m V_s^2]^2}{2 \rho V_s \left[ 1 + \frac{1}{3} (V_F / V_s)^2 (\omega / \omega_p)^2 \right]}. \quad (2.14)$$

There is a great similarity between (2.12), (2.13) and those expressions derived in the absence of the magnetic field when  $ql < 1$ .

### III. DISCUSSION

The mechanism by which the electrons transfer energy from the external electric field to the acoustic wave when the drift velocity is greater than the sound wave velocity is similar to that discussed previously<sup>2</sup> in the absence of the dc magnetic field. The maximum value of the absorption coefficient is independent of the value of the relaxation time as long as  $\omega_c \tau \gg 1$ . In the limit of

very long relaxation times, the value of the drift velocity at which this maximum occurs approaches the sound velocity. However, since we have used the dc values of the mobility and the diffusion coefficient, our expressions are valid only when  $\omega\tau < 1$ . When the screening of the transverse currents break down, the transverse electric fields induced by the acoustic wave vanish. Under this condition, we have a situation similar to that discussed by Dumke and Haering for semimetals.<sup>4</sup>

In a typical semiconductor the deformation potential constant  $C$  is of the order of 10 eV,  $\tau = 10^{-11}$  sec at low temperatures, and  $V_s = 10^5$  cm/sec. With an electron density of  $10^{18}$  electrons/cm<sup>3</sup>, expression (2.14) has the value  $\alpha_{\max} = \pm 10^3$  at  $\omega = 10^9$  sec<sup>-1</sup>, but this maximum occurs at unattainably high electric fields. For  $H = 10^4$  G and electric fields of order 10 V, we get an amplification factor of less than unity. For lower electron densities, the situation becomes more favorable. When  $n_0 = 10^{14}$  electrons/cm<sup>3</sup>, expression (2.14) has the value  $\alpha_{\max} = \pm 10$  for  $\omega = 10^9$  sec<sup>-1</sup>,  $\alpha_{\max} = \pm 5 \times 10^3$  for  $\omega = 10^{10}$  sec<sup>-1</sup>, and  $\alpha_{\max} = \pm 10^6$  for  $\omega = 10^{11}$  sec<sup>-1</sup>. The maximum in this case occurs for values of  $V_H/V_s$  of order unity while for the case of higher densities, the maximum occurs for  $V_H/V_s$  of order  $10^2$ . Therefore, at high sound

frequencies and low carrier densities, the amplification factor can become very large.

Physically, for these high frequencies and low-carrier densities, the screening of the ionic currents by the electronic currents breaks down. When this occurs in semiconductors,<sup>5</sup> the ultrasonic absorption, and when  $V_H/V_s > 1$ , the ultrasonic amplification, increase quite rapidly with frequency and the amplification can attain high values. For higher frequencies the breakdown of screening becomes complete and the amplification begins to saturate with frequency as long as the condition  $\omega\tau < 1$  holds. It is in this frequency range, i.e., where the breakdown of the screening is complete, that the amplification factor can become quite large.

*Note added in proof.* The condition for the screening of the transverse current to break down is equivalent to the vanishing of the Hall field accompanying the sound wave. In the case considered by Dumke and Haering,<sup>4</sup> the vanishing of the Hall field was accomplished because there were equal concentrations of electrons and holes. However, at high enough sound frequencies, namely those frequencies at which the transverse screening breaks down, the Hall field will vanish even if there is only one type of carrier present

## Effects of Electron Correlations on the Properties of Alkali Metals\*

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(Received 26 December 1962)

The effects of electron correlations on the single-particle processes in alkali metals have been calculated incorporating the more appropriate band theory masses at the Fermi surface rather than those at the bottom of the conduction band, as previously used. This modification substantially improves the agreement between theory and experiment.

IN previous papers,<sup>1,2</sup> the effects of electron correlations on the low-temperature specific heat and the Pauli paramagnetic spin susceptibility were calculated by means of a momentum transfer interpolation procedure designed to obtain results relevant to the region of metallic densities. The approximations used were the random phase approximation (RPA) for long-wavelength interactions and second-order perturbation theory with the neglect of parallel spin interactions for the short-wavelength interactions. The latter approximation corresponds to the physical consideration that electrons of parallel spin do not get close enough to interact via short-range forces. As Pines has shown,<sup>3</sup>

the influence of band structure on the correlation corrections is small for the alkali metals and the effects of the lattice can be taken into account by the introduction of the lattice effective mass into the kinetic energy term. However, the effective masses which were introduced both in Pines' work and in I correspond to masses evaluated at the bottom of the conduction band. For properties such as low-temperature specific heat and Pauli paramagnetism, one is concerned with single-particle excitations in the immediate vicinity of the Fermi surface. A *proper* account of the lattice would *not* correspond to the band parameters evaluated at the bottom of the band but rather the parameters evaluated at the Fermi level itself. These latter parameters, with considerations of Fermi-surface distortion, have recently been calculated by Ham.<sup>4</sup> It is the purpose of this note

\* Supported by the National Science Foundation.

<sup>1</sup> S. D. Silverstein, Phys. Rev. **128**, 631 (1962) (referred to as I).

<sup>2</sup> S. D. Silverstein (to be published).

<sup>3</sup> D. Pines, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1955), Vol. 1, p. 367.

<sup>4</sup> F. S. Ham, Phys. Rev. **128**, 2524 (1962).