very long relaxation times, the value of the drift velocity at which this maximum occurs approaches the sound velocity. However, since we have used the dc values of the mobility and the diffusion coefficient, our expressions are valid only when $\omega\tau < 1$. When the screening of the transverse currents break down, the transverse electric fields induced by the acoustic wave vanish. Under this condition, we have a situation similar to that discussed by Dumke and Haering for semimetals.[4]

In a typical semiconductor the deformation potential constant C is of the order of 10 eV, $\tau = 10^{-11}$ sec at low temperatures, and $V_s = 10^5$ cm/sec. With an electron density of 10^{18} electrons/cm^3, expression (2.14) has the value $\alpha_{max} = \pm 10^3$ at $\omega = 10^9$ sec^{-1}, but this maximum occurs at unattainably high electric fields. For $H = 10^4$ G and electric fields of order 10 V, we get an amplification factor of less than unity. For lower electron densities, the situation becomes more favorable. When $n_0 = 10^{14}$ electrons/cm^3, expression (2.14) has the value $\alpha_{max} = \pm 10$ for $\omega = 10^9$ sec^{-1}, $\alpha_{max} = \pm 5 \times 10^3$ for $\omega = 10^{10}$ sec^{-1}, and $\alpha_{max} = \pm 10^5$ for $\omega = 10^{11}$ sec^{-1}. The maximum in this case occurs for values of V_H/V_s of order unity while for the case of higher densities, the maximum occurs for V_H/V_s of order 10^2. Therefore, at high sound

frequencies and low carrier densities, the amplification factor can become very large.

Physically, for these high frequencies and low-carrier densities, the screening of the ionic currents by the electronic currents breaks down. When this occurs in semiconductors,[5] the ultrasonic absorption, and when $V_H/V_s > 1$, the ultrasonic amplification, increase quite rapidly with frequency and the amplification can attain high values. For higher frequencies the breakdown of screening becomes complete and the amplification begins to saturate with frequency as long as the condition $\omega\tau < 1$ holds. It is in this frequency range, i.e., where the breakdown of the screening is complete, that the amplification factor can become quite large.

Note added in proof. The condition for the screening of the transverse current to break down is equivalent to the vanishing of the Hall field accompanying the sound wave. In the case considered by Dumke and Haering,[4] the vanishing of the Hall field was accomplished because there were equal concentrations of electrons and holes. However, at high enough sound frequencies, namely those frequencies at which the transverse screening breaks down, the Hall field will vanish even if there is only one type of carrier present

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Effects of Electron Correlations on the Properties of Alkali Metals*

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The effects of electron correlations on the single-particle processes in alkali metals have been calculated incorporating the more appropriate band theory masses at the Fermi surface rather than those at the bottom of the conduction band, as previously used. This modification substantially improves the agreement between theory and experiment.

IN previous papers,[1,2] the effects of electron correlations on the low-temperature specific heat and the Pauli paramagnetic spin susceptibility were calculated by means of a momentum transfer interpolation procedure designed to obtain results relevant to the region of metallic densities. The approximations used were the random phase approximation (RPA) for long-wavelength interactions and second-order perturbation theory with the neglect of parallel spin interactions for the short-wavelength interactions. The latter approximation corresponds to the physical consideration that electrons of parallel spin do not get close enough to interact via short-range forces. As Pines has shown,[3]

the influence of band structure on the correlation corrections is small for the alkali metals and the effects of the lattice can be taken into account by the introduction of the lattice effective mass into the kinetic energy term. However, the effective masses which were introduced both in Pines' work and in I correspond to masses evaluated at the bottom of the conduction band. For properties such as low-temperature specific heat and Pauli paramagnetism, one is concerned with single-particle excitations in the immediate vicinity of the Fermi surface. A *proper* account of the lattice would *not* correspond to the band parameters evaluated at the bottom of the band but rather the parameters evaluated at the Fermi level itself. These latter parameters, with considerations of Fermi-surface distortion, have recently been calculated by Ham.[4] It is the purpose of this note

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[1] S. D. Silverstein, Phys. Rev. **128**, 631 (1962) (referred to as I).
[2] S. D. Silverstein (to be published).
[3] D. Pines, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, Inc. New York, 1955), Vol. 1, p. 367.
[4] F. S. Ham, Phys. Rev. **128**, 2524 (1962).

to apply these new Fermi surface parameters to the correlation contributions and to show that one now gets quite reasonable agreement with experiment. The agreement for the Pauli susceptibility is better than that for the specific heat. This is easily understood because only the effects of electron-electron interactions were considered; while these are the dominant contributing factors in the former process,[5] one must also consider phonon corrections in the specific heat process.

In I, we expressed the low-temperature specific-heat ratio by (for $m^*/m = 1$)

$$C/C_0 = [1 + \tfrac{1}{2}(\alpha r_s)^2 \sum{}' (1)]^{-1}. \tag{1}$$

Here, $\sum{}' (1)$ corresponds to the derivative of the electron self-energy. Curves were plotted for C/C_0 as a function of r_s and the results for a given lattice effective mass could then be established by the use of the relation[6]

$$\frac{C^*}{C_0} = \left(\frac{m}{m^*} + \frac{C_0}{C} - 1\right)^{-1}. \tag{2}$$

In Fig. 1 we plot a similar curve for the Pauli susceptibility. This was derived by means of the momentum-transfer interpolation procedure coupled with a variational calculation about a polarized ground
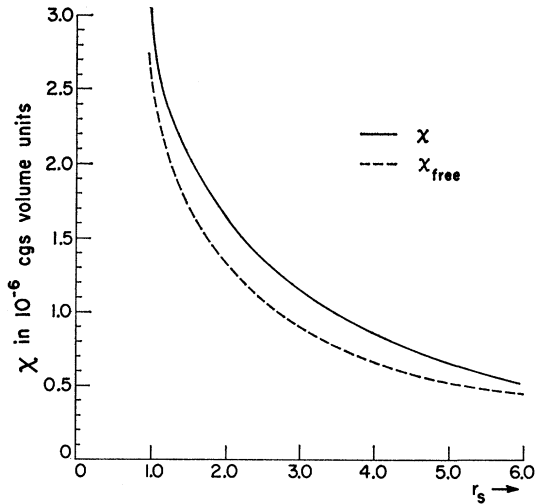

FIG. 1. Plot of susceptibility for $m^*/m = 1$.

[5] D. Pines (private communication).

[6] In I, the numerical results quoted were for the ratio of the interacting specific heat with an effective mass to the free-electron value, also with an effective mass. We represented this by $(C/C_0)^*$. The results quoted in the present note correspond to the conventional definition where C_0 is the completely free electron value.

TABLE I. Results for alkali metals. The theoretical values given are at 0° K.

	Li	Na	K	Rb	Cs
r_s	3.22	3.96	4.87	5.18	5.57
$(m^*/m)_{\text{av F.S.}}$[a]	1.66	1.00	1.09	1.21	1.76
$(C^*/C_0)_{\text{theo}}$	1.96	1.15	1.35	1.56	2.86
$(C/C_0)_{\text{exp}}$[b]	2.20	1.27	1.27	1.30	1.59
$\chi_{\text{theo}}^* \times 10^{+6}$	2.20[c]	0.86	0.73	0.78	1.15
$\chi_{\text{exp}} \times 10^{+6}$	2.08 ± 0.1[d]	$\begin{cases} 0.95 \pm 0.1^{\text{d}} \\ 0.89 \pm 0.04^{\text{e}} \\ 1.13 \pm 0.05^{\text{f}} \end{cases}$			

[a] See F. S. Ham (reference 4); these values correspond to averaged "thermal masses" which take into account Fermi surface (F.S.) distortion.
[b] Best values from compilation of all data extrapolated to 0°K.
[c] $\chi_{\text{theo}}^*(\text{Li}) = 2.16 \times 10^{-6}$ cgs volume units at room temperature.
[d] See reference 7.
[e] R. T. Schumacher and W. E. Vehse, Bull. Am. Phys. Soc. 4, 296 (1960).
[f] R. T. Schumacher and W. E. Vehse, J. Phys. Soc. Japan (Suppl.) B-1, 460 (1962).

state; the polarization parameter being determined as the extremum condition on the ground-state energy. The values of the susceptibility for a given effective mass can be obtained from this curve by the use of

$$\chi^* = \frac{\chi}{1 + (m/m^* - 1)\chi/\chi_{\text{free}}}. \tag{3}$$

In Table I we give the results for the alkali metals. We see that the theoretical results, except for the specific heat of Cs, are in quite reasonable accord with the experimental values. Presuming there is no cause for the alteration of the theoretical band masses, the large deviation for Cs most likely arises from one of the two following causes: (1) a large negative interference term in the masses arising from the electron-phonon interaction, or (2) the Fermi surface of Cs is distorted in a manner such that a free-electron model for the calculation of correlation effects is just not appropriate. An experiment which would help to determine the cause is a measurement of Pauli susceptibility for Cs by the method of Schumacher and Slichter[7] (remembering that phonons play a negligible role in this process). Spin resonaance has been observed for Cs in ammonia solution,[8] and with a pure enough sample at low enough temperatures, the experiment could presumably be performed for a bulk sample.

The author would like to thank Dr. F. S. Ham for permission to use his band theory results prior to publication.

[7] R. T. Schumacher and C. P. Slichter, Phys. Rev. 101, 58 (1956).
[8] R. A. Levy, Phys. Rev. 102, 31 (1956).