Low-Field Galvanomagnetic Coefficients of Gallium*

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Low-field galvanomagnetic measurements have been made on gallium single crystals at 77°K and the eighteen independent coefficients specified in a phenomenological equation determined. The coefficients are highly anisotropic. The resistivities differ by a factor of 7.4, the Hall constants by a factor of 10.3, and the magnetoresistance coefficients differ by a factor of 160. The possibility of fitting a "many-valley" model to the data is considered but it is concluded that the use of this model is not appropriate for gallium.

INTRODUCTION

It has been the practice to present the results of low-field galvanomagnetic measurements in terms of a phenomenological relation in which the resistivity is expanded in powers of the magnetic field and the independent nonzero coefficients are determined by applying crystal symmetry and the Onsager relations.

A method of determining these coefficients has been published for all the crystal point groups by Kao and Katz[1] and explicitly for gallium (Point Group D_{2h}) by Yahia and Marcus.[2] For gallium the electric field may be expressed to the order B^2 as

$$E_i=J_i(\rho_{ii}+\rho_{iiii}B_i^2+\rho_{iijj}B_j^2+\rho_{iikk}B_k^2)+J_j(\rho_{ijk}B_k+2\rho_{ijji}B_iB_j)+J_k(-\rho_{ikj}B_j+2\rho_{ikki}B_iB_k). \quad (1)$$

After applying the Onsager relations which require that $\rho_{ijk}=-\rho_{jik}$ and $\rho_{ijji}=\rho_{jiji}$,[3] Eq. (1) yields eighteen independent coefficients.

The three resistivities and the three Hall coefficients measured at 77°K have been reported by Yahia and Marcus[2] and a preliminary report of this work has been made.[4] In this work we measured the galvanomagnetic effects in gallium at 77°K and calculated all of the eighteen independent coefficients in the phenomenological relation to order B^2.

EXPERIMENTAL DETAILS

Single crystals 1 mm square and 20 mm long were prepared from 99.999% pure gallium obtained from Alcoa and oriented so that the crystallographic axes were aligned with the specimen axes to within one degree. In addition to the two current leads, eight potential leads were attached to each crystal so that the potential measured between one pair of leads could be checked against that measured between a similar pair on the same crystal. The crystals are identified by a letter which represents the axis along which the current flowed. The identification of the axis follows the notation of Bradley[5] ($b_0<a_0<c_0$).

The potentials were measured by a standard dc technique. The signal was amplified and plotted on an x-y recorder as a function of either magnetic field strength or magnetic field direction. Since any change in the strength or direction of the field induced a transient potential in the measuring circuit, the data were taken pointwise, but the points were taken close enough together so that in effect, a continuous curve was plotted. Because the raw data were quasicontinuous, the reduced curves presented are continuous and show no data points.

For specific details on the preparation and mounting of crystals, the measuring circuits, and the method of taking data see Reed and Marcus.[6]

EXPERIMENTAL RESULTS

Both rotation and field dependence curves for the Hall voltage, magnetoresistance, and planar Hall voltages were taken on the crystals measured. Typical rotation curves are shown in Figs. 1 to 3, and typical deviation of the field dependence curves from a straight line is due to higher order terms coming into effect at higher fields, the cubic terms in the Hall data being about 4% of the linear terms at 5 kG and the quartic terms in the planar Hall and magnetoresistance data being about 4% of the quadratic terms at 25 kG squared. The presence of a quartic term can be seen in the transverse magnetoresistance rotation curve for $J\|c$ [Fig. 2(b)] where the higher order terms cause the deviation of this curve at $B\|b$ from the expected $A\cos^2\theta+B\sin^2\theta$ dependence.

The eighteen coefficients were computed by fitting a cubic equation to the Hall field dependence data and a quartic equation to the planar Hall and magnetoresistance field dependence data by the method of "least squares." Because at the highest fields, still higher order terms were apparent, the coefficients were

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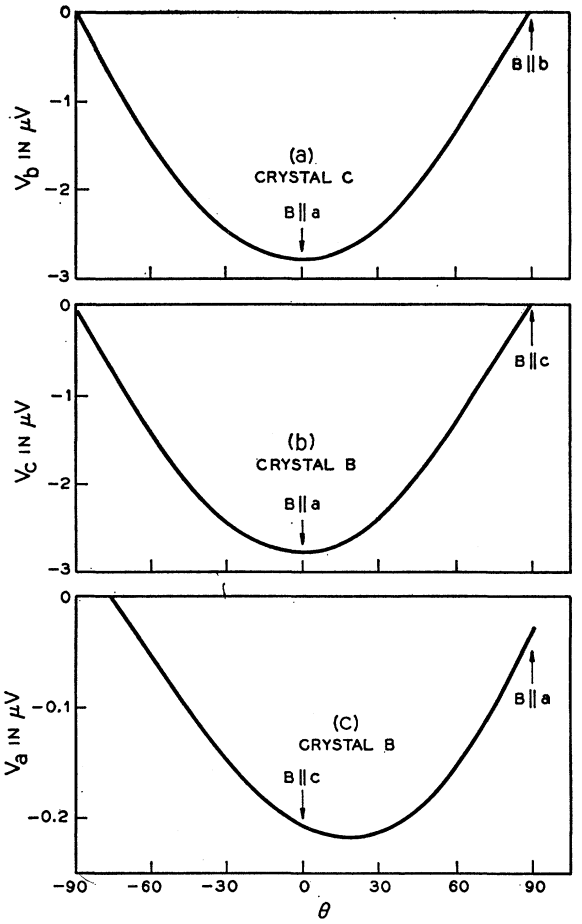

FIG. 1. Hall voltage vs magnetic field direction, $I=4$ A, $B=10$ kG, $T=77°$K; (a) Crystal C, (b) Crystal B, (c) Crystal B.

only calculated for magnetic fields up to 10 kG. These coefficients are given in Table I.

TABLE I. Galvanomagnetic coefficients for gallium at 77°K.

$\rho_{aa}=3.0\times10^{3}$	$\rho_{bb}=1.4\times10^{3}$	$\rho_{cc}=10.4\times10^{3}$
$\rho_{abc}=-0.6\times10^{-3}$	$\rho_{cab}=-7.1\times10^{3}$	$\rho_{bca}=-7.6\times10^{-3}$
$\rho_{aaaa}=0.4\times10^{-7}$	$\rho_{aabb}=1.4\times10^{-7}$	$\rho_{aacc}=3.1\times10^{-7}$
$\rho_{bbaa}=0.8\times10^{-7}$	$\rho_{bbbb}=0.2\times10^{-7}$	$\rho_{bbcc}=3.4\times10^{-7}$
$\rho_{ccaa}=24\times10^{-7}$	$\rho_{ccbb}=32\times10^{-7}$	$\rho_{cccc}=6.4\times10^{-7}$
$\rho_{abba}=-0.2\times10^{-7}$	$\rho_{acca}=0.4\times10^{-7}$	$\rho_{bccb}=0.8\times10^{-7}$
$[\rho_{ij}]=ab$ Ω-cm	$[\rho_{ijk}]=ab$ Ω-cm/G	$[\rho_{ijkl}]=ab$ Ω-cm/G^2

The variation of purity, mounting, etc., from crystal to crystal was minimized by calculating all eighteen coefficients from the measurements made on only three crystals. Although the coefficients given in Table I are for the three crystals, other crystals were measured, and the coefficients calculated for them agree with the values in the table to within the experimental error. The error in these measurements is due to misalignment of the magnetic field, measurement of the voltages, measurement of the crystal dimensions, placement of the leads, and misalignment of the crystal and specimen axes. The effect of misalignment of the crystals and misplacement of the leads is very apparent in Fig. 1(c) where it can be seen that the maximum of the Hall rotation curve occurs when the magnetic field makes an angle of 20° with the c axis. Yahia and Marcus[2] have shown that the large anisotropy of the Hall constants combines with a small rotation of the crystal axis to produce the above effect. Since the one-degree misalignment of the crystal is not enough to account

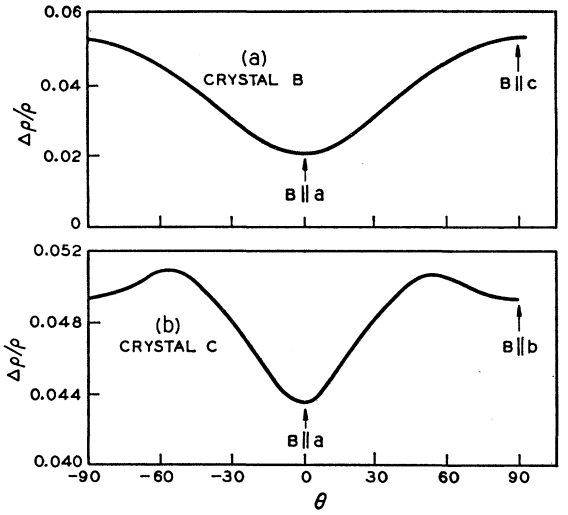

FIG. 2. Magnetoresistance vs magnetic field direction $I=2$ A. $B=18.1$ kG, $T=77°$K; (a) Crystal B, (b) Crystal C.

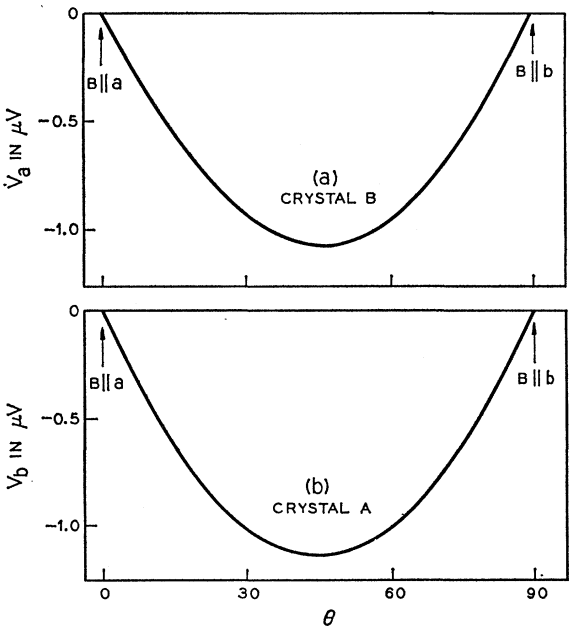

FIG. 3. Planar Hall voltage vs magnetic field direction, $I=4$ A, $B=12$ kG, $T=77°$K; (a) Crystal B, (b) Crystal A.

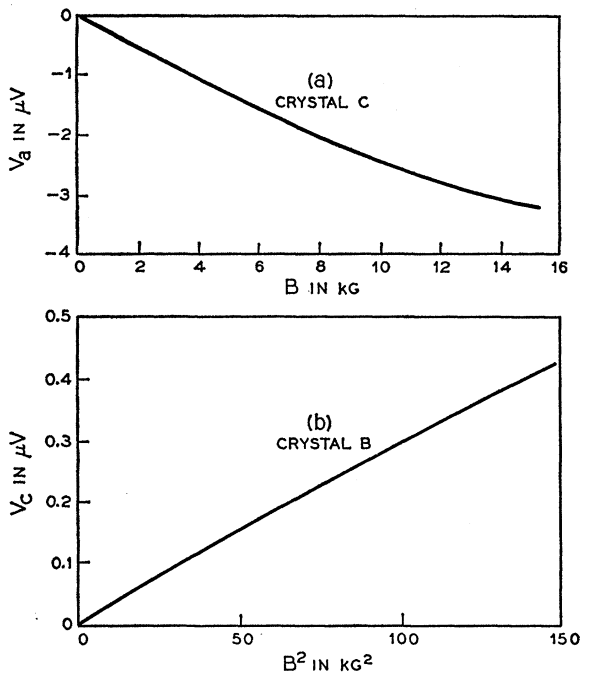

FIG. 4. (a) Hall voltage vs magnetic field strength, Crystal C; $I=4$ A, $T=77°$K, $B\|b$ axis. (b) Planar Hall voltage vs magnetic field strength Crystal B; $I=4$ A, $T=77°$K, field is 45° away from b axis in bc plane.

numerically for the shift observed, it is calculated that the leads must be out of alignment by 1 deg. This is not unreasonable since a 1-deg misalignment is produced if the leads are misplaced by one-half of the wire diameter. If these various sources of error are taken into account, the estimated error is $\pm 0.1\times 10^{3}$ for the resistivities, $\pm 0.2\times 10^{-3}$ for the Hall constants, and $\pm 0.2\times 10^{-7}$ for the magnetoresistance and planar Hall coefficients except for ρ_{ccaa} and ρ_{ccbb} where the error is $\pm 2\times 10^{-7}$. The field dependence curves are shown in Figs. 4 and 5.

DISCUSSION OF THE DATA

Several points should be noted about these coefficients. The first is the large anisotropy. The resistivities differ by a factor of 7.4, the Hall constants differ by a factor of 10.3, and the magnetoresistance coefficients differ by a factor of 160. The second is that the longitudinal magnetoresistance ρ_{cccc} is larger than the magnetoresistance for the current in either the a or b directions. The longitudinal effect is usually smaller than the transverse effect. The final point is that the Onsager relations were tested in the three crystals and were found to check to within the experimental error. This is not a trivial result since the coefficients in most crystals are equal from crystal symmetry and not from the Onsager relations. Gallium is not of the few metals which has low enough symmetry to effectively test these relations. Figures 1(a) and 1(b) and Figs. 3(a) and 3(b) illustrate the validity of these relations for the Hall coefficients ρ_{bca} and ρ_{cba} and the planar Hall coefficients ρ_{abba} and ρ_{baba}. For the field in a given direction it is seen that the interchange of the current and electric field directions does not affect the measured voltages and hence the coefficients.

A "many-valley" model of the Fermi surface, developed primarily by Herring,[7] Abeles and Meiboom,[8] and Drabble and Wolfe,[9] has been used successfully to interpret the low-field galvanomagnetic effects in semiconductors and in the semimetals bismuth[8] and antimony.[10] Since the data for gallium at 77°K fit the phenomenological expansion of the resistivity, it is of interest to consider this model.

From his analysis of the low-field de Haas–van Alphen data, Shoenberg[11] has proposed the use of three ellipsoids, each having its axes along the crystallographic axes. If this model represents the complete Fermi surface, the predicted longitudinal magnetoresistance for all three axes is zero. However, the measurements show that none of the longitudinal coefficients is zero so that this arrangement cannot represent the total surface. Since the low-field de Haas–van Alphen effect measures only the small pieces of the Fermi surface, it is still possible that these ellipsoids form a small band which is only one of several bands contributing to the magnetoresistance.

A general one-band model, consisting of eight ellip-

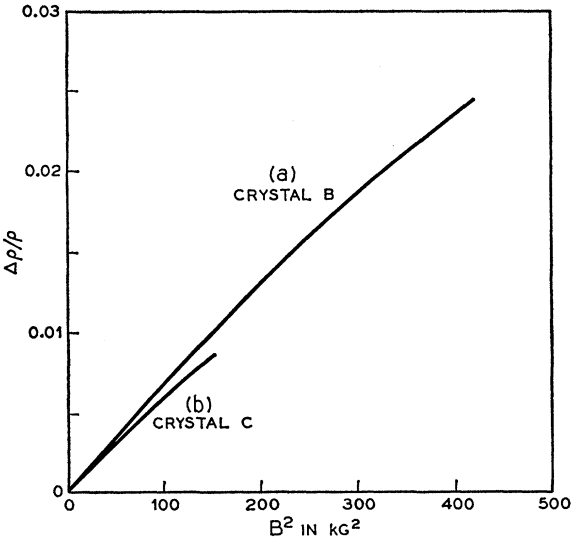

FIG. 5. Magnetoresistance vs magnetic field strength $I=2$ A, $T=77°$K. (a) Transverse magnetoresistance, crystal B, $B\|a$ axis. (b) Longitudinal magnetoresistance, crystal C, $B\|c$ axis.

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soids with 7 adjustable parameters, has been tried without success. This result is not unexpected since the galvanomagnetic measurements at[6] 4.2°K indicate that gallium has at least two bands and an equal number of holes and electrons. At present, a general two-band model with fifteen variables has not been calculated due to the difficulty in solving the simultaneous equations. However, it is not expected that a suitable fit would be found even if the equations for the two-band model were solved. Galvanomagnetic[6] and magnetoacoustic[12] measurements suggest that the Fermi surface of gallium approximates the free-electron surface and this surface is far from ellipsoidal. It should be remembered that although a given set of ellipsoids has a corresponding set of galvanomagnetic coefficients, the converse is not true and it may not be possible to represent a set of galvanomagnetic coefficients by a set of ellipsoids.

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APPENDIX

Since most theoretical models calculate conductivities rather than resistivities, it is necessary to invert the resistivity tensor. The relations between the resistivity and conductivity coefficients are as follows:

$$\sigma_{ii}=\rho_{ii}^{-1},$$
$$\sigma_{ijk}=\rho_{ijk}/\rho_{ii}\rho_{jj},$$
$$\sigma_{iiii}=-\rho_{iiii}/\rho_{ii}^{2},$$
$$\sigma_{iijj}=-\rho_{iijj}/\rho_{ii}^{2}-\rho_{kij}^{2}/\rho_{ii}^{2}\rho_{kk},$$
$$\sigma_{ijij}=-\rho_{ijij}/\rho_{ii}\rho_{jj}+\tfrac{1}{2}\rho_{jki}\rho_{kij}/\rho_{ii}\rho_{jj}\rho_{kk}.$$

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Ferromagnetic Resonance of Holmium Nitride

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Taking into account the quantum mechanical contraction of the spin moment due to the partial quenching of the angular momentum of the trivalent holmium ions, the effective g factor of the ferromagnetic compound HoN is obtained by means of a spin-wave approximation and neglecting the effect of the sixth-order cubic crystalline field. It is also assumed that only the two lowest lying states of each ion need be considered, and that there is no accidental crossover between them. Using a parameter which is estimated from the effective Bohr magneton number observed, the value of the effective g factor is predicted to be about 8.

IT has been reported that many of the heavy rare earth compounds have a cubic structure and become ferromagnetic at low temperatures.[1–3] The characteristic magnetic structures of the rare earth nitrides having an NaCl structure were reported by Wilkinson and his associates[1] and the intermetallic compounds with a cubic Laves structure were investigated by Bozorth and others,[2] and Williams *et al.*[3]

Kittel,[4] Van Vleck,[5] and Dillon snd Walker[6] reported the ferromagnetic resonance phenomena in rare earth iron garnets. Cooper *et al.*[7] and Niira[8] reported analyses of the magnetic behavior of heavy rare earth metals using a spin-wave approximation. One of the present authors has calculated the temperature dependence of the spontaneous magnetization of holmium nitride.[9]

The ferromagnetic resonance phenomenon in holmium nitride is of great theoretical interest to physicists. The present paper treats the ferromagnetic resonance of HoN by employing the same simple model as that used in a previous paper by one of the present authors,[9] in which the quantum-mechanical contraction of the effective spin moment due to the partial quenching of the angular momentum was essentially taken into account. In the authors' model it is assumed simply that there are only the two lowest lying states in each ion and further that there is no accidental crossover[10] between them.[9]

Next consider a ferromagnetic system composed of N trivalent holmium ions which are subjected to a cubic crystalline field and which interact with one another by an exchange energy. At absolute zero temperature,

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