soids with 7 adjustable parameters, has been tried without success. This result is not unexpected since the galvanomagnetic measurements at[6] 4.2°K indicate that gallium has at least two bands and an equal number of holes and electrons. At present, a general two-band model with fifteen variables has not been calculated due to the difficulty in solving the simultaneous equations. However, it is not expected that a suitable fit would be found even if the equations for the two-band model were solved. Galvanomagnetic[6] and magnetoacoustic[12] measurements suggest that the Fermi surface of gallium approximates the free-electron surface and this surface is far from ellipsoidal. It should be remembered that although a given set of ellipsoids has a corresponding set of galvanomagnetic coefficients, the converse is not true and it may not be possible to represent a set of galvanomagnetic coefficients by a set of ellipsoids.

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ACKNOWLEDGMENTS

We would like to thank J. H. Condon for his assistance with the experimental problems and discussions of the data, and Dr. C. R. Kannewurf for discussions on the "many-valley" theory.

APPENDIX

Since most theoretical models calculate conductivities rather than resistivities, it is necessary to invert the resistivity tensor. The relations between the resistivity and conductivity coefficients are as follows:

$$\sigma_{ii}=\rho_{ii}^{-1},$$
$$\sigma_{ijk}=\rho_{ijk}/\rho_{ii}\rho_{jj},$$
$$\sigma_{iiii}=-\rho_{iiii}/\rho_{ii}^{2},$$
$$\sigma_{iijj}=-\rho_{iijj}/\rho_{ii}^{2}-\rho_{kij}^{2}/\rho_{ii}^{2}\rho_{kk},$$
$$\sigma_{ijij}=-\rho_{ijij}/\rho_{ii}\rho_{jj}+\tfrac{1}{2}\rho_{jki}\rho_{kij}/\rho_{ii}\rho_{jj}\rho_{kk}.$$

PHYSICAL REVIEW VOLUME 130, NUMBER 3 1 MAY 1963

Ferromagnetic Resonance of Holmium Nitride

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(Received 19 November 1962)

Taking into account the quantum mechanical contraction of the spin moment due to the partial quenching of the angular momentum of the trivalent holmium ions, the effective g factor of the ferromagnetic compound HoN is obtained by means of a spin-wave approximation and neglecting the effect of the sixth-order cubic crystalline field. It is also assumed that only the two lowest lying states of each ion need be considered, and that there is no accidental crossover between them. Using a parameter which is estimated from the effective Bohr magneton number observed, the value of the effective g factor is predicted to be about 8.

IT has been reported that many of the heavy rare earth compounds have a cubic structure and become ferromagnetic at low temperatures.[1–3] The characteristic magnetic structures of the rare earth nitrides having an NaCl structure were reported by Wilkinson and his associates[1] and the intermetallic compounds with a cubic Laves structure were investigated by Bozorth and others,[2] and Williams *et al.*[3]

Kittel,[4] Van Vleck,[5] and Dillon snd Walker[6] reported the ferromagnetic resonance phenomena in rare earth iron garnets. Cooper *et al.*[7] and Niira[8] reported analyses of the magnetic behavior of heavy rare earth metals using a spin-wave approximation. One of the present authors has calculated the temperature dependence of the spontaneous magnetization of holmium nitride.[9]

The ferromagnetic resonance phenomenon in holmium nitride is of great theoretical interest to physicists. The present paper treats the ferromagnetic resonance of HoN by employing the same simple model as that used in a previous paper by one of the present authors,[9] in which the quantum-mechanical contraction of the effective spin moment due to the partial quenching of the angular momentum was essentially taken into account. In the authors' model it is assumed simply that there are only the two lowest lying states in each ion and further that there is no accidental crossover[10] between them.[9]

Next consider a ferromagnetic system composed of N trivalent holmium ions which are subjected to a cubic crystalline field and which interact with one another by an exchange energy. At absolute zero temperature,

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the effective spin moment of all the rare earth ions is directed along the molecular field caused by the exchange energy. If the relaxation of the rare earth ions is not very great, each energy level of Ho^{+3} can be well defined. When the over-all splitting of the ground multiplet of the trivalent holmium ions in the compound is considered to be small compared with the separation of the excited multiplets from the ground multiplet,[5] the total angular momentum J_i for the ground multiplet of the ith Ho^{+3} ion is a good quantum number.

On the other hand, the exchange energy

$$-A\sum_{i\neq j}(\mathbf{J}_i\mathbf{J}_j)$$

also causes a dynamical effect upon the spin system, in which A is the exchange integral between the nearest neighboring ions. This implies that excitation of a rare earth ion induced by an applied small microwave field can propagate through the entire crystal and that in the crystal a certain kind of spin wave would be created which is slightly affected by the exchange stiffness but is affected by the anisotropic field to a larger degree.

In an external magnetic field H applied parallel to the quantization axis, i.e., ⟨100⟩, the total Hamiltonian $\mathcal{H}$ involves the unperturbed one-ion Hamiltonians $\mathcal{H}_{0i}$ composed of the cubic crystalline potential $V(\mathbf{J}_i)$, the Zeeman energy, and the perturbation $\mathcal{H}'$. The Zeeman energy is proportional to the molecular field $-2AZ_0\langle\langle J^z\rangle\rangle$ parallel to ⟨100⟩, where Z_0 is the coordination number. The perturbation $\mathcal{H}'$ involves the terms of the dynamical exchange effect defined by $-A\sum_{i\neq j}(\mathbf{J}_i\mathbf{J}_j)+2AZ_0\langle\langle J^z\rangle\rangle\sum_i J_i{}^z$ and the Zeeman energy $-g_J\mu_B H\sum_i J_i{}^z$ due to H, where g_J is the Landé g factor and μ_B the Bohr magneton. The energy quantum of this spin system can be calculated from the difference ΔE between the two energies of the ground and the first excited states for $\mathcal{H}$. The two energies mentioned above are calculated by using the following two wave functions: for the ground state

$$\Psi_g=\varphi_g(1)\varphi_g(2)\cdots\varphi_g(N), \tag{1}$$

and for the first excited state

$$\Psi_e=\frac{1}{\sqrt{N}}\sum_i\varphi_g(1)\varphi_g(2)\cdots\varphi_e(i)\cdots\varphi_g(N), \tag{2}$$

in which the normalized wave functions

$$\varphi_g=\sum_m a_m|m\rangle,\quad\text{and}\quad\varphi_e=\sum_n b_n|n\rangle \tag{3}$$

correspond to the ground state and first excited state eigenfunctions obtained by solving the secular equations for the unperturbed Hamiltonians $\mathcal{H}_{0i}$, respectively.[11] In Eq. (3), a_m and b_n are the linear combination factors of the wave functions $|m\rangle$ and $|n\rangle$, in which m and n stand for the z components of the total angular momentum of the free ionic states. The ΔE obtained is composed of two terms, one of which corresponds to the anisotropy and the other is proportional to the external magnetic field,

$$\Delta E=E_{\text{anis}}+g_{\text{eff}}\mu_B H, \tag{4}$$

where g_{eff} is the effective Landé g factor obtained in the form

$$g_{\text{eff}}=g_J(\textstyle\sum_m m|a_m|^2-\sum_n n|b_n|^2). \tag{5}$$

If the cubic crystalline field completely disappears, the energy difference is reduced to the well-known Zeeman energy, $g_J\mu_B H$. Neglecting the effect of the sixth-order cubic crystalline field, the numerical calculation of g_{eff} is carried out by using data for the wave functions tabulated by the present authors in a previous paper.[11] The result is shown in Fig. 1, where the parameter u stands for the ratio of the intensity of the fourth-order cubic crystalline field to the effective exchange field.[11] When the mixing degree u of the cubic crystalline potential with the exchange energy is increased, g_{eff} increases gradually at first from a value of 5/4, and after reaching a certain point $u\approx0.5$ it increases very rapidly and approaches a constant value of about 9 until the value $u\approx0.8$ is reached. When u increases still further, g_{eff} decreases sharply to zero.

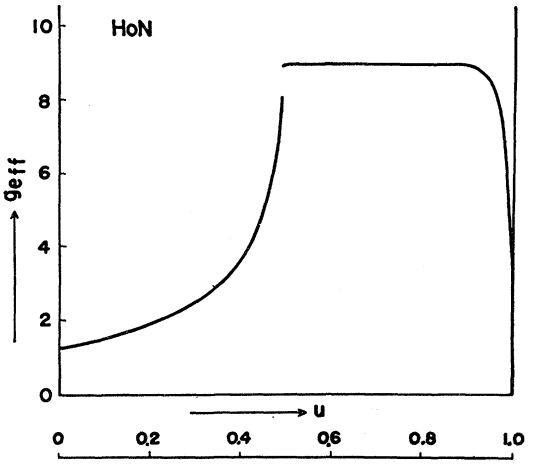

FIG. 1. Effective g factor of HoN at zero temperature. The parameter expresses the degree of mixing of the fourth-order cubic crystalline potential with the exchange field. The abrupt change at the point $u\approx0.5$ is due to the fact that there is an accidental crossover between the first excited state and second excited state which occurs at this point.

We have already obtained the parameter dependence of the effective Bohr magneton numbers of Ho^{+3} in the cubic crystalline potential and effective exchange field.[11] Using the effective Bohr magneton number observed by Wilkinson and his collaborators[1] for the calculated value of the Bohr magneton number,[11] we have a value of about 0.9 for the parameter u.[9] The predicted value of g_{eff} is calculated to be about 8.

The effect of the sixth-order cubic crystalline field on the value of g_{eff} in several rare earth compounds with cubic structure is discussed in detail in another paper.

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