agrees quite well with three other quite different types of calculations, the pure p-shell calculations, the approach of Warburton and Pinkston, and the approach of Talmi and Unna, gives strength to the shell-model assignments of the energy levels which are given in this paper.

It should be stressed that the disagreement between the positions of the calculated energy levels and the positions of the experimentally observed energy levels is most probably due to the neglect of the deformation and core-excitation of the C^{12} core and not due to ignorance of the parameters ν_p, ν_s, and ν_d of the harmonic oscillator wave functions.

It is to be noted from Table IV that the eigenfunctions for practically all the states are quite pure jj two-particle wave functions. This fact is also true of the unlisted eigenfunctions. This purity of the eigenfunctions appears to have a direct connection with the conjecture of Talmi and Unna[8] that it is possible to use pure jj wave functions and an effective potential to calculate energy eigenvalues. That is, in some manner which is not completely clear, the effective potential seems to include some of the more important aspects of configuration mixing.

Sebe[27] has recently calculated the positions and nuclear properties of the low-lying negative-parity states in N^{14} using a model in which a proton is coupled to a C^{13} core. The C^{13} core was assumed to exist in either the ground state or first excited state of C^{13} and the wave functions for these "basic" core states were obtained from an intermediate shell-model calculation.

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[27] T. Sebe (to be published).

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Low-Energy Process $\gamma+p\rightarrow K^{+}+\Sigma^{0}$*

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The reaction $\gamma+p\rightarrow\Sigma^{0}+K^{+}$ is discussed using the model developed in a previous paper. For odd-$K\Sigma$ parity the differential cross section can be accounted for by the one-nucleon pole term and the K and K^{*} exchange terms. With this model it is very difficult to fit the data for even-$K\Sigma$ parity. The coupling constant found for odd-$K\Sigma$ parity is $g_{\Sigma NK}^{2}/4\pi\approx4.5$, very close to the value $g_{\Lambda NK}^{2}/4\pi\approx4.0$ found in the previous paper.

I. INTRODUCTION

In a previous paper[1] a model was constructed for the photoproduction process $\gamma+p\rightarrow K^{+}+\Lambda^{0}$ at low energies. This model was based on the approximation of neglecting faraway singularities as viewed from the "physical region." The close resemblance in kinematics of the class of strange particle production processes, viz., $\gamma+N\rightarrow K+Y$ and $\pi+N\rightarrow K+Y$, suggests that the same model should hold for all of them. In the following the model is applied to $\gamma+p\rightarrow K^{+}+\Sigma^{0}$.

The terms to be taken in our calculation would thus be the one-nucleon term in the direct channel (s channel),[2] as well as the K^{+} and K^{*} exchange terms. Since there is no evidence to date of any enhancement in a particular multipole state of the $K\Sigma$ system above the production threshold, we shall not have contributions due to such enhancements. It cannot be overemphasized that this very simple model would not be adequate as the energy gets higher. It is our hope, however, that it will give a description of what is happening in the low-energy region and serves as a guide in the high-energy region.

Now let us turn to the experimental side. Up to the present only very scanty data exist for $\gamma+p\rightarrow K^{+}+\Sigma^{0}$. Several measurements of this process were made before 1960 at California Institute of Technology and at Cornell.[3] Recently new data became available from the work done at Cornell.[4] We will compare our model with the new data. The experiments are still proceeding and

* Supported in part by the Office of Naval Research.
[1] T. K. Kuo, Phys. Rev. **129**, 2264 (1963). This paper will hereafter be referred to as I.
[2] The 3−3 resonance, for simplicity, is neglected. When more experimental information becomes available, we should put in its contribution.
[3] A summary of these can be found in F. Turkot, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 369.
[4] R. L. Anderson, E. Gabathuler, D. Jones, B. D. McDaniel, and A. J. Sadoff, Phys. Rev. Letters **9**, 131 (1962).

further data will undoubtedly offer us a clearer picture in the near future.

In Sec. II we explain our notation and give the expression for $d\sigma/d\Omega$. The comparison with experiments is made in Sec. III. Section IV will be devoted to discussions of the results and related subjects.

II. KINEMATICS AND DIFFERENTIAL CROSS SECTIONS

The notation used is that of I, changing everything associated with Λ^0 to that with Σ^0. Thus, the four-momenta for the photon, the nucleon, the K-meson, and the Σ^0 hyperon will be k, p_1, q, and p_2, respectively. The c.m. system variable is used throughout, as in I.

The parity of the Σ hyperon has not been definitely established, although there is strong evidence favoring its being odd[5] [i.e., $P(K\Sigma)=-1$]. In this paper we carry out the calculation for both parities. As the odd-parity formalism was presented in I, we give only that for even parity in the following. Even-parity quantities will be indicated with a prime. The T matrix can be written

$$T'=\sum_{i=1}^{4} A_i'\mathfrak{M}_i', \tag{1}$$

where

$$\mathfrak{M}_i'=-\gamma_5\mathfrak{M}_i, \tag{2}$$

and the $\mathfrak{M}_i'$'s have been defined in I. Further, the differential cross section can be expressed as

$$\frac{d\sigma}{d\Omega}=\frac{q}{k}|\chi_f^+\mathfrak{F}'\chi_i|^2, \tag{3}$$

and $\mathfrak{F}'$ will take the form

$$\mathfrak{F}'=i\boldsymbol{\sigma}\cdot(\hat{k}\times\hat{\epsilon})\mathfrak{F}_1'-\boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\sigma}\cdot\hat{\epsilon}\mathfrak{F}_2'+i\sigma\cdot\hat{k}\sigma\cdot(\hat{k}\times\hat{\epsilon})\mathfrak{F}_3' +i\boldsymbol{\sigma}\cdot\hat{q}\hat{q}\cdot(\hat{k}\times\hat{\epsilon})\mathfrak{F}_4', \tag{4}$$

which is obtained from the corresponding quantity $\mathfrak{F}$ by the replacement $\hat{\epsilon}\rightarrow\hat{k}\times\hat{\epsilon}$ and $\hat{k}\times\hat{\epsilon}\rightarrow-\hat{\epsilon}$. This amounts to an interchange of even and odd parties of the photon (magnetic multiples $\leftrightarrow$ electric multipoles), and in turn amounts to an interchange of the parity of the $K\Sigma$ system. Thus, the multipole decomposition of $\mathfrak{F}_i'$ is simply obtained from (11) of I by changing $M_{l^\pm}$ and $E_{l^\pm}$ into $E_{l^\pm}$ and $M_{l^\pm}$, respectively. The relations between $\mathfrak{F}_i'$ and A_i', however, are somewhat more cumbersome than those for the odd-parity case. They will not be recorded here since they are not used in the following calculation. Simpler relations are obtained by adapting the spinor matrices to the forms which are obtained from $\bar{u}\mathfrak{M}_i'u$. Such relations can be found in the paper by Fayyazuddin.[6] The translation of one notation into the other is easy.

[5] R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters **8**, 175 (1962).

[6] Fayyazuddin, Phys. Rev. **123**, 1882 (1961). See also S. Hatsukade and H. J. Schnitzer, *ibid.* **128**, 468 (1962). Note the different definition of $\mathfrak{M}'_i$ amplitudes used in this reference.

Let us now turn to the differential cross sections. For $P(K\Sigma)=-1$, the various terms are just given by formulas (14), (15), and (24) of I, always remembering to change g_Λ into g_Σ and m_Λ into m_Σ, etc. Note also that K^* has recently been determined to be a vector meson.[7] For $P(K\Sigma)=+1$, we have merely to make the replacement $\mathfrak{M}_i\rightarrow\mathfrak{M}_i'$ and $m_\Sigma\rightarrow-m_\Sigma$ in all expressions.

Using the usual technique of calculating traces, the differential cross section can be written in the following form:

$$\frac{d\sigma}{d\Omega}=\frac{1}{4}\frac{e^2g_\Sigma^2}{(4\pi)^2}\frac{q}{k}\frac{1}{s}\bigg(X_0+\frac{C_1}{t-m_{K^*}^2}X_1+\frac{C_2}{t-m_{K^*}^2}X_2 +\frac{C_1^2}{(t-m_{K^*}^2)^2}X_3+\frac{C_1C_2}{(t-m_{K^*}^2)^2}X_4+\frac{C_2^2}{(t-m_{K^*}^2)^2}X_5\bigg), \tag{5}$$

where [for $P(K\Sigma)=-1$,]

$$X_0=1+\frac{t-m_K^2}{s-m_N^2}+2\frac{q^2\sin^2\theta}{(t-m_K^2)^2}(\Delta^2-m_K^2) +(2m_N\mu_P)\bigg(1-\frac{m_\Sigma}{m_N}+\frac{t-m_K^2}{s-m_N^2}\bigg)+\mu_P^2(\Delta^2-t), \tag{6}$$

$$X_1=-\tfrac{1}{2}\mu_Pq^2\sin^2\theta(s-m_N^2)-\mu_Pt(s-m_N^2) +(t-m_K^2)\bigg[-\tfrac{1}{2}m_N(1+2m_N\mu_P)\frac{t-m_K^2}{s-m_N^2} -\tfrac{1}{2}\mu_Pt+\tfrac{1}{2}\Delta[1+\mu_P(3m_N+m_\Sigma)]\bigg], \tag{7}$$

$$X_2=\tfrac{1}{2}(1-\Delta\mu_P)(s-m_N^2)q^2\sin^2\theta+(1-\Delta\mu_P)t(s-m_N^2) +(t-m_K^2)\bigg\{-\frac{\Delta m_N}{2}(1+2\mu_Pm_N)\frac{t-m_K^2}{s-m_N^2} +t[1+(m_N-\tfrac{1}{2}\Delta)\mu_P]-\frac{M\Delta}{2}(1-\Delta\mu_P)\bigg\}, \tag{8}$$

$$X_3=-\tfrac{1}{2}(s-m_N^2)^2q^2\sin^2\theta-t(s-m_N^2)^2 +(M\Delta-t)(s-m_N^2)(t-m_K^2) +(\tfrac{1}{4}\Delta^2-m_N^2-\tfrac{1}{4}t)(t-m_K^2)^2, \tag{9}$$

$$X_4=\tfrac{1}{2}M(-t+\Delta^2)(t-m_K^2)^2, \tag{10}$$

$$X_5=\tfrac{1}{2}q^2\sin^2\theta(s-m_N^2)^2t+t^2(s-m_N^2)^2 +t(s-m_N^2)(t-m_K^2)(t-M\Delta) +(t-m_K^2)^2[\tfrac{1}{4}M^2\Delta^2-\Delta(\tfrac{1}{4}\Delta+m_N)t]. \tag{11}$$

[7] W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters, **9**, 131 (1962).

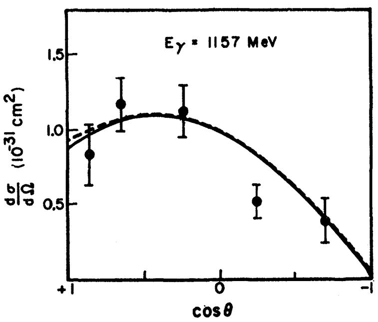

FIG. 1. Angular distribution of K^+ meson compared with fits. Solid and dashed curves correspond to sets (1) and (2), respectively, (see text), for the case $P(K\Sigma)=-1$.

In these formulas $C_i=(e/4\pi)g_\Sigma C_i'$; also $\Delta=m_\Sigma-m_N$, $M=m_\Sigma+m_N$.

For $P(K\Sigma)=+1$, we should perform the replacement:

$$m_\Sigma\rightarrow -m_\Sigma\quad(\Delta\rightarrow -M,\ M\rightarrow -\Delta)$$

everywhere.

III. COMPARISON WITH EXPERIMENT

We have calculated $d\sigma/d\Omega$ numerically from formula (5) and compared it to the differential cross section measured at $E_\gamma=1157$ MeV as given by reference 4. It turns out that the calculated $d\sigma/d\Omega$ is of the form $a+b\cos\theta+C\cos^2\theta$; higher powers in $\cos\theta$ have small coefficients. This implies that the one point at $\cos\theta=-0.24$ in reference 4 cannot possibly be fitted. Indeed, a least-square analysis of the data[8] shows that $\cos^4\theta$ is required to bring it in order. This might mean either that the measured point is in error, or that large D waves are already generated at the rather low energy of $q=210$ MeV.[9] (The K-meson wavelength $\simeq 1$ F.) We shall take the first point of view and determine the three parameters in our model (g_Σ, C_1, C_2) by fitting the data up to $\cos^2\theta$. As can be seen from (5), the values of C_1 and C_2 will be found as solutions to a pair of coupled quadratic equations. We shall discuss the results for $P(K\Sigma)=\pm 1$ separately.

A. $P(K\Sigma)=-1$

In this case we find that there are two sets of values of C_1 and C_2 that give good fits. They are:

$$(1)\quad g_\Sigma^2/4\pi=4.5,\quad C_1=-1.34\times10^{-3}(\text{MeV})^{-1},\quad C_2=0.63\times10^{-6}(\text{MeV})^{-2}.$$

$$(2)\quad g_\Sigma^2/4\pi=4.4,\ C_1=2.05\times10^{-3}(\text{MeV})^{-1},\quad C_2=-0.90\times10^{-6}(\text{MeV})^{-2}.$$

Figure 1 gives the fits for these choices. The excitation function, $(d\sigma/d\Omega)_{\cos\theta=0}$, is also calculated and is presented in Fig. 2. It is seen that set (1) and set (2) give very similar results. It turns out that our choices do not give an S wave threshold rise. This is satisfying when compared with several old data points.

For future use we have plotted $d\sigma/d\Omega$ for $q=100$ MeV and $q=300$ MeV in Fig. 3. Note that at $q=300$ MeV, $\cos^3\theta$ terms become important. Choice (2) gives a larger $\cos^3\theta$ term. However, the two sets give close results, and at the present level of experimental accuracy it is rather difficult to single out one in favor of the other.

B. $P(K\Sigma)=+1$

In this case we find that a reasonable fit cannot be obtained. This occurs because the coupled quadratic equations in C_1 and C_2 do not have real solutions owing to the fact that none of the terms of $d\sigma/d\Omega$ in (5) have a large enough coefficient for $(-\cos^2\theta)$. For example, letting $C_1=C_2=0$, then $d\sigma/d\Omega\propto(10+0.4\cos\theta-\cos^2\theta)$. In this connection we note that $P(K\Sigma)=+1$ was an explanation for the $\sin^2\theta$-type angular distribution.[3] This result, besides other assumptions, relies heavily on the anomalous magnetic moment of hyperons. According to our model, however, $P(K\Sigma)=+1$ actually does not fit into the framework. It is also worthwhile to note that even if we ignore the problem of angular distribution, $g_\Sigma^2/4\pi$ would take the uncomfortably small value ~0.04 if we want to get the correct order of magnitude of the measured cross section.

IV. DISCUSSIONS AND REMARKS

We have seen that the model suggests a rather close resemblance between Σ and Λ. We found that it is rather difficult to reconcile $P(K\Sigma)=+1$ with the meas-

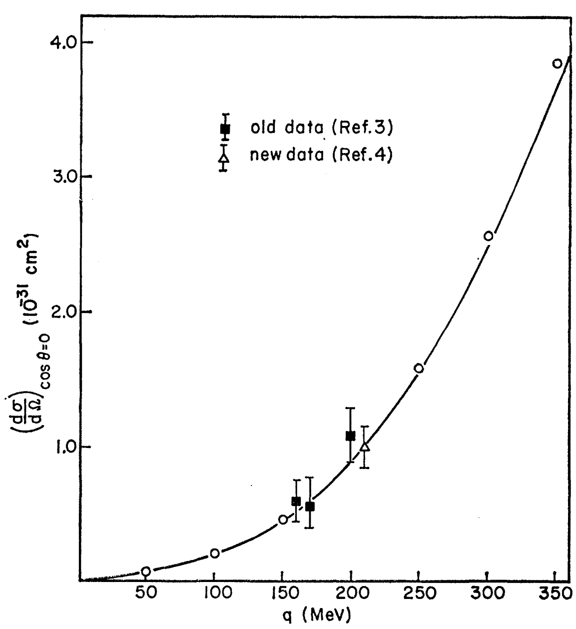

FIG. 2. Excitation function. Solid curve corresponds to the choice of set (1). Circles are values for set (2).

[8] R. L. Anderson, Ph.D. thesis, Cornell University, 1962 (unpublished).

[9] Cf., however, F. Grard and G. A. Smith, Phys. Rev. **127**, 607 (1962); and F. S. Crawford, Jr., F. Grard, G. A. Smith, *ibid.* **128**, 368 (1962) in which the reaction $\pi^++p\rightarrow\Sigma^++K^+$ was studied. At $p_\pi=1170$ MeV/c ($q\cong250$ MeV), D waves are important. Probably the same is occurring here in the photoproduction.

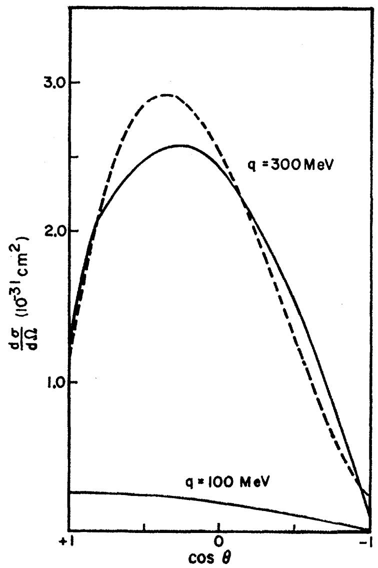

FIG. 3. Differential cross sections at q = 300 and 100 MeV. Parameter choice (1) is represented by solid lines. At q = 100 MeV the two choices (1) and (2) almost coincide.

ured differential cross section. If we employ $P(K\Sigma) = -1$, then we get $g_\Sigma^2/4\pi \simeq 4.5$, to be compared with $g_\Lambda^2/4\pi = 4.0$ as obtained in I.

Another interesting point about K-meson production is its threshold behavior. We have obtained a non-S wave rise near threshold, using the C_i values obtained from angular distributions. On the other hand, S-wave excitation persists in $\gamma + p \rightarrow K^+ + \Lambda^0$. In this connection we recall the well-known photopion production situation in which π^+ production is characterized by an S wave rise vs π^0 production, for which the S-wave contribution is very small $[\sim O(m_\pi/m_N)^2]$ (Kroll-Ruderman theorem[10]). In the dispersion theory language this comes about because for photopion production the nearest singularity is the pole at $s = m_N^2$, which is due both to the direct one-nucleon pole and to the one-nucleon exchange pole.[11] This means that the threshold behavior is determined by the second-order perturbation expansion. The above result then follows easily. For K-meson production, however, the threshold photon c.m. energy is about 600 MeV, causing a very large recoil contribution. The singularity distribution is not simple. Thus, we get a different behavior for K^+-meson production, and this is expected also of K^0-meson production.

Finally, we would like to make a few remarks about the group of reactions $\pi + N \rightarrow Y + K$. The preliminary production angular distributions show a variety of patterns.[12] As we have seen, the production distribution is really the combined effect of s- and t-channel contributions. If we assume a strong $T = \frac{3}{2}$ interaction of the K-π system (denoted as K' in the following) together with K^* which has $T = \frac{1}{2}$, then, according to our model, the angular distribution is mainly determined by the interference of s-channel contributions with those of K' and K^* exchange. Thus the K^0 meson in $\pi^- + p \rightarrow \Lambda^0 + K^0$ peaks forward because of K^* exchange. If we assume a negative coupling coefficient for K', then its interference contributions would account for the K^+ backward peak in $\pi^- + p \rightarrow \Sigma^- + K^+$, since in this case only $T = \frac{3}{2}$ objects can be exchanged. With these coupling strengths for the exchange of $T = \frac{1}{2}$ and $T = \frac{3}{2}$ objects, we can get an estimate of the angular distributions for $\pi^- + p \rightarrow \Sigma^0 + K^0$ and $\pi^- + p \rightarrow \Sigma^+ + K^+$ at low energies by isospin arguments, provided that we assume similar s-channel contributions in all cases. The result is that the K meson should peak backward in the former, and that it should be more or less flat in the latter process. These statements are approximately satisfied by existing data.

Some support for $K - \pi$ $T = \frac{3}{2}$ interaction comes from the form of the isospin crossing matrix coupling the $T = \frac{1}{2}$ and $T = \frac{3}{2}$ channels[13]:

$$\frac{1}{3}\begin{pmatrix} -1 & 4 \\ 2 & 1 \end{pmatrix}.$$

It is seen that a resonance in $T = \frac{1}{2}$ would give strong attraction in $T = \frac{3}{2}$, and vice versa. The splitting of the two isospin states is due to a $T = 1$ $\pi\pi \rightarrow K\bar{K}$ interaction. It would certainly be interesting to carry out a calculation of this coupled $T = \frac{1}{2}$ and $T = \frac{3}{2}$ scattering problem.

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[10] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).

[11] The photopion production singularities can be found in J. Kennedy and T. D. Spearman, Phys. Rev. **126**, 1596 (1962).

[12] R. H. Dalitz, Lectures at the Summer School in Theoretical Physics at Bangalore, India, (1961).

[13] B. W. Lee, Phys. Rev. **120**, 325 (1960); M. Gourdin, Y. Noirot, and P. Salin, Nuovo Cimento **18**, 651 (1960).