and

$$\Delta F_2=(1.793)^{-1}(1+v)^{-1}[(\Delta G_M)^2+(\Delta G_E)^2 -2r(\Delta G_E)(\Delta G_M)]^{1/2}. \quad (7)$$

For $q^2\approx 10\text{F}^{-2}$, $\Delta G_M\approx\Delta G_E$ from Table III, $r\approx -1.0$ and $v=q^2/90\ll 1$. Equation (6) gives $\Delta F_1<\Delta G_E$ while Eq. (7) gives $\Delta F_2>\Delta G_M$. For $q^2\approx 25$ F^{-2}, ΔF_M is much smaller than any of the other 3 errors, ΔF_1, ΔF_2, or ΔF_E.

Bishop[3] and HMW[4] have recently discussed the approximate equality of the form factors F_E and F_M. (A historical note: It was observed[10] in 1960 that F_2 decreased more rapidly with q^2 than does F_1. Then new form factors F_E and F_M were defined, with the result that F_E decreases more rapidly than F_1 and F_M decreases less rapidly than F_2. This might make it possible to reunite the split form factors.) This equality does seem valid[2] in the range $0<q^2\leq 2.98$ F^{-2} and by definition must hold precisely for $q^2=0$. However, according to our analysis in Table III, they appear unequal in the range $7\leq q^2\leq 25$ F^{-2}. The last two columns give the ratio F_E/F_M, with its standard error. (In calculating the standard error of F_E/F_M it is important to use the strong negative correlation between the errors in F_E and F_M.) The four highest values of q^2 (16.5, 18, 21.5, and 24.9 F^{-2}) give a ratio F_E/F_M which is larger than unity by an amount which is statistically significant, namely, 1.4, 2.4, 2.1, and 0.6 standard errors, respectively. In no case is F_E/F_M significantly less than unity.

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[10] R. Hofstadter, *Nuclear and Nucleon Structure*, (W. A. Benjamin, Inc., New York, 1963), p. 61–63.

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p-p Higher Partial Waves and the 3-Meson Model

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The pion, the ρ–ω vector pair, and a scalar 2π resonance are taken to explain the long-range nuclear forces. The calculated values are fitted to *p–p* phase-shift solutions for various values of the unknown mass and coupling constants.

1. INTRODUCTION

If the one-meson exchange forces from the pion, the ρ and ω vector particles, and a scalar meson or resonance are combined with unitarity, one can get satisfactory fits to the nucleon-nucleon scattering phase shifts.[1,2] Bryan, Dismukes, and Ramsay[1] have done a *p-p* calculation of this kind, solving the Schrödinger equation by iteration, while Scotti and Wong[2] have fitted *p-p* and *n-p* data with a dispersion-theoretical approach. These fits have been made to the lower partial waves or to the experimental observables; it is of some interest, however, to examine the model's predictions for the higher partial waves. True, the higher angular momentum waves are comparatively ill-determined by a phase-shift analysis; on the other hand, they depend only on the relatively long-range forces and should therefore be especially amenable to a Born approximation, that is, first-order calculation. In terms of elastic unitarity, which relates the scattering matrix to its absolute value squared, the high-l waves are quite small, therefore leading one to hope that their squares can be neglected. If the model *does* represent the higher waves accurately, it implies both that the inelastic unitarity contributions are small, and that forces of the range of $\frac{1}{2}$ or $\frac{1}{3}$ of a pion Compton wavelength are adequately represented by the π, ρ, ω, and scalar mesons. In this connection, it should be noted that although there is no experimental agreement on indications that a scalar resonance exists,[1] such a "particle" may be a reasonable method of approximating the cross-channel S-wave contributions from two-pion exchange, just as the ρ meson provides the main part of the cross-channel P-wave forces. Whether such an approximation is sensible, of course, depends on the unknown details of the dynamics. The approximation of cuts by poles is a standard technique in S-matrix theory and may or may not work. The hypothesis of a scalar meson is presented here as just that, a hypothesis. A successful use of the scalar meson in phenomenology would tend, of course, to indicate that it is a useful approximation for certain pion-exchange forces.

2. ANALYSIS

In this note, the parameters of the three-meson model[1] are adjusted to fit the Livermore group phase-shift

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[1] R. Bryan, C. Dismukes, and W. Ramsay, Nucl. Phys. (to be published).
[2] A. Scotti and D. Wong, Phys. Rev. Letters **10**, 142 (1963).

TABLE I. Values of 310-MeV matrix element $\alpha^x \times 10^3$ ($\alpha \simeq 2\bar{\delta}^N$ in radians) from Livermore phase-shift solutions vs α_{calc} for various values of M_s, g_s^2, and g_v^2.

Livermore phase-shift solutions at 310 MeV	$^3F_4$ α^x	$^3F_4$ α_{calc}	Matrix elements$\times 10^3$ ϵ_4 α^x	ϵ_4 α_{calc}	$^3H_4$ α^x	$^3H_4$ α_{calc}	Parameters giving correct $^1G_4$ and $^3F_3$ phase-analysis values M_s	g_s^2	g_v^2
		120		−57		24	3μ	3.6	36
No. 1	85.1	140	−34.8	−56	60.8	26	3.5μ	8.8	49
		170		−56		27	4μ	21	65
		110		−58		24	3μ	2.5	13
No. 4	118	140	−33.1	−57	20.9	26	3.5μ	7.2	24
		165		−57		27	4μ	17	37
		100		−58		23	3μ	2.5	19
No. 5	87.6	115	−56.7	−57	20.8	25	3.5μ	5.9	27
		130		−57		25	4μ	14	39
		110		−58		24	3μ	2.4	7.3
No. 6	52.1	125	−50.4	−58	10.4	25	3.5μ	5.8	16
		150		−57		26	4μ	14	26
OPEC values		32.9		−58.2		19.9			

solutions[3] for the $^3F_3$, $^3F_4$, $^1G_4$, ϵ_4, and $^3H_4$ phases at 310 MeV. The solutions considered are those labeled 1, 4, 5, and 6. Solutions 2 and 3 are quite similar to solution 1 and are not looked at here.

The procedure followed is to calculate the scattering matrix elements α_l, $\alpha_{l,l}$, $\alpha_{l,l+1}$, $\alpha_{l,l-1}$, and α^j for each meson field, following the procedure of Cziffra *et al.*,[4] where the α for the pion is also given. Then these α are compared to α^x, the phase-shift solution. [The connection of α^x with the phase shifts is given in reference 5: one must correct for Coulomb scattering and angular momentum mixing. For the small phase shifts considered, these corrections are small, and α^x is about equal to twice the nuclear (−bar) phase shift.[5]] Then the part of α^x not due to α^π determines the scalar and vector parameters:

$$\alpha_{l,j}{}^x = \alpha_{l,j}{}^\pi + \alpha_{l,j}{}^s + \alpha_{l,j}{}^v, \tag{1}$$

$$\alpha_{l,j}{}^x - \alpha_{l,j}{}^\pi = g_s^2 f_{sl,j}(M_s) + g_v^2 f_{vl,j}. \tag{2}$$

The α^s and α^v are given explicitly by Perring and Phillips.[6] The scalar mass M_s is an unknown parameter, and the vector mass $M_v = 780$ MeV$= M_\omega \simeq M_\rho$. The scalar coupling constant g_s^2 and the vector coupling constant

$$g_v^2 = g_\omega^2 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 g_\rho^2$$

are the other two parameters. α^π was calculated using $g_\pi^2 = 14.4$, $\mu = 137.5$ MeV. The values assumed for M_s were 3μ, 3.5μ, and 4μ, as suggested by the lower partial-wave analysis and by fragmentary experimental indications.[1,2] For each value of M_s, the Eq. (2) for α_4 and α_{33} ($^1G_4$ and $^3F_3$) at a lab energy of 310 MeV were solved to give g_s^2 and g_v^2. (See Table I.) For each solution, the values of α_{34} ($^3F_4$), α^4 (ϵ_4), and α_{54} ($^3H_4$) were calculated. These values are compared in the table with the phase-shift solutions for those parameters. The OPEC (one-pion exchange contribution) values are given for comparison.

3. CONCLUSIONS

The table shows that the three-meson model can be made to agree best with solutions 5 and 6. In fact, if one takes the values $M_s = 3\,\mu$, $g_s^2 = 1.9$, $g_v^2 = 11$, one can fit the $^3F_3$ and $^3F_4$ phases of solution 5, while getting an $\alpha(^1G_4)$ of 50 compared to $\alpha^x(^1G_4) = 54.0$. One particular fit such as this is, of course, an *extremely* speculative matter, since the phase-shift solutions involve large uncertainties themselves. It is interesting to note, however, the general result that, with the exception of $^3H_4$ in solution 6, the three-meson matrix elements *correct* the one-pion exchange contributions for *all* solutions instead of making them worse. Since one cannot rely on the phase-shift solutions, even such a general correction may be specious; nevertheless, the result may be taken as encouraging. What really should be done is to make a new phase-shift search including the vector and scalar contributions explicitly, just as was done with the pion in reference 4. If the model is correct, one would expect that the number of acceptable solutions should decrease.

One can see from the present work some of the qualitative features of a three-meson modified phase-shift solution. For example, a solution such as the YLAM of Breit *et al.*,[7] is unlikely, since their $^1G_4$ value is set to OPEC, while the $^1G_4$ scalar meson matrix element is

[3] Livermore group, solutions communicated by D. Wong.
[4] S. P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).
[5] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).
[6] J. Perring and R. Phillips, Harwell report AERE-R 4077 (1962).
[7] G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960).

rather large for $g_s^2\sim 1$ and its vector element is rather insensitive to g_v^2. A similar criticism of a $^1G_4$ based on OPEC has already been made by the Yale group themselves on the grounds of a comparison with experiment.[8] It is interesting to speculate that here OPEC should be based on ϵ_4, since the behavior of this phase shift is quite unusual in this model. Its scalar-meson matrix element (for $g_s^2=1$) is very small at 310 MeV, and, in fact, disappears in the nonrelativistic limit. Its vector-meson element, though larger than the scalar (for $g_s^2=g_v^2$), is also small because of the short range of the ρ and ω; therefore, $\alpha(\epsilon_4)$ should be quite close to its OPEC value. The table shows this insensitivity to g_s^2 and g_v^2 values.

The $^3H_4$ phase shift shows the normal attenuation of scalar and vector matrix elements at high l; it will probably be close to its OPEC value if the ratios of g_s^2 to g_v^2 are similar to those shown in the table. Just like all the matrix elements considered, the vector contribution is opposite in sign to that of the scalar, so that, for instance, for $M_s=3\,\mu$, $^3H_4$ is equal to $(^3H_4)_{\rm OPEC}$ if

$$g_s^2=0.0370\,g_v^2$$

and greater than OPEC for larger g_s^2.

[8] G. Breit, M. H. Hull, Jr., K. E. Lassila, and H. N. Ruppel, Phys. Rev. Letters **5**, 274 (1960) and Proc. Natl. Acad. Sci. **46**, 1649 (1960).

As for the ^3F and $^1G_4$ waves, one set of values from low l work,[1] i.e., $g_s^2(M_s=4\,\mu)=16$ and $g_v^2=34$, gives $\alpha(^3F_4)=0.125$. This result agrees with the table in asserting that the phase-shift solutions 1, 5, and 6 give too small a $^3F_4$ phase shift. The modified phase-shift analysis would be a test of this conjecture.

The conclusions given here would be modified slightly by the inclusion of the $J=0^-$, $T=0$ particle of mass $\simeq 4\,\mu$, the η meson. This particle increases slightly the absolute value of the $^1G_4$ and $^3F_4$ phase shifts, and decreases that of the $^3F_3$ wave; the resultant g_s^2 and g_v^2 would be somewhat less than those given in the table.

Further modifications would involve the inclusion of a derivative-coupling vector term. The Dirac term given, however, seems to be by far the most important in p-p scattering, consonant with the apparent dominance of ω exchange in the nucleon form factor results.[1]

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Branching Ratios of Reactions of π^- Mesons Stopped in Hydrogen and Deuterium*

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We measure the Panofsky ratio $P=\omega(\pi^-+p\rightarrow\pi^0+n)/\omega(\pi^-+p\rightarrow\gamma+n)$ and the branching ratio $S=\omega(\pi^-+d\rightarrow n+n)/\omega(\pi^-+d\rightarrow\gamma+n+n)$ by stopping π^- mesons in liquid hydrogen and liquid deuterium and detecting the γ rays produced. A high-resolution γ-ray spectrometer of the 180-deg-focusing type is employed. Sixty-six Geiger tubes and nine scintillation counters are used in the spectrometer to define the electron-positron orbits, providing an intrinsic instrument resolution of 0.8%. The values we obtain for the branching ratios are $P=1.51\pm0.04$ and $S=3.16\pm0.12$. This value for P is in good agreement with that obtained in previous measurements, while the value for S is significantly larger than previous results. With regard to the conventional phenomenological analysis of S-wave pion physics, the Panofsky ratio is in good agreement, whereas the value obtained in this experiment for the branching ratio S is considerably larger than predicted.

I. INTRODUCTION

When a π^- meson comes to rest ($\beta<0.01$) in liquid hydrogen, nuclear capture occurs in approximately 10^{-12} sec through one of the following channels:

$$\pi^-+p\rightarrow\pi^0+n, \qquad (1)$$

$$\pi^-+p\rightarrow\gamma+n, \qquad (2)$$

or

$$\pi^-+p\rightarrow e^-+e^++n. \qquad (3)$$

Reaction (1), mesonic capture, produces a 0.41-MeV neutron and a π^0 meson with $\beta=0.21$. The lifetime for decay of the π^0 is approximately 2×10^{-16} seconds and leads to one of the following final states:

$$n+\gamma+\gamma, \qquad (1a)$$

$$n+\gamma+e^++e^-, \qquad (1b)$$

or

$$n+e^++e^-+e^++e^-. \qquad (1c)$$

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