Spin Dependence of the Thermal Neutron Cross Section of Co^{59}†

R. I. SCHERMER
Brookhaven National Laboratory, Upton, New York
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Transmission experiments have been carried out with polarized, monochromatic neutrons and polarized Co^{59} nuclei using a polycrystalline sample of cobalt metal. By studying the transmission as a function of temperature and neutron energy, the effects of magnetic and nuclear scattering may be separated, and the spin dependence of both the capture and scattering cross sections determined. It is found that $(78.3\pm1.0)\%$ of the thermal capture is into $I+\frac{1}{2}=4$ states, the remainder being into $I-\frac{1}{2}$ states. This establishes the contribution of a bound level to the thermal cross section. The scattering cross section is $(87\pm1)\%$ due to $I-\frac{1}{2}$ states. These results are discussed in terms of the resonance structure of Co^{59}.

I. INTRODUCTION

A NUMBER of experiments have already been performed on the interaction of polarized slow neutrons and polarized nuclei.[1–4] The major purpose of these experiments was to assign spins to slow neutron resonances and, in the most recent work, to measure hyperfine interaction constants. The work was carried out exclusively on samples in which neutron capture far outweighed neutron scattering, and was generally performed at or near the resonance energy. In this paper a situation is considered in which the cross section is determined by far-removed resonances and in which scattering is important. To further complicate the problem, the nuclei are contained in a ferromagnetic medium. For slow neutrons, the total angular momentum J of the compound nucleus can only be either $J=I+\frac{1}{2}$ or $J=I-\frac{1}{2}$, where I is the spin of the target nucleus. The quantity of interest is the fraction of the cross section due to each spin state.

In the next section we derive the expression for the cross section for polarized neutrons incident on polarized nuclei contained in a magnetic sample. It will be shown that, by proper treatment of the experimental data, the magnetic and nuclear contributions can be separated. The "transmission effect" [defined in Eq. (14)] separates into a temperature-independent term, representing mainly interference between nuclear and magnetic scattering, and a temperature-dependent, purely nuclear term. In addition, measurements at different energies, at which the ratio of capture-to-scattering is different, allows us to calculate separately the spin dependence of the capture and the scattering.

We shall see that the experiment measures essentially the difference between the cross sections for the two spin states. In favorable cases, such as the present one, in which these almost cancel, the experiment is extremely sensitive. It yields the percentage of cross section in each spin state to a high accuracy, and absolute values whose accuracy is determined by the accuracy of measurements using unpolarized neutrons.

II. THEORY

A. Evaluation of the Total Cross Section

We want to evaluate the total cross section for neutrons of a given spin state passing through a magnetic material containing polarized nuclei. The problem has already been discussed by Rose,[5] without magnetic interactions, and by Halpern and Johnson,[6] without considering nuclear polarization. In addition, there is a great body of work on scattering of neutrons by bound nuclei (see, for instance, reference 7).

If we are interested in the cross section far from a resonance, neutron absorption will be unaffected by the fact that the target nucleus is bound in a lattice. The absorption cross section, σ_a, is thus given by Rose's expression

$$\sigma_a=\frac{I}{2I+1}(1-f_Nf_n)\sigma_{a-}+\frac{(I+1)}{2I+1}(I+1+f_Nf_n)\sigma_{a+}. \quad (1)$$

f_n and f_N are, respectively, the neutron and nuclear polarizations, and σ_{a+}, σ_{a-} are the cross sections for absorption into the states $J=I+\frac{1}{2}$ and $J=I-\frac{1}{2}$, respectively. The nuclear polarization is defined as $f_N=\langle I_z\rangle/I$ with the z axis taken in the direction of the external polarizing field H_0. Similarly, $f_n=\langle S_z\rangle/S$. It is convenient to write Eq. (1) separately for neutrons in the $m_s=+\frac{1}{2}$ and $m_s=-\frac{1}{2}$ states ($f_n=+1$ and -1, respectively).

$$\sigma_a=\sigma_{0a}\pm f_N\sigma_{pa}, \quad (2a)$$

$$\sigma_{0a}=\frac{I+1}{2I+1}\sigma_{a+}+\frac{I}{2I+1}\sigma_{a-}=\sigma_a{}^{(I+\frac{1}{2})}+\sigma_a{}^{(I-\frac{1}{2})}, \quad (2b)$$

† Work performed under contract with the U. S. Atomic Energy Commission.

[1] H. Postma, H. Marshak, V. L. Sailor, F. J. Shore, and C. A. Reynolds, Phys. Rev. **126**, 979 (1962). Also contains references to concurrent and earlier work at other laboratories.

[2] H. Marshak, H. Postma, V. L. Sailor, F. J. Shore, and C. A. Reynolds, Phys. Rev. **128**, 1287 (1962).

[3] V. L. Sailor, R. I. Schermer, F. J. Shore, C. A. Reynolds, H. Marshak, and H. Postma, Phys. Rev. **127**, 1124 (1962).

[4] H. Marshak, Bull. Am. Phys. Soc. **7**, 305 (1962); also G. Brunhart, H. Marshak, C. A. Reynolds, V. L. Sailor, R. I. Schermer, and F. J. Shore, *ibid.* **7**, 305 (1962).

[5] M. E. Rose, U. S. Atomic Energy Commission Document AECD-2183 (1948).

[6] O. Halpern and M. H. Johnson, Phys. Rev. **55**, 898 (1939).

[7] L. S. Kothari and K. S. Singwi, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1959), Vol. 8, p. 109.

$$\sigma_{pa}=\frac{I}{2I+1}(\sigma_{a+}-\sigma_{a-})=\frac{I}{I+1}\sigma_a{}^{(I+\frac{1}{2})}-\sigma_a{}^{(I-\frac{1}{2})}. \quad (2c)$$

σ_{0a} is the cross section for the unpolarized case. σ_{pa} is the "polarization cross section" and is the quantity directly measured in the experiment, as is seen below.

The scattering is calculated in the Born approximation, using a potential of the form

$$V=-(2\pi\hbar^2/m)\textstyle\sum_\nu(b_{c\nu}+b_{i\nu}\mathbf{I}_\nu\cdot\mathbf{S})\delta(\mathbf{r}-\mathbf{r}_\nu) \\ -(2\pi\hbar^2/m)\textstyle\sum_j\mathbf{Q}_j\cdot\mathbf{S}, \quad (3)$$

in which the νth nucleus is located at position $\mathbf{r}_\nu$, and the second sum is over the unpaired electrons in the crystal. $\mathbf{Q}$ is a magnetic interaction operator whose detailed properties do not concern us. b_c and b_i are, respectively, the coherent and incoherent amplitudes for the unpolarized case:

$$b_c=\frac{I+1}{2I+1}b_++\frac{I}{2I+1}b_-, \\ b_i=\frac{2}{2I+1}(b_+-b_-), \quad (4)$$

in which b_+ and b_- are the scattering amplitudes for the two possible spin states. We have an incident plane wave neutron with wave vector $\mathbf{k}$ and spin state χ_s interacting with a system of nuclei in spin state $\chi_I=\prod_\nu\chi_{I_\nu}$ contained in a lattice in a magnetic state M to give a scattered neutron $\mathbf{k}'$ in spin state χ_s', leaving the nuclei in a spin state χ_I' and the lattice in state M'. At the moment we specifically exclude phonons from the analysis, and consider only elastic scattering with the spin system.

$$|\mathbf{k}|=|\mathbf{k}'| \quad \text{and} \quad M=M'.$$

The nuclear terms resulting from the potential (3) have already been given by Rose.[5] We consider a monatomic, monoisotopic lattice with small enough absorption so that b_+ and b_- are essentially real quantities. After averaging over initial and final states, the purely nuclear scattering per atom and per unit solid angle is given by

$$d\sigma_{s,\mathrm{nuc}}=\frac{1}{n}|\sum_\nu b_ce^{i\mathbf{K}\cdot\mathbf{r}_\nu}|^2\left[1+\left(\frac{f_Nb_iI}{2b_c}\right)^2\left(1+4\frac{f_n}{If_N}\frac{b_c}{b_i}\right)\right] \\ +\tfrac{1}{4}b_i{}^2I(I+1)\left[1-\frac{f_N}{I+1}(f_NI+f_n)\right], \quad (5)$$

where $\mathbf{K}=\mathbf{k}-\mathbf{k}'$ and n is the number of atoms in the crystal. For an ordinary ferromagnet, the operator Q has the property that

$$\textstyle\sum_j\langle M'e^{i\mathbf{k}'\cdot\mathbf{r}_j}|\mathbf{Q}_j|Me^{i\mathbf{k}\cdot\mathbf{r}_j}\rangle=2\sum_\nu e^{i\mathbf{K}\cdot\mathbf{r}_\nu}p_\nu(\mathbf{K})\mathbf{q}. \quad (6)$$

The sum on the right-hand side is over atoms. $p(\mathbf{K})$ is the usual magnetic scattering amplitude[8] $p(\mathbf{K})=(\gamma e^2/mc^2)Sf(\mathbf{K})$, where S is the effective ionic spin and $f(\mathbf{K})$ the form factor. $\mathbf{q}=\mathbf{K}(\mathbf{K}\cdot\mathbf{n})-\mathbf{n}$ where $\mathbf{n}$ is a unit vector in the direction of magnetization. The purely magnetic elastic scattering per unit solid angle and per atom is then given by the familiar term[6]

$$d\sigma_{s,\mathrm{mag}}=(p^2q^2/n)|\textstyle\sum_\nu e^{i\mathbf{K}\cdot\mathbf{r}_\nu}|^2. \quad (7)$$

The interference term is given by

$$d\sigma_{s,\mathrm{int}}=(2/n)\textstyle\sum_{\nu\nu'}e^{i\mathbf{K}\cdot(\mathbf{r}_\nu-\mathbf{r}_{\nu'})}(f_npb_cq_z+pq_zb_iIf_N), \quad (8)$$

where q_z is the projection of $\mathbf{q}$ on the applied field H_0. The first term is that derived by Halpern and Johnson,[6] and gives the interference between the magnetic scattering and nuclear scattering for unpolarized nuclei. The second term is the only new result of our analysis, and gives the interference between the magnetic scattering and the increased coherent nuclear scattering resulting from the nuclear polarization. It should be noted that this new term *does not change sign* with the neutron polarization. We now specify that we are interested in the total scattering cross section, i.e., we collect Eqs. (5), (7), and (8) and integrate over all angles. We further state that we are interested in neutron wavelengths short enough so that essentially all Bragg planes contribute. From Eq. (5) we get,

$$\sigma_{s,\mathrm{nuc}}=4\pi\left[\frac{I+1}{2I+1}b_+{}^2+\frac{I}{2I+1}b_-{}^2 \right. \\ \left. +f_Nf_n\frac{1}{2I+1}(b_+{}^2-b_-{}^2)\right]. \quad (9)$$

Defining $\sigma_{s\pm}=4\pi b_\pm{}^2$, Eq. (9) may be written as

$$\sigma_{s,\mathrm{nuc}}=\sigma_{0s,\mathrm{nuc}}\pm f_N\sigma_{ps,\mathrm{nuc}}. \quad (10)$$

The symbols are defined by analogy with Eq. (2), with subscript "s" (for scattering) replacing subscript "a" everywhere. We denote the result of integrating Eqs. (7) and (8) over all angles as $\sigma_{0s,\mathrm{mag}}$ and $\sigma_{s,\mathrm{int}}=\pm\sigma_{ps,\mathrm{int}}+f_N\sigma'_{ps,\mathrm{int}}$, respectively. Adding Eqs. (2a) and (10), $\sigma_{0s,\mathrm{int}}$ and $\sigma_{0s,\mathrm{mag}}$, we get the total cross section, σ_T for the two possible neutron orientations:

$$\sigma_T=\sigma_{0a}+\sigma_{0s,\mathrm{nuc}}+\sigma_{0s,\mathrm{mag}} \\ +f_N\sigma'_{ps,\mathrm{int}}\pm\sigma_{ps,\mathrm{int}}\pm f_N(\sigma_{pa}+\sigma_{ps}) \\ =\sigma_{0T}+f_N\sigma'_{ps,\mathrm{int}}\pm\sigma_{ps,\mathrm{int}}\pm f_N\sigma_{pT}. \quad (11)$$

B. The Transmission Effect

The following calculation has been given in detail by Postma, *et al.*[1] It is described briefly here because there are some modifications for the present case. We consider separately the transmission through the sample of neutrons in the two orientations. If the number of

[8] G. E. Bacon, *Neutron Diffraction* (Oxford University Press, New York, 1955), p. 149.

neutrons in these states is w_+ and w_-, respectively, then

$$w_\pm=\tfrac{1}{2}(1\pm f_n). \quad (12)$$

If $f_n>0$, $w_+>w_-$ and we will refer to the "parallel case"; the other situation is called "antiparallel." For a layer of target of thickness dt, we have the following simultaneous differential equations:

$$dw_\pm=[-w_\pm N(\sigma_{0T}+f_N\sigma'_{ps,\text{int}}\pm\sigma_{ps,\text{int}} \pm f_N\sigma_{pt}\pm D(w_--w_+)]dt. \quad (13)$$

Here there are N nuclei per cm^3 of sample, and D^{-1} is the mean free path for spin reversal. Equations (13) are solved in the parallel case with Eqs. (12) as initial conditions, and in the antiparallel case with $-\phi f_n$ replacing f_n in Eqs. (12). (ϕ is the efficiency for flipping the neutron spin in our apparatus.) The total count rate is then $w_{p,a}=(w_++w_-)_{p,a}$, the subscripts referring to the two cases. The transmission effect is then defined as

$$\mathcal{E}=(w_p-w_a)/(w_p+w_a). \quad (14)$$

Equation (14) as derived applies only to monochromatic neutrons. The spectrometer resolution is taken into account by separately convoluting the numerator and denominator of Eq. (14) with the instrumental resolution function. The resulting expression contains all the coefficients in Eq. (13). However, in our case (far from resonance) the cross sections vary slowly and monotonically with energy and the resolution correction can be ignored. This has the important effect that only the terms of Eq. (13) which change sign with the neutron polarization enter into $\mathcal{E}$. The result,[1] for the case valid here in which $[N^2(\sigma_{ps,\text{int}}+\sigma_{pT})^2+D^2]^{1/2}t\ll 1$ is

$$\mathcal{E}=-\frac{f_n}{1+Dt}\frac{(1+\phi)}{2}Nt(\sigma_{ps,\text{int}}+f_N\sigma_{pT}), \quad (15)$$

in which t is the sample thickness.

Equation (15) is not yet complete, since it does not include the effect of higher order contamination of the neutron beam. If f_α is the fraction of the neutron count rate due to the α order, and τ_α is the transmission of the unpolarized sample for that order, then the observed transmission is given by

$$\tau_{\text{obs}}=\sum_\alpha f_\alpha\tau_\alpha.$$

The observed effect is related to $\mathcal{E}$ of Eq. (15) by

$$\mathcal{E}_{\text{obs}}=\mathcal{E}_1\frac{f_1\tau_1}{\tau_{\text{obs}}}\left[1+\sum_{\alpha>1}\frac{\mathcal{E}_\alpha f_\alpha\tau_\alpha}{\mathcal{E}_1 f_1\tau_1}\right]=h\mathcal{E}_1, \quad (16)$$

where $\mathcal{E}_\alpha$ is the transmission effect for αth order neutrons.

Neglecting all but magnetic hyperfine interactions, the nuclear polarization will be given by the Brillouin function $f_N=B_{7/2}(A/kT)$, where A/k is the hyperfine coupling constant in temperature units, T the absolute temperature and 7/2 is the spin of Co^{59}. For the values of A/k and T involved here, it is an excellent approximation to take $f_N\approx A(I+1)/3kT=3A/2kT$. The error in the approximation at the lowest temperature used (0.095°K) is $\sim 1\%$. We, thus, have the observed effect as a function of temperature:

$$\mathcal{E}_{\text{obs}}=-\frac{f_n h}{1+Dt}\frac{(1+\phi)}{2}Nt\left(\sigma_{ps,\text{int}}+\sigma_{pT}\frac{3A}{2kT}\right) =\mathcal{E}_{\text{elect}}+\mathcal{E}_{\text{nuc}}/T. \quad (17)$$

It is expected that $\mathcal{E}_{\text{elect}}$ will vary little with temperature in the region of interest, which is well below the Curie point. The two terms may thus be distinguished by a measurement of $\mathcal{E}_{\text{obs}}$ as a function of temperature.

So far we have neglected inelastic scattering processes. All inelastic events involving the spin system will be incoherent and thus not interfere with the nuclear scattering. These events may depend on the neutron polarization, however, and thus will comprise a small correction to $\mathcal{E}_{\text{elect}}$. Phonon processes modify all the purely nuclear terms in the same way. At sufficiently low temperatures, the phonon processes are accounted for by simply neglecting the Debye-Waller factor on the elastic scattering. The error is then of order $[(A+1)/A]^2\sim 1.03$ for Co^{59}. We shall ignore this correction, since the scattering cross section is not known to comparable accuracy.

III. DESCRIPTION OF EXPERIMENT

The experimental procedure is essentially the same as that described previously.[1-3] Polarized neutrons are passed through a sample of Co metal held at low temperature in a high magnetic field by means of a demagnetization cryostat mounted on the arm of a polarized neutron spectrometer. Counts are taken with the neutrons polarized alternately parallel and antiparallel to the applied field. The data are analyzed in terms of the transmission effect, $\mathcal{E}$, Eq. (14).

A. Sample Preparation and Cooling

As in previous work,[2] the refrigerating salt consisted of 250 g of iron ammonium alum, connected to a 0.95°K He^4 bath through two Pb heat switches and a guard salt containing 350 g of iron ammonium alum. The salts were grown on copper wires which served as the heat conduction link to the sample. The sample was a flat slab of Co metal, 1.250 in.×0.772 in.×0.200 in. thick ($Nt=4.521\times 10^{22}$ atoms/cm^2). The metal was 99.8% pure, the major impurities being Ni and Fe. A 470-Ω resistor[9] coated with Apiezon N grease was pressed into a hole drilled in each end of the slab.

This was quite a massive sample (23.232 g of Co). It took ~1 h after demagnetization for the effect, $\mathcal{E}$, to reach its maximum value. That is, the sample took ~1 h

[9] Obtained from Speer Carbon Company, Bradford, Pennsylvania.

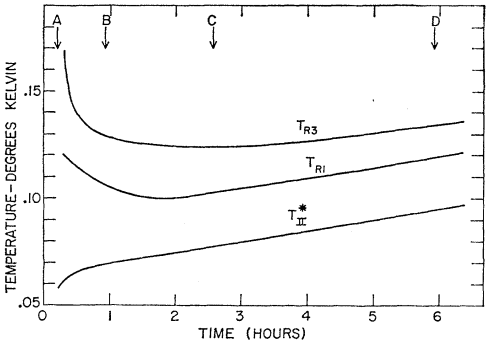

FIG. 1. Typical temperature-time curves. Adiabatic demagnetization was completed at time A. T_{II}^* is the magnetic temperature of the refrigerating salt. T_R is a resistor temperature, calculated using Eq. (18); T_{R1} is for the resistor attached to the salt; T_{R3} is for the resistor imbedded in the top of the sample. Neutron data was accepted in the interval BD. Temperature data in the interval CD was used to correct resistor temperature to thermodynamic temperature.

to cool, compared to times too short to measure in previous work.[3] Further, there is a certain amount of unavoidable vibration of the sample in the applied magnetic field. The resultant eddy-current-heating caused the refrigerating salt to warm up at a rate of $\sim 4.5\times10^{-3}$ °K/h, compared to $\sim 1.5\times10^{-3}$ °K/h for more favorable samples.[2] However, the time spent at low temperatures was still more than adequate to permit excellent counting statistics with only one demagnetization for each temperature point.

B. Thermometry

Magnetic susceptibility of both salts was measured using a dc ballistic method. An ac bridge was used to measure the resistance of the two resistors imbedded in the salt and an additional 470 Ω resistor[9] attached to the wires at the top of the cooling salt. All the secondary thermometers were calibrated separately between 4.2 and 0.95°K against the He^4 vapor pressure. All three resistors had essentially the same behavior. Typical temperature-time curves are shown in Fig. 1. The resistor temperature T_R was calculated using the equation,

$$T_R = A\ \log R/(\log R - B)^2. \tag{18}$$

It is seen that although the salt appeared to reach equilibrium within 40 min after demagnetization, the resistors cooled for almost two hours and then began to warm up. The effect $\mathcal{E}$ grew in at an intermediate rate. Only neutron counts taken after the resistors had cooled sufficiently were used in the final analysis (time B in Fig. 1). About 5 h of data ($\sim 1.5\times10^7$ counts at a neutron energy of 0.0725 eV) was taken for each low-temperature point. The high-temperature points ($T \geq 0.95$°K) are for even longer counting times, generally overnight.

The final quoted temperatures were derived as follows: T^* was corrected to spherical shape and then to thermodynamic temperature T using the data of Kurti and Simon.[10] Since the salt temperature was also measured by a resistor (T_{R1}), we were able to plot a correction curve to reduce T_{R1}, and thus presumably all the resistor temperatures, to T. Only temperature data taken after the resistors had begun to warm up was used in constructing this correction curve (time C in Fig. 1).

If the alternate data of Cooke *et al.*[11] had been used for the T^*-T conversion, the temperatures would be somewhat lower. The end result would be a value of σ_{pT} some 10% smaller, which will be seen shortly to make little difference in the final results.

IV. RESULTS AND ANALYSIS

A. Transmission Effect

$\mathcal{E}$ was measured at a neutron energy of 0.0725 eV at five temperatures and at 2.11 eV for two temperatures. The results are summarized in Table I. The 0.0725-eV results are plotted in Fig. 2. A least-squares fit gives, at 0.0725 eV,

$$\mathcal{E} = -(0.821\pm0.018)\times10^{-2} + (0.166\pm0.004)\times10^{-2}/T;$$

a weighted average of the 2.11-eV data yields

$$\mathcal{E} = -(0.165\pm0.026)\times10^{-2}/T,$$

assuming that $\mathcal{E}_{\text{elect}}=0$ at this energy.

As seen from Eq. (17), a number of auxiliary quantities must be determined before σ_{pT} can be calculated from the observed $\mathcal{E}$. These quantities are discussed individually.

B. Calculation of Auxiliary Quantities

1. The quantity $f_n[(1+\phi)/2]$ has been directly measured in the energy range 0.06–1 eV by analyzing the beam polarization with a second crystal. We take

$$f_n[(1+\phi)/2] = 0.88\pm0.03 \quad \text{at} \quad E = 0.0725 \text{ eV},$$

and

$$f_n[(1+\phi)/2] = 0.90\pm0.03 \quad \text{at} \quad E = 2.11 \text{ eV}.$$

TABLE I. Transmission effect as a function of neutron energy and sample temperature.

E (eV)	T (°K)	ε (%)
0.0725	0.095	+0.898±0.026
	0.205	+0.024±0.025
	0.95	−0.641±0.017
	4.2	−0.781±0.029
	77	−0.840±0.016
2.11	0.092	−1.79 ±0.10
	0.95	−0.175±0.050

[10] Tabulated in *American Institute of Physics Handbook*, edited by Dwight Gray (McGraw-Hill Book Company, Inc., New York, 1957), Vol. 4, p. 18.

[11] A. H. Cooke, H. Meyer, and W. P. Wolf, Proc. Roy. Soc. (London) **A233**, 536 (1956).

The second figure represents an extrapolation, and has been used previously.[1-3]

2. A measurement of neutron polarization with and without the sample gives[1]

$$\frac{f_n(\text{sample in})}{f_n(\text{sample out})}=e^{-2Dt}.$$

We, thus, measured $Dt=0.199\pm0.002$ at 0.0725 eV. We expect[1] that $D\sim1/E$, and, thus, calculate $Dt=0.037\pm0.001$ at 2.11 eV.

3. For cross sections which vary slowly with energy, as in the present case, the higher order contamination correction, h, carries little weight. It is sufficient to assume that only second-order neutrons contribute. From preliminary measurements we have made of f_2 as a function of energy on the spectrometer actually used here, we get $f_2\sim0.06$ at 0.0725 eV, $f_2\sim0.05$ at 2.11 eV, and we estimate

$$h=0.95\pm0.01 \quad \text{at} \quad 0.0725 \text{ eV},$$
$$h=0.98\pm0.01 \quad \text{at} \quad 2.11 \text{ eV}.$$

4. The most accurate measurement of the hyperfine constant is undoubtedly the nuclear magnetic resonance result of Gossard and Portis,[12] $A=217.2$ Mc/sec at 0°K. This applies to fcc cobalt. Our sample is probably a mixture of fcc and hcp phases. The results of Arp *et al.*[13] show that the splitting is only $\sim2\%$ larger in the fcc than in the hcp phase. We take this 2% as an error on A, and get finally, in temperature units,

$$|A/k|=(1.04\pm0.02)\times10^{-2} \text{ °K}.$$

With $\mu=+4.583$ nm, this is an effective field of 217.5 kOe. μ is assumed positive from shell-model considerations. The sign of H_{eff} has been determined directly, and is *negative*.[14] This is important. The sign of $\mathcal{E}_{\text{nuc}}$ depends only on the signs of μ, H_{eff}, and σ_{pT} and we are eventually interested in the sign of σ_{pT}.

5. The A value quoted above is that in zero applied field. As shown by Marshall,[15] the actual field is given by

$$H=H_{\text{eff}}+H_{\text{local}}$$
$$=H_{\text{eff}}+H_0-DM_S+(4\pi/3)M_S+H'.$$

H', the residual Lorentz field for noncubic symmetry is negligible. $M_S=1446$ Oe. The average applied field H_0 during the experiment was 17.7 kOe. The demagnetizing factor D has been calculated assuming the sample is an ellipsoid with axes by the rectangular dimensions. The field was applied parallel to the short edge of the sample face (0.772 in.). Using the curves given by Osborn,[16] we get $D/4\pi=0.171$. Finally, $H_{\text{local}}=(20.6\pm1)$ kOe, and $A'/k=-(0.93\pm0.02)\times10^{-2}$ °K in the applied field.

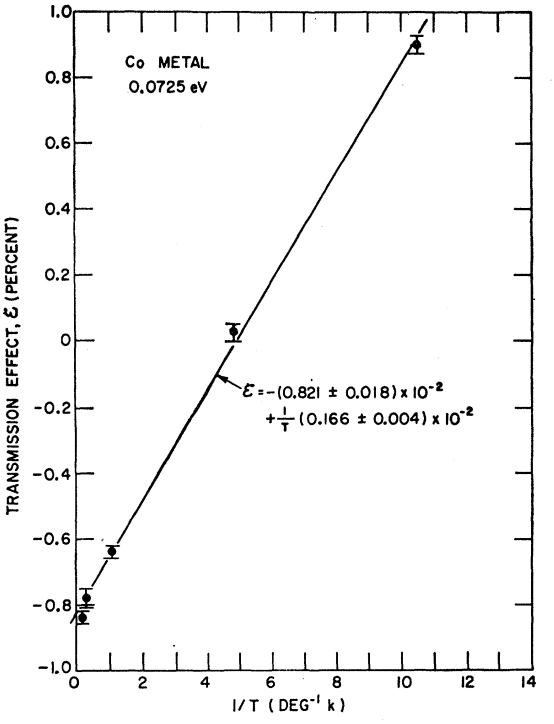

FIG. 2. Transmission effect at 0.0725 eV in cobalt metal as a function of temperature. The solid line is a least-squares fit of the data.

6. Because hcp cobalt is magnetically hard and anisotropic, we must check that the full nuclear polarization is actually seen along the field direction. (In Ho-In,[3] only about 50% of the true polarization was observed.) There does not seem to be a published magnetization curve for polycrystalline Cobalt for fields >10 kOe. However, we have two approximate calculations to show that this effect may be ignored.

Bozorth[17] gives the correlation (in our notation)

$$M/M_S\approx1-8K_1^2/105M_S^2H_0^2.$$

With[18] $K_1=7.1\times10^6$ ergs/cm³ (at 77°K) and with $M_S=1446$ Oe as before, $M/M_S\approx0.994$.

We can also get an estimate from our depolarization data. In $H_0=20$ kOe, $B=H+4\pi M=38$ kg, which gives a Larmor frequency for a neutron of 100 Mc/s. A 0.073-eV neutron thus moves a distance $\delta_L=3.4\times10^{-3}$ cm during one Larmor precession, which we expect to be many magnetic domains. We, therefore, take Eq. (18) of reference 1, which we write in the form

$$D\approx4\pi^2\langle B_i^2\delta_i^2\rangle_{\text{av}}n/B^2\delta_L^2,$$

[12] A. C. Gossard and A. M. Portis, Suppl. J. Appl. Phys. **31**, 205 (1960).
[13] V. Arp, D. Edmonds, and R. Petersen, Phys. Rev. Letters **3**, 212 (1959).
[14] J. G. Dash, R. D. Taylor, P. P. Craig, D. E. Nagle, D. R. F. Cochran, and W. E. Keller, Phys. Rev. Letters **5**, 152 (1960).
[15] W. Marshall, Phys. Rev. **110**, 1280 (1958).
[16] J. A. Osborn, Phys. Rev. **67**, 351 (1945).
[17] R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand, Inc., New York, 1951), p. 581.
[18] R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand, Inc., New York, 1951), p. 568.

where there are n domain/cm. In each domain of thickness δ_i the field component perpendicular to the applied field is B_i. The most unfavorable case is for small δ_i, but even for $\delta_i=10^3$ Å, which should be a lower limit, we get, using our measured value of $D=0.4$ cm^{-1}, $\langle B_i^2/B^2\rangle=0.01$. If θ is the angle between H_0 and B, then $\langle\cos\theta\rangle=0.995$. We thus expect to observe between 99 and 100% of the polarization in each domain, and we take $(f_N)_{\text{obs}}=3A'/2kT$ within the limits of error.

Inserting all these constants into Eq. (17) for $\mathcal{E}_{\text{nuc}}$ we finally arrive at the values

$$\sigma_{pT}=+3.69\pm0.19 \text{ b} \quad \text{at} \quad 0.0725 \text{ eV},$$
$$\sigma_{pT}=-3.04\pm0.24 \text{ b} \quad \text{at} \quad 2.11 \text{ eV}.$$

V. INTERPRETATION OF THE σ_{pT} MEASUREMENTS

A. Expected Value for σ_{pT}

We could, in principle, assume an energy variation for σ_{ps} and σ_{pa} and treat the data on σ_{pT} at the two energies as simultaneous equations for these quantities. However, since the 0.0725-eV data is more precise, it is better to work with just this value and use the 2.11-eV result as a consistency check.

We take the following scattering parameters for Co^{59} as known: $\sigma_{0s}=6.7\pm0.4$ b, constant over the region of interest,[19] $b_c=+0.25\times10^{-12}$ cm,[20] $\zeta=4\pi b_c^2=0.78$ b. The incoherent scattering cross section $s=\sigma_{0s}-\zeta=5.9$ b. There are two possible pairs of values (b_+,b_-) which will give the observed values of σ_{0s} and b_c. They are given by the simultaneous solution[21] of Eqs. (2b) and (4), modified according to the discussion of Eqs. (9) and (10).

$$b_+=b_c\pm\left(\frac{s}{4\pi}\frac{I}{I+1}\right)^{1/2},$$
$$b_-=b_c\mp\left(\frac{s}{4\pi}\frac{I+1}{I}\right)^{1/2}. \tag{19}$$

This gives the two possibilities for b_+, b_-, and σ_{ps} given in Table II. The low-energy capture of Co^{59} is dominated by the 130 eV resonance whose Breit-Wigner parameters are given in Table III.[22] From these parameters we determine that well over one-half of σ_{0a} at the two energies of interest is due to the 130-eV level. Since this level has $J=1+\frac{1}{2}$, we expect $\sigma_{pa}>0$ at both energies.

TABLE II. Possible values of b_+, b_-, and σ_{ps}. The values $\sigma_{0s}=6.75$ b, $b_c=+0.25\times10^{-12}$ cm are assumed known.

Case	$b_+\times10^{-12}$ (cm)	$b_-\times10^{-12}$ (cm)	σ_{ps} (b)
1	0.85±0.02	−0.53±0.03	2.43±0.26
2	−0.35±0.02	1.03±0.03	−5.16±0.33

TABLE III. Breit-Wigner parameters[a] for the 130-eV resonance in Co^{59}.

$E=130.2$ eV	$J=I+\frac{1}{2}=4$
$\Gamma_n=5.13\pm0.10$ eV	$2g\Gamma_n=5.77\pm0.10$ eV
$\Gamma=5.36\pm0.30$ eV	
$\Gamma_\gamma=0.40\pm0.04$ eV, assuming $\sigma_{0a}(2200 \text{ m/sec})=38.2$ b	

[a] Reference 22.

A number of closely spaced resonances have been reported above 4.3 keV.[23] If we assume that these levels have a Γ_γ comparable to that of the 130-eV resonance, then these levels give a negligible contribution to σ_{0a}.

B. Analysis of the Observed σ_{pT} into Spin States

We have, at 2.11 eV, $\sigma_{pT}=\sigma_{ps}+\sigma_{pa}=-3.04\pm0.24$ b. From the considerations of part A, this negative value is consistent only with the choice $\sigma_{ps}=-5.16\pm0.33$ b (Table II, case 2). We can then calculate σ_{pa} at both energies. Using Eqs. (2b) and (2c) we may write

$$\frac{\sigma_{pa}}{\sigma_{0a}}=\frac{2I+1}{I+1}\frac{\sigma_a^{(I+\frac{1}{2})}}{\sigma_{0a}}-1. \tag{20}$$

We, thus, may calculate $\sigma_a^{(I+\frac{1}{2})}/\sigma_{0a}$, the fraction of the thermal absorption cross section due to $I+\frac{1}{2}$ states. Using the 0.0725-eV data, we find this is (78.3±1.0)%. In calculating this number, σ_{0a} has been taken as exactly 38.2 b, with no error. The above standard deviation thus represents the uncertainty introduced by just the present experiment. If we use the 2.11 eV data we find $\sigma_a^{(I+\frac{1}{2})}/\sigma_{0a}=(84.3\pm5.4)\%$ which agrees within error with the above number. By a similar analysis it is found that (87±1)% of the scattering cross section is due to $I-\frac{1}{2}$ states. The standard deviation here is entirely due to the uncertainty in σ_{0s}.

In addition, the results are only consistent with the negative sign for A/k. Suppose we had taken $A/k>0$, thus reversing the signs of both the observed σ_{pT}. The 2.11-eV data would then force the choice $\sigma_{ps}>0$ (Table II, case 1), and $\sigma_{pa}(2.11 \text{ eV})>0$. It would then be impossible for σ_{pT} to change sign between 2 and 0.07 eV, in contradiction with the observation.

C. Discussion of the Spin-Dependent Cross Sections

From the ratio $\sigma_a^{(I+\frac{1}{2})}/\sigma_{0a}=0.783$ calculated above, we would predict a value of $\sigma_a^{(I+\frac{1}{2})}=17.7\pm0.2$ b at

[19] C. S. Wu, L. J. Rainwater, and W. W. Havens, Jr., Phys. Rev. **71**, 174 (1947).
[20] W. L. Roth, Phys. Rev. **110**, 1333 (1958).
[21] G. E. Bacon, *Neutron Diffraction* (Oxford University Press, New York, 1955), p. 33.
[22] A. Jain and R. Chrien [private communication (to be published)].
[23] R. E. Coté, L. M. Bollinger, and J. M. LeBlanc, Phys. Rev. **111**, 288 (1958).

0.0725 eV. Using the parameters of Table III, we find that the 130-eV level accounts for 14.4±1.5 b of this. The difference, 3.3±1.5 b, is significant and may indicate small systematic errors in either or both of the experiments. In particular, if we had used the temperature data of Cooke *et al.*[11] as discussed in Sec. III B, the difference would be reduced. We may take the present result as setting an upper limit of $\Gamma_\gamma=0.49\pm0.01$ eV for the 130-eV level.

However, none of the positive energy levels can possibly account for $\sigma_a{}^{(I-\frac{1}{2})}$. There, thus, must be a contribution to the cross section from negative energy levels, predominately $(I-\frac{1}{2})$ in character. Indeed it is possible to explain the observed result quantitatively with an $I-\frac{1}{2}$ resonance of reasonable size about 100 eV below the binding energy.

The largely $(I-\frac{1}{2})$ nature of the scattering is nicely consistent with the above picture. In the energy region of interest we are below an $(I+\frac{1}{2})$ level (destructive interference) and above an $(I-\frac{1}{2})$ level (constructive interference). To explain the small value of $\sigma_s{}^{(I+\frac{1}{2})}$ quantitatively, it is necessary to demand that a reasonable part of the resonance structure in the keV region be $(I+\frac{1}{2})$ levels. The reason for this condition is as follows. For each spin state we have

$$\sigma_s{}^{(J)}=4\pi g\left|R'_J+\sum_k \frac{\lambda_{0k}\Gamma_{nk}/2}{(E-E_{0k})+i\Gamma/2}\right|^2, \qquad (21)$$

where the sum is over all resonances in the same spin state. If we assume that only the 130-eV level contributes to the sum, we find that $R'_{I+\frac{1}{2}}=11.4$ or 4.35 F. Neither of these values is admissable from the standpoint either of potential scattering measurements made in this mass region[24] or of the optical model[25] which predict $R'=5$ to 7 F. This situation may be remedied by the inclusion, in the sum appearing in Eq. (21), of additional positive energy levels (bound levels increase the discrepancy). The calculation cannot be performed because the Γ_n values of the resonances above 4.3 keV have not been measured. However, we can indicate what a "reasonable part" of this structure in the $I+\frac{1}{2}$ state involves. Before these resonances were resolved, levels were reported[26] at 4.7 keV with $2_g\Gamma_n=320$ eV and 7.8 keV with $2_g\Gamma_n=240$ eV. By simply assuming that this fictitious 4.7 keV level has $J=I+\frac{1}{2}$, we get a reasonable value of $R'_{I+\frac{1}{2}}=6.44$ F.

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[24] K. Seth, D. Hughes, R. Zimmerman, and R. Garth, Phys. Rev. **110**, 692 (1958).

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