Electrodisintegration of H^3 and He^3†

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The electrodisintegration cross sections for 400-MeV electrons on H^3 and He^3 were calculated using the Born approximation for the motion of the electrons and allowing both nucleon-deuteron and three-nucleon disintegrations. The contributions due to these two processes were found to be comparable for both nuclei, assuming the processes to be uncoupled. The cross sections for both processes were found to be essentially the sum of the cross sections for elastic scattering off the free fragments multiplied by a function characterizing the momentum distribution of the fragment in the ground state of the target nucleons. The effects of the final-state interactions between the fragments were considered and found to reduce the cross sections appreciably relative to the result obtained by neglecting these interactions.

INTRODUCTION

MUTO and Sebe have calculated the high-energy ($\sim$400 MeV) electrodisintegration cross section for[1] He^4 using the following assumptions: (1) The effects of the scattered electron at the nucleus may be represented by the Møller potentials[2]; (2) the nuclear ground-state wave functions may be represented by Gaussians in the squares of the internucleon separations; (3) the predominant reactions are one-particle dissociations; (4) the motion of the nuclear fragments may be treated in the first Born approximation.

The Møller potentials result from regarding the initial and final electron states as plane waves. At bombarding energies of 400 MeV with targets of small atomic number, distortion effects should be negligible and the first assumption may be used. Assumption (2) is made for convenience.

Assumption (3) will be dropped for the following reason. Muto and Sebe chose a range parameter for He^4 corresponding to an rms charge radius of 1.69×10^{-13} cm in order to fit the experimental electrodisintegration data.[3] This radius is very close to the range of accepted experimental values $[(1.61-1.68)\times10^{-13}$ cm$]$ but it should be corrected for the finite size of the proton, in which case calculations based on the assumption that multifragment final states may be ignored lead to results which disagree with observation. The inclusion of other possible breakup reactions could conceivably correct this situation. Hence, a more detailed investigation of the possibility of multiparticle breakup is in order.

The disintegration of He^4 can lead to five different final states if multifragment processes are allowed. The $A=3$ nuclei admit to only two possibilities:

$$\begin{aligned} H^3 &\rightarrow n+H^2 \\ &\rightarrow 2n+p, \end{aligned}$$

with an analogous scheme for He^3. In order to investigate the possibility that multifragment reactions may contribute substantially, we shall investigate the cross sections for H^3 and He^3 taking into account both possible reactions. If the two-particle contribution does not clearly dominate the cross section in this case, it will indicate that multifragment reactions must be considered for an adequate discription of the He^4 disintegration.

Inclusion of the complete breakup reaction for H^3 and He^3 introduces an additional complication into the calculations. If only the two-fragment breakup is considered, assumption (4) is reasonable to a good approximation. For an incident electron energy of 400 MeV and a scattering angle of 60° (a typical experimental situation) the nuclear excitation energy is roughly 60 MeV. If all this energy is manifest in the relative motion of two particles, it is reasonably accurate to ignore their interactions, that is, to use the Born approximation. However, with three fragments sharing this energy, it is unlikely that we will be able to ignore interactions between them. Therefore, corrections for final-state interactions among the nuclear fragments will be made.

The cross sections for H^3 and He^3 will both be considered here, since it turns out that the effects of the final-state interactions are substantially different for these two nuclei.

PRELIMINARY DISCUSSION

The qualitative aspects of the electrodisintegration of He^3 will be reviewed briefly before presentation of the actual calculations. For simplicity we shall discuss only the static Coulomb portion of the interaction.

The wavelength of the incident electron is small compared to the average separation of the nucleons in the He^3 nucleus at beam energies on the order of 400 MeV. For that reason, if an electron interacts strongly with one of the protons, it is unlikely that it will interact strongly with the other proton and we may conclude that large-angle scattering will be characterizable in some sense as a two-body collision.

If the proton undergoing the collision were at rest, the electron scattered through a given angle would possess

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[1] T. Muto and T. Sebe, Progr. Theoret. Phys. (Kyoto) **18**, 621 (1957).
[2] W. Heitler, *Quantum Theory of Radiation* (Clarendon Press, Oxford, 1954), 3rd ed., pp. 231–237.
[3] R. Hofstadter, Rev. Mod. Phys. **28**, 214, 223 (1956).

a unique final energy. However, the proton is contained in a nucleus and has a momentum distribution characteristic of the structure of that nucleus. The component of the proton's velocity parallel to the direction of the incident electron's momentum will in effect vary the bombarding energy and result in a distribution of scattered energies at a given angle, the features of this distribution reflecting the momentum distribution of the struck proton.[3] The important interaction in this process, then, is that between an electron and a "free" proton, rather than that between an electron and the nucleus.

Note that the previous discussion does not indicate whether the struck proton will be ejected from the nucleus or not. It may take up all the momentum transferred and be ejected (disintegration) or it may "drag along" the rest of the nucleus (elastic scattering). Of course, the higher the momentum transfer, the more likely disintegration becomes. Note also that if the neutron is "near" the struck proton and instantaneously in a momentum state relative to the proton similar to an instantaneous state occurring in a deuteron while the electron is being scattered, the two-body collision may be electron deuteron rather than electron proton, that is, the scattering particle which is ejected in disintegration could conceivably be a deuteron.

If the disintegration proceeds via the electron-deuteron scattering, the other fragment is obviously a proton. If, however, the electron interacts with a proton, the remainder of the nucleus in the final state could appear either as a deuteron or as a proton and neutron. The relative ratio of these two processes is determined by whether the relative momentum distribution of the neutron and proton is "nearer" a deuteron or free (interacting) particles. The phase-space factors are also different for the two processes.

The portion of the cross section attributed to the three-fragment breakup should be essentially twice that for a single proton with the appropriate momentum distribution. The two-fragment reaction should contain scattering off a proton with a contribution due to the scattering off a deuteron. Of course, the deuteron will scatter less effectively due to its "softer" charge and in addition will contribute at a different scattered electron energy. In addition, these remarks presuppose that the two reactions are independent. This will be assumed throughout.

The interaction between the recoiling, high-energy fragment which has performed as the scatterer and the remaining fragment or fragments should be small for large excitation energies as previously indicated. The most important correction will be that for the force between the two remaining nucleons in the three-fragment breakup. These nucleons may have a small relative velocity and inclusion of their interaction could have a sizable effect on the three-fragment portion of the cross section.

THE INTERACTION

The electromagnetic interaction between the electron and the nucleons can be represented for our purposes by[4,5]

$$\begin{aligned} H' = e\sum_{j=1}^{A}\Big\{ & \tfrac{1}{2}[1+\tau(j)_3]\phi(\mathbf{r}_j,t) - \left(\frac{i\hbar}{2Mc}\right) \\ & \times[\mu_+ + \tau(j)_3\mu_-]\boldsymbol{\sigma}_j\cdot[\nabla_j\times\mathbf{A}(\mathbf{r},t)] - \tfrac{1}{2}\left(\frac{\hbar q}{2Mc}\right)^2 \\ & \times\{2[\mu_+ + \tau(j)_3\mu_-] - \tfrac{1}{2}[1+\tau(j)_3]\}\phi(\mathbf{r}_j,t)\Big\}. \qquad (1) \end{aligned}$$

The vector $\mathbf{r}_j$ is the position of the jth nucleon measured from the laboratory system. ϕ and $\mathbf{A}$ are the Møller potentials produced by the scattered electron. The quantity $\tau(j)_3$ is the z component of the isotopic spin of the jth nucleon. Operating on a proton spin function it yields $+1$; operating on a neutron spin function it yields -1. The isotopic spin formalism will not actually be used here. Equation (1) has been written employing $\tau(j)_3$ for compactness. The quantities μ_+ and μ_- are defined by

$$\mu_+ = (\mu_p + \mu_n)/2$$
$$\mu_- = (\mu_p - \mu_n)/2,$$

where μ_p and μ_n are the static magnetic moments of the proton and neutron, respectively. The Møller potentials may be taken to be

$$\left.\begin{aligned} \phi(\mathbf{r}_j,t) &= \langle u_f | u_0\rangle \\ \mathbf{A}(\mathbf{r}_j,t) &= \langle u_f|\boldsymbol{\alpha}|u_0\rangle \end{aligned}\right\} \times\left(-\frac{4\pi e}{[q^2-(\Delta E/\hbar c)^2]}\right)e^{i(\mathbf{q}\cdot\mathbf{r}_j-\Delta E\,t/\hbar)}, \qquad (2)$$

where $|u_0\rangle$ and $|u_f\rangle$ are the electron spinors in the initial and final states and the operator $\boldsymbol{\alpha}$ is the Dirac current density. The momentum transfer $\hbar\mathbf{q}$ and the energy transfer ΔE are defined in terms of the initial electron momentum $\mathbf{p}_0$ and the final momentum $\mathbf{p}$. For these high energies the rest mass of the electron may be neglected, so that we may write

$$\begin{aligned} \Delta E &= E_0 - E = c(p_0 - p), \\ \hbar\mathbf{q} &= \mathbf{p}_0 - \mathbf{p}. \end{aligned} \qquad (3)$$

If the initial and final nuclear states are designated $|0\rangle$ and $|f\rangle$, respectively, the cross section for a transition induced by H' is proportional to $|\langle f|H'|0\rangle|^2$

[4] L. Durand III, Phys. Rev. **115**, 1020 (1959). This is the interaction used by V. Z. Jankus, Phys. Rev. **102**, 1586 (1956) with the convection term dropped and the Darwin-Foldy term included. The former term is small in the neighborhood of the peak cross section; Durand has shown that the latter term may contribute substantially.

[5] L. van Hove and K. W. McVoy (unpublished).

averaged over initial electron and nucleon spins and summed over final spins. The electron spin sums can be performed at once. If the results of this operation are denoted by $(|H'_{f0}|^2)_a$, then we find that

$$(|H'_{f0}|^2)_a=A^2\Big\{|Q|^2\cos^2(\theta/2)+|\mathbf{S}|^2\sin^2(\theta/2)+\frac{c^2}{2E_0E}[(\mathbf{S}^*\cdot\mathbf{p}_0)(\mathbf{S}\cdot\mathbf{p})+(\mathbf{S}^*\cdot\mathbf{p})(\mathbf{S}\cdot\mathbf{p}_0)]+\frac{c}{2E_0E}[E_0(Q^*\mathbf{S}\cdot\mathbf{p}+Q\mathbf{S}^*\cdot\mathbf{p})+E(Q^*\mathbf{S}\cdot\mathbf{p}_0+Q\mathbf{S}^*\cdot\mathbf{p})]\Big\}, \quad (4)$$

where

$$\cos\theta=\mathbf{p}_0\cdot\mathbf{p}/(p_0p),$$

$$Q=\sum_j\langle f|e^{i\mathbf{q}\cdot\mathbf{r}_j}|0\rangle\{\tfrac{1}{2}(1+\tau(j)_3)-\tfrac{1}{2}(\hbar q/2Mc)^2\times[2(\mu_++\tau(j)_3\mu_-)-\tfrac{1}{2}(1+\tau(j)_3)]\}, \quad (5)$$

$$\mathbf{S}=\sum_j\{[-(i\hbar/2Mc)(\mu_++\tau(j)_3\mu_-)]\times\boldsymbol{\sigma}_j\times\mathbf{q}\langle f|e^{i\mathbf{q}\cdot\mathbf{r}_j}|0\rangle\},$$

and

$$A^2=\frac{16\pi^2e^4}{[q^2-(\Delta E/\hbar c)^2]^2}.$$

The time factor in the definitions of Q and $\mathbf{S}$ has been omitted since it will ultimately cancel against a time factor from the nuclear wave functions.

The quantity in Eq. (4) will not only be averaged over nucleon spins but also over directions of nucleon momenta, that is, only the vector $\mathbf{q}$ will survive. For that reason, we may replace (4) by the expression[5,6]

$$(|H'_{f0}|^2)_a\to A^2\cos^2(\theta/2)\Big\{|Q|^2-\left(\frac{\Delta E}{\hbar cq}\right)\times[Q\hat{q}\cdot\mathbf{S}^*+Q^*\hat{q}\cdot\mathbf{S}]+\tfrac{1}{3}[2\tan^2(\theta/2)+1]|\mathbf{S}|^2-\tfrac{1}{2}\left[2\tan^2(\theta/2)-1+3\left(\frac{\Delta E}{\hbar cq}\right)^2\right]\times[(\hat{q}\cdot\mathbf{S}^*)(\hat{q}\cdot\mathbf{S})-\tfrac{1}{3}|\mathbf{S}|^2\Big\}, \quad (6)$$

where $\hat{q}$ is a unit vector in the direction of the momentum transfer. The terms containing $\hat{q}\cdot\mathbf{S}$ are zero, and if we neglect $[\Delta E/(\hbar cq)]^2$ compared to one, we obtain

$$(|H'_{f0}|^2)_a=A^2\cos^2(\theta/2)\times\{|Q|^2+\tfrac{1}{2}[2\tan^2(\theta/2)+1]|\mathbf{S}|^2\}, \quad (7)$$

which is the form we shall use for subsequent calculations.

[6] R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

DISINTEGRATION OF H^3

The H^3 nucleus can break up into a proton and two neutrons or a neutron and a deuteron. The two processes will be regarded as independent. The wave functions for H^3 and H^2 are taken as Gaussians in the nucleon separations. At first, the fragments in the final state are treated as independent. Corrections for the final-state interactions are made later in the paper.

The initial state of the H^3 nucleus can be represented in the form

$$\psi_0^m(1[2,3])=\phi_0(1\{2,3\})\chi_{1/2}^m(1[2,3]), \quad (8)$$

where 1 represents the proton, 2 and 3 the neutrons, enclosure in square brackets denotes antisymmetry in exchange, and enclosure in curly brackets denotes symmetry in exchange. ϕ_0 depends only on the space coordinates of the nucleons. $\chi_{1/2}^m$ is the total spin function consisting of the two neutrons in a singlet state and the proton coupled to a total spin 1/2.

2*n*-*p* REACTION

This process may proceed with the neutrons coming off in either a singlet or triplet spin state. The wave functions in these two cases may be written as

$$\psi_{s\frac{1}{2}}^{m_j}(1[2,3])=(2)^{-1/2}[v(1)w(2,3)+v(1)w(3,2)]\chi_{s\frac{1}{2}}^{m_j}(1[2,3]), \quad (9)$$

and

$$\psi_{tj}^{m_j}(1[2,3])=(2)^{-1/2}[v(1)w(2,3)-v(1)w(3,2)]\chi_{tj}^{m_j}(1\{2,3\}).$$

Here, $v(1)$ describes the motion of the proton, $w(2,3)$ that of the neutrons. $\chi_{s\frac{1}{2}}^{m_j}$ represents the singlet state for the two neutrons coupled to the proton spin for a total spin of 1/2. $\chi_{tj}^{m_j}$ is the triplet two-neutron state coupled to the proton state for a total spin $j=1/2$ or 3/2. Note that the time dependence of these wave functions has not been included. As previously remarked, this time dependence will combine with the time dependence in the Møller potentials to produce a delta function in the energy.

n-H^2 REACTION

The wave function for the deuteron is represented by $\psi_H(1,2)\chi_1^m(1,2)$ where ψ_H is a Gaussian in $|\mathbf{r}_1-\mathbf{r}_2|$. The final-state wave function for this process then has the form.

$$\psi_j^{m_j}(1[2,3])=(2)^{-1/2}[\psi_H(1,2)v(3)\chi_j^{m_j}(1,2,3)-\psi_H(1,3)v(2)\chi_j^{m_j}(1,3,2)], \quad (10)$$

where $\chi_j^{m_j}(1,2,3)$ stands for particles 1 and 2 coupled to spin 1 and then coupled to the spin 1/2 of particle 3 to a total spin j.

THE FORM OF THE CROSS SECTIONS

The differential cross section for an electron to be scattered into an element of solid angle $d\Omega_p$ and an energy interval dE while the nucleus disintegrates into N fragments is

$$\left(\frac{d^2\sigma}{dEd\Omega_p}\right)_N = I_0^{-1}(2\pi/\hbar)\left(\frac{\rho(\mathbf{p})}{cd\Omega_p}\right)\int (|H'_{f0}|^2)_A \times\rho(\mathbf{k}_1)dE_1\rho(\mathbf{k}_2)dE_2\cdots \times\cdots\rho(\mathbf{k}_{N-1})dE_{N-1}\,\delta(\text{energy}). \quad (11)$$

$\rho(\mathbf{p})$ is the density of final electron states and I_0 is the incident flux of electrons, equal to c/v where v is the volume of normalization (chosen here as unity). If the motion of the nuclear fragments is referred to the center of mass of the nucleus, there will be $N-1$ independent momenta for N fragments. The center of mass of the nucleus will recoil with momentum $\hbar\mathbf{q}$, the momentum transfer specified by choice of $\mathbf{p}$ and $d\Omega_p$. The subscript A on the square of the matrix element denotes an average over the electron and nucleon spins. The integral sign implies integration over all differential quantities to the right. Integration over dE_1 (for instance) yields conservation of energy. The energy balance obtained can be written in the following form:

$$E_0 = E + (2AM)^{-1}(p_0^2+p^2-2\mathbf{p}_0\cdot\mathbf{p}) + E_B + \hbar^2k_1^2/(2\mu_1) + \hbar^2k_2^2/(2\mu_2) + \cdots, \quad (12)$$

where E_0 and E are the initial and final electrons energies, respectively. The second term on the right-hand side of (12) is the recoil energy of the nucleus. The mass of the nucleus has been written as AM, where A is the atomic weight and M is the mass of a proton (the mass of the neutron is taken equal to M also). μ_1, μ_2, $\cdots$ are the reduced masses of the fragments in the final state. E_B is the binding energy absorbed in the disintegration.

The density of electron states $\rho(\mathbf{p})$ is (including the recoil of the nucleus)

$$\rho(\mathbf{p}) = \frac{p^2}{(2\pi\hbar)^3}\left[1+\frac{2E_0}{AMc^2}\sin^2(\theta/2)\right]^{-1}\left(\frac{W}{AMc^2}\right), \quad (13)$$

where

$$W = [A^2M^2c^4 + \hbar^2c^2q^2]^{1/2}.$$

If the recoil energy of the nucleus is small, $W/AMc^2 \simeq 1$. Using this approximation and factoring out $A^2\cos^2(\theta/2)$ [see Eq. (7)], the expression for the cross section may be written

$$\left(\frac{d^2\sigma}{dEd\Omega_p}\right)_n = \sigma_M \int [A^{-2}\cos^{-2}(\theta/2)(|H'_{f0}|^2)_A] \times\rho(\mathbf{k}_1)\rho(\mathbf{k}_2)dE_2\cdots\rho(\mathbf{k}_{N-1})dE_{N-1}, \quad (14)$$

where σ_M is the Mott cross section for scattering off a point charge

$$\sigma_M = \left(\frac{e^2}{2E_0}\right)^2 \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\left[1+\frac{2E_0}{AMc^2}\sin^2(\theta/2)\right]^{-1}, \quad (15)$$

and the integration on dE_1 has been performed.

The result in Eq. (14) is still perfectly general and applicable to any multifragment disintegration. We may specialize it now to our particular choice of spin dependence without yet being committed to a specific spatial behavior. Equations (8), (9), and (10) will be used to calculate the nucleon-deuteron and three-nuclear cross sections for both H^3 and He^3. The He^3 wave functions may be obtained from those for H^3 by reversing the labeling of protons and neutrons. These cross sections may be written together in the following manner:

For the three-fragment reaction,

$$\left(\frac{d^2\sigma}{dEd\Omega_p}\right)_3 = \sigma_M[ZF_p+(A-Z)F_n]\int\rho(\mathbf{l})\rho(\mathbf{k})dE_l \times\left|\int v(1)^*\omega(2,3)^*e^{i\mathbf{q}\cdot\mathbf{r}_1}\psi_0(1,2,3)d\tau\right|^2; \quad (16)$$

and for the two-fragment reaction,

$$\left(\frac{d^2\sigma}{dEd\Omega_p}\right)_2 = \sigma_M\Big\{F_{\mathrm{H}}\int\rho(\mathbf{k})\Big|\int\psi_{\mathrm{H}}^*(1,2)v(3)^* \times(e^{i\mathbf{q}\cdot\mathbf{r}_1}+e^{i\mathbf{q}\cdot\mathbf{r}_2})\psi_0(1,2,3)d\tau\Big|^2 + [(Z-1)F_p+(A-Z-1)F_n] \times\int\rho(\mathbf{k})|\psi_{\mathrm{H}}^*(1,2)v(3)^* \times e^{i\mathbf{q}\cdot\mathbf{r}_3}\psi_0(1,2,3)d\tau|^2\Big\}, \quad (17)$$

where

$$\begin{aligned} F_p &= 1+(\hbar q/2Mc)^2[2\mu_p^2\tan^2(\theta/2)+(\mu_p-1)^2],\\ F_n &= (\hbar q/2Mc)^2\mu_n^2(2\tan^2(\theta/2)+1),\\ F_{\mathrm{H}} &= \{1+(\hbar q/2Mc)^2(2\tan^2(\theta/2)+1) \times[\tfrac{2}{3}(\mu_p+\mu_n)^2+\tfrac{1}{3}(\mu_p-\mu_n)^2]\}. \end{aligned} \quad (18)$$

These equations result from discarding several small terms in (4) and all interference terms, that is, cross terms between the scattering from different fragments. The largest value of such terms are less than 0.1% of the maximum value of the terms retained in the present calculation. This result has already been anticipated in the qualitative discussion. It should be noted that for smaller momentum transfers these terms must be retained. (For $\theta=60°$, $E_0=100$ MeV, the scattering is strongly coherent.) A rule of thumb would be: if the elastic cross section is large compared to the peak electrodisintegration cross section, the interference terms are large; in the opposite case, these terms are small.

THE WAVE FUNCTIONS

The space parts of the wave functions in Eqs. (8), (9), and (10) will now be written down. Let $\mathbf{R}$ be the coordinate of the center of mass of the nucleus relative to the lab system, $\mathbf{r}_1$ the coordinate of the dissimilar nucleon, $\mathbf{r}_2$ and $\mathbf{r}_3$ the coordinates of the similar nucleons measured from the nuclear center of mass. Then, with the definitions

$$\begin{aligned}\boldsymbol{\rho}&=\mathbf{r}_1-\mathbf{r}_2,\\ \mathbf{r}&=\tfrac{3}{2}\mathbf{r}_3,\end{aligned} \tag{19}$$

the wave functions will be written

$$\begin{aligned}\phi_0(1,2,3)&=N_0e^{-\beta(\frac{1}{2}\rho^2+\frac{2}{3}r^2)},\\ \psi_H(1,2)v(3)&=N_2e^{-\beta'(\frac{1}{2}\rho^2)}e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\mathbf{q}\cdot\mathbf{R}},\\ v(1)w(2,3)&=N_3e^{i\mathbf{k}\cdot\mathbf{r}}e^{i\mathbf{q}\cdot\mathbf{R}}\exp(i\mathbf{l}\cdot\boldsymbol{\rho}).\end{aligned} \tag{20}$$

N_0, N_2, and N_3 are normalization constants, defined to normalize the functions in (16) for integration over $d\boldsymbol{\rho}\cdot d\mathbf{r}\cdot d\mathbf{R}$.

The parameters β and β' can be related to the mean square charge radii of H^3 and H^2 in the following way:

$$\begin{aligned}\beta&=\tfrac{1}{2}(\langle r(\mathrm{H}^3)^2\rangle)^{-1},\\ \beta'&=\tfrac{3}{8}(\langle r(\mathrm{H}^2)^2\rangle)^{-1}.\end{aligned} \tag{21}$$

The values[6,7]

$$\begin{aligned}(\langle r(\mathrm{H}^3)^2\rangle)^{1/2}&=2.26\times10^{-13}\text{ cm},\\ (\langle r(\mathrm{H}^2)^2\rangle)^{1/2}&=2.86\times10^{-13}\text{ cm},\end{aligned} \tag{22}$$

have been used in these calculations, corrected for the finite size of the proton with a radius of 0.77×10^{-13} cm.[8]

THE CROSS SECTIONS EVALUATED

Substituting the wave functions in (20) into Eqs. (16) and (17) the cross sections for the two processes may be evaluated in a straightforward fashion. The densities $\rho(\mathbf{l})$ and $\rho(\mathbf{k})$ are free-particle densities as quoted by Schiff[9] with one provision: Since $\mathbf{l}$ is the wave number for the relative motion of two identical particles, an additional factor of 1/2 must be put into $\rho(\mathbf{l})$ to insure that the momentum states are not counted twice.[10]

Conservation of energy yields the following relations for the two processes: In the case of the two-fragment disintegration,

$$E_0=E+\hbar^2q^2/(2AM)+E_s+\hbar^2k^2/(2\mu_k),\quad \mu_k=\tfrac{2}{3}M;$$

and for the three-fragment breakup,

$$E_0=E+\hbar^2q^2/(2AM)+E_B+\hbar^2k^2/(2\mu_k)+\hbar^2l^2/(2\mu_l),\qquad \mu_l=\tfrac{1}{2}M,$$

where E_s is the separation energy of the ejected nucleon and E_B is the binding energy of the nucleus. Note that in performing the integral on dE_l in (16) it must be remembered that k is a function of l as expressed above. The integral is performed at constant E.

The expressions for the cross sections become

$$\left(\frac{d^2\sigma}{dEd\Omega_p}\right)_3=\sigma_M[ZF_p+(A-Z)F_n]\left\{\left(\frac{3f(E)}{\hbar^2q^2\beta}\right)\times e^{-q^2/3\beta}e^{-f(E)/\beta}I_2\left(\frac{q}{\beta}[4f(E)/3]^{1/2}\right)\right\}, \tag{23}$$

where

$$f(E)=\frac{M}{\hbar^2}\left[\frac{3\hbar^2k^2}{4M}+\frac{\hbar^2l^2}{M}\right]=\frac{M}{\hbar^2}\left[\Delta E-E_3-\frac{\hbar^2q^2}{2AM}\right], \tag{24}$$

and

$$\begin{aligned}\left(\frac{d^2\sigma}{dEd\Omega_p}\right)_2=&\,\sigma_M[(Z-1)F_p+(A-Z-1)F_n]\\ &\times\left\{\left(\frac{3^{3/2}\times4}{\hbar^2q\pi}\right)\left(\frac{\beta\beta'^{3/2}}{(\beta+\beta')^3}\right)e^{-[q^2/3+f(E)/\beta]}\right.\\ &\left.\times\sinh\left(\frac{q}{\beta}[4g(E)/3]^{1/2}\right)\right\}\\ &+\sigma_MF_{\mathrm{H}}\left\{\left(\frac{3^{3/2}8}{\hbar^2q\pi}\right)\left(\frac{\beta\beta'^{3/2}}{(\beta+\beta')^3}\right)e^{-q^2/12\beta}\right.\\ &\left.\times e^{-q^2/3\beta}\sinh\left(\frac{q}{2\beta}[4g(E)/3]^{1/2}\right)\right\},\end{aligned} \tag{25}$$

where

$$g(E)=\frac{M}{\hbar^2}\left[\frac{3\hbar^2k^2}{4M}\right]=\frac{M}{\hbar^2}\left[\Delta E-E_2-\frac{\hbar^2q^2}{2AM}\right]. \tag{26}$$

E_3 is the binding energy for complete disintegration, while E_2 is the separation energy for one nucleon. For purposes of curve plotting, the values of E_3 and E_2 for both nuclei will be averaged and this average value used in both $g(E)$ and $f(E)$. In that case $g(E)=f(E)$ and we are neglecting the "average deuteron binding energy," a small correction at the energies of interest. I_2 is modified Bessel function of the first kind of order two and arises directly from the integration.

DISCUSSION OF THE UNCORRECTED CROSS SECTIONS

Insight can be gained into the form of the cross sections by evaluating the integral

$$\int v(1)^*w(2,3)^*e^{i\mathbf{q}\cdot\mathbf{r}_1}\psi_0(1,2,3)d\tau,$$

[7] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 204.
[8] R. Hofstadter, *High Energy Electron Scattering Tables* (Stanford University Press, Stanford, California, 1960), p. 62.
[9] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., 1955), 2nd ed., p. 200.
[10] The author is indebted to William F. Ford for pointing out this factor.

which appears in Eq. (15). Using the functions in (20), this quantity is found to be proportional to

$$e^{-l^2/(2\beta)}e^{-3(\mathbf{k}-\frac{2}{3}\mathbf{q})^2/(8\beta)}.$$

This quantity is large when $\mathbf{k}=\frac{2}{3}\mathbf{q}$ and l is small. Reference to the definition of $\mathbf{k}$ and $\mathbf{l}$ is (20) and consideration of the kinematics involved show that this corresponds to one nuclear taking off all the momentum transfered. Therefore, the cross section will be a maximum near the electron energy corresponding to scattering off a free nucleon (corrected for the binding energy of the nucleus and the excitative energy involved in the relative motion of the other two fragments). Note that it cannot be zero since this would yield a zero density of final states.

In the two-fragment breakup, there will be a peak near the position of the free nucleon recoil and another peak at the free deuteron position. This latter peak will be small because of the relative "softness" of the deuteron's charge and current distributions.

The information thus gathered about either process may be summarized as follows: The cross section will be equal to the sum of the cross section for elastic scattering by the various fragments multiplied by a function characteristic of the nucleon momentum distribution in the ground state.

The cross sections for H^3 are shown in Fig. 1. The contributions to the cross section due to the two possible reactions are shown, together with their sum. These are displayed for $E_0=400$ MeV, $\theta=60°$, and $q=1.83\times10^{13}$ cm^{-1}. A similar plot for He^3 using the same scale for the cross section is given in Fig. 2. Although the momentum transfer $\hbar\mathbf{q}$ varies as a function of E, it has been evaluated at the peak of the cross section and treated as a constant. Its variation is only about 5% across the peak so that this is a good approximation.

The ordinates for all the graphs are plotted in the same (arbitrary) units. The normalization was chosen for convenience.

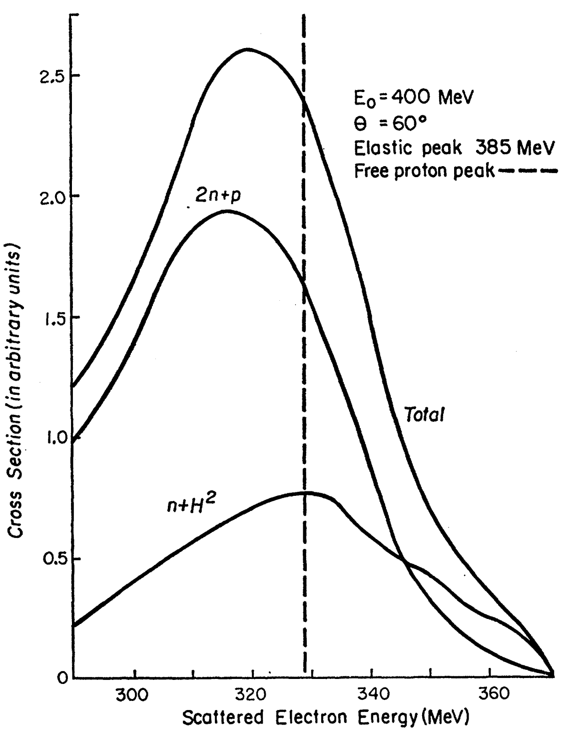

FIG. 1. H^3 cross section (uncorrected).

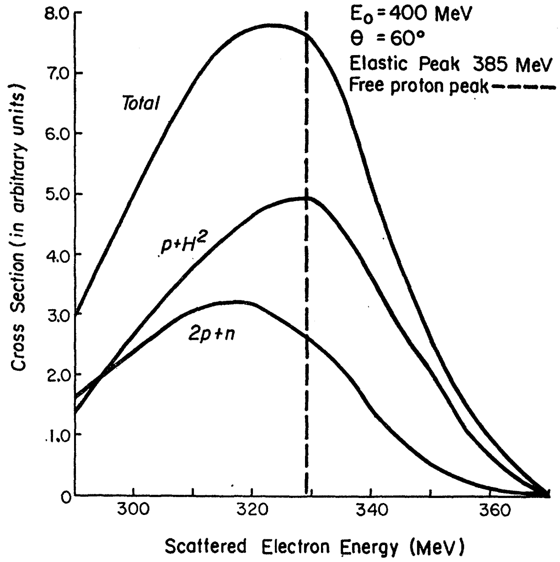

FIG. 2. He^3 cross section (uncorrected).

FINAL-STATE CORRECTIONS

The cross sections in the previous section have been calculated assuming the fragments do not interact in the final state. An estimate of the actual situation will now be made for $E_0=400$ MeV and $\theta=60°$. The results obtained should be applicable as long as the peak cross section occurs for a reasonably large nuclear excitation energy.

The peak cross section in this case occurs for an excitation energy $[\hbar^2 f(E)/M]$ of about 50 MeV. In the three-fragment breakup, this energy is shared unequally between the nucleons: One takes a great deal of the energy, the other two only a small amount. [The correction made here will be constructed on the basis of the situation in the neighborhood of the peak cross section. It will not be accurate on the high (electron) energy tail of the peak. However, the correction has been directed to giving the peak height accurately so this restriction will be ignored.] To estimate the effects of interactions, we shall assume that only the latter two nucleons interact. Then the final-state motion can be characterized by a plane wave times a function describing the interaction two-nucleon motion. To the extent that the interaction of these two nucleons with the fast nucleon can be ignored, it will not be necessary to correct the

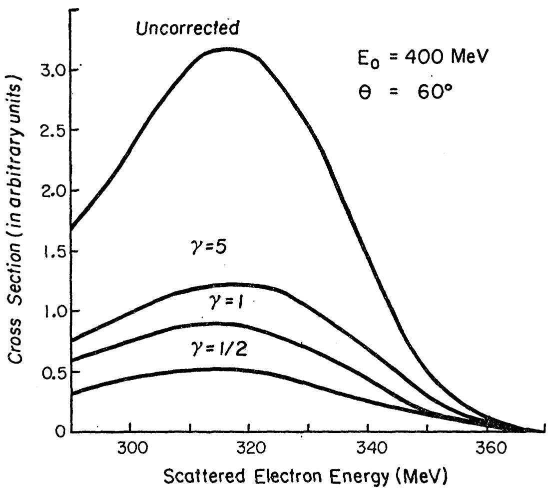

FIG. 3. He^3, corrected three-fragment cross section.

two-fragment breakup at all. Ignoring the correlation between the fast nucleon and the two remaining nucleons in either case should not strongly affect the relative proportion of the two reactions. This proportion should be most strongly altered by the correction which will be made.

The three-fragment cross sections for the two nuclei can be written in the following form:

$$\left(\frac{d^2\sigma}{dEd\Omega_p}\right)_3=\frac{4\sqrt{3}}{\pi^4}\left(\frac{M}{\hbar^2 q}\right)\sigma_M\int_0^{[f(E)]^{1/2}} dl\, l^2 e^{-[f(E)-l^2]/\beta}$$
$$\times\sinh\left(\frac{q}{\beta}\{4[f(E)-l^2]/3\}^{1/2}\right)$$
$$\times\Bigg\{[(2-Z)F_p+(Z-1)F_n]$$
$$\times\left|\int\psi_s{}^*(\rho)e^{-\beta\rho^2/2}d\varrho\right|^2$$
$$+2[(Z-1)F_p+(2-Z)F_n]$$
$$\times\left[\frac{3}{4}\left|\int\psi_t{}^*(\rho)e^{-\beta\rho^2/2}d\varrho\right|^2\right.$$
$$\left.+\frac{1}{4}\left|\int\psi_s{}^*(\rho)e^{-\beta\rho^2/2}d\varrho\right|^2\right]\Bigg\}. \quad (27)$$

ψ_s describes the singlet motion of the interacting nucleons, ψ_t the triplet. The definitions made in Eq. (19) have been used here. The form of Eq. (20) can be described as follows: Consider the bombardment of H^3 for which $Z=1$. If the scattering is off the proton (the F_p term), the neutrons are left in the singlet state. If the scattering is off either neutron, the proton and remaining neutron come out in a weighted mixture of the singlet and triplet states.

The effective range theory for ψ_s and ψ_t will be used, wherein

$$\psi_s\to e^{-i\delta_s}\frac{\sin(l\rho+\delta_s)}{l\rho},$$
$$\psi_t\to e^{-i\delta_t}\frac{\sin(l\rho+\delta_t)}{l\rho}, \quad (28)$$

where

$$\delta_s=\cot^{-1}[1/(la_s)-r_sl/2],$$

and

$$\delta_t=\cot^{-1}[1/(la_t)-r_tl/2].$$

The values of the scattering lengths and effective ranges are those quoted by Evans.[11] Only the s waves have been included. This is the only angular momentum state which contributes since the final-state wave functions are multiplied by an s wave in every case.

The wave functions in (28) both possess a singularity at $\rho=0$ which is nonphysical and may contribute significantly. This is removed by multiplying ψ_s and ψ_t by a factor $(1-e^{-\gamma\rho})$.

Computation of the integrals in (27) were done using a Burrough's 220 computer for $\gamma=1/2$, 1, 2, 5, and 50 [in units of $(2\beta)^{1/2}$ where $(2\beta)^{1/2}=0.943\times10^{13}$ cm^{-1}]. The results for H^3 and He^3 are shown in Figs. 3 and 4. The curves for $\gamma=50$ were omitted since they are essentially the same as those for $\gamma=5$. The curves for

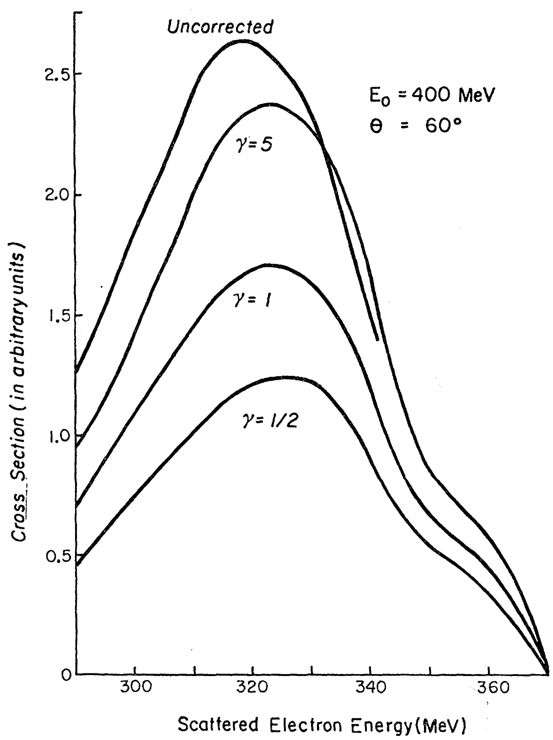

FIG. 4. H^3, total corrected cross section.

[11] R. D. Evans, *The Atomic Nucleus* (McGraw-Hill Book Company, Inc., New York, 1955), p. 329.

$\gamma=2$ were omitted to keep the plots simple. The momentum transfer was assumed constant at the value previously quoted.

The total corrected cross sections for the two nuclei are shown in Figs. 4 and 5 labeled by the various γ values. These curves are the sum of the corrected three fragment cross sections and the uncorrected two-particle cross sections. Note that with $\delta_s=\delta_t=0$ and $\gamma=\infty$, the expression in (27) reduces to that given in (23).

RESULTS OF THE CORRECTIONS

The effects of including the final-state interactions are substantial as can be seen in Figs. 3 and 6 and, hence, must be included to gain an accurate estimate of the electrodisintegration cross section.

The most striking feature is the large reduction of the He^3 three-particle cross section relative to that for H^3 when $\gamma=5$. [Since the results for $\gamma=5$ and $\gamma=50$ are essentially the same for each nucleus, we may conclude that for $\gamma=5$ the exponential correction factor in (29) is effectively zero.] This result is attributable to the fact that the triplet interaction, which is present in a large proportion in the case of He^3, reduces the cross section much more strongly than does the singlet interaction which dominates for H^3. (Note that there is no *a priori* reason to choose the same γ value to characterize the small separation behavior of the singlet and triplet states. This has been done for simplicity only. Also, the energy dependence of γ has been neglected.)

At higher bombarding energies, the final-state interactions should become less important. The maximum contribution to the dl integral in (27) will come at successively higher values of l. As l becomes larger, the phase shift gets smaller, reducing the effects of its presence.

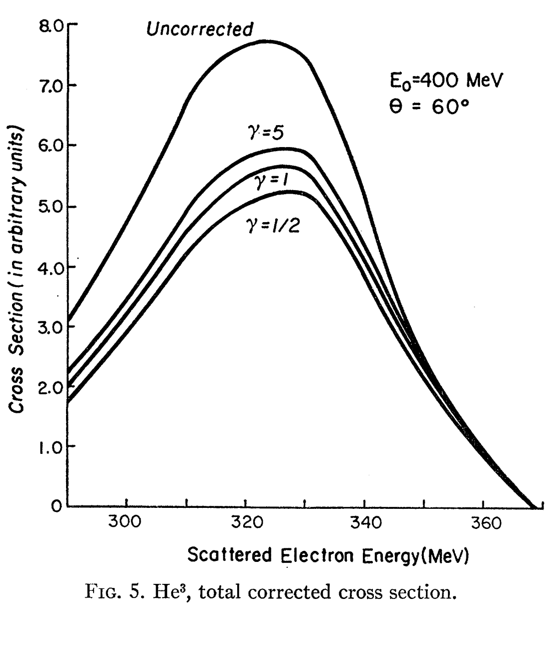

FIG. 5. He^3, total corrected cross section.

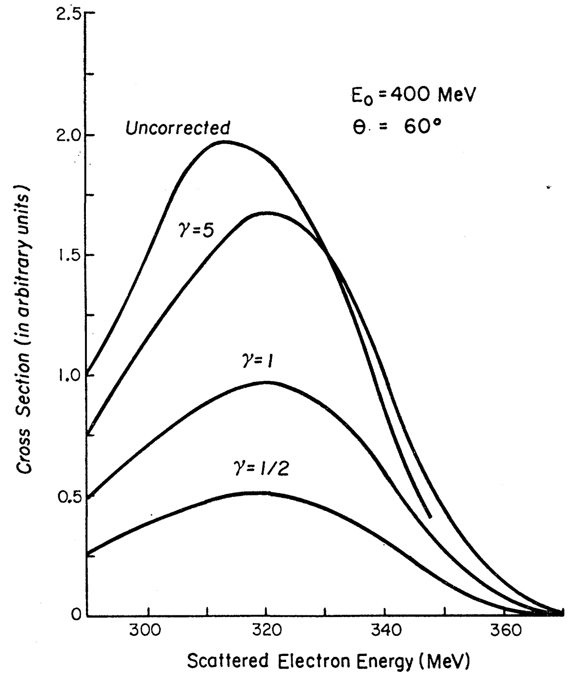

FIG. 6. H^3, corrected three-fragment cross section.

At lower bombarding energies the corrections should become larger, and the interaction in the final state should result in the domination of the cross section by the two-fragment breakup.

DISCUSSION OF THE APPROXIMATIONS

It is somewhat difficult to summarize the approximations made in order to provide bounds on the usefulness of the formulas obtained here. For the calculations of the uncorrected cross sections, the region where 300 MeV$\leq E_0\leq$500 MeV, $30°\leq\theta\leq 90°$ should be well represented. (These limits are imposed by the assumptions regarding the form of the interaction.) The final-state corrections should be appropriate for large excitation energies, as previously noted. The treatment given here should not be expected to reproduce the high-energy tail in any case.

The He^3 three-particle reaction yielded an uncorrected cross section which was essentially two times the one-proton cross section (neglecting the neutron spin scattering). This will hold for q values such that interference is small. For $\theta=60°$, $E_0=100$ MeV, the interference is large and the result is essentially four times the one-proton cross section (again neglecting the neutron). Hence, except for very high energies, the three-fragment cross section will not have the relatively simple form quoted here.

One reservation to be attached to the results herein, stems from regarding the two disintegration modes as independent. The author hopes to clarify the importance, if any, of coupling between the processes in a subsequent publication.

CONCLUSIONS

We have established here, to the extent that all the approximations made are justifiable, that the complete disintegration may contribute significantly to the electrodisintegration cross section for H^3 or He^3 using electron energies on the order of 400 MeV. It has also been shown that the final-state interactions in this multifragment reaction essentially control the amount of this contribution, especially the small separation behavior of the wave function of the two low-energy fragments. This wave function has been approximated rather crudely here for explorative purposes, with a parameter γ specifying its behavior within the range of the interaction. As can be seen from Figs. 4 and 5, the cross sections depend rather strongly on γ and hence we should try to indicate the value of γ most appropriate to the actual behavior of the two-nucleon system.

We may obtain an estimate of γ by equating the wave function used here to the solution of the appropriate square-well potential, both evaluated at zero separation. This will ensure that our wave function will start at $\rho=0$ with the same value as the solution to the square well and will also have the same asymptotic form. This, of course, will yield γ as a function of the energy of relative motion of the two nucleons. If we evaluate this for small energies, the appropriate values of γ become $\simeq 1.0$ for the singlet state and $\simeq 1.5$ for the triplet state [in units of $(2\beta)^{1/2}$].

The corrected curves for these γ values could now be plotted but in view of the crudity of the estimate, we only note that we have obtained an idea of the limits within which we could expect to find the cross section. It should be emphasized that γ has no simple physical significance but was introduced as a convenient means to determine the importance of the shape of the wave function for the low-energy nucleons within the range of the interaction. Since small values of this parameter are indicated by the estimate above (and, therefore, substantial corrections to the cross sections), a more accurate version of this wave function should be used, but it is doubtful if this is justified until experimental data on the disintegration cross sections are available. Presumably such detailed calculations could yield a means for quantitative investigation of the ground states of H^3 and He^3.

The calculations of Muto and Sebe[1] indicated the possibility of a "bump" on the high-energy tail of the inelastic continuum for He^4 due to scattering off the heavy fragments, in agreement with the tentative report of such structure by Hofstadter.[12] It is interesting to note that such a feature is present in the curves in 4 and 5. More recent experiments on He^4 have failed to display this structure.[13] If the multifragment processes in He^4 do not "swamp" the reactions resulting in He^3+n or H^3+p then such a structure is indicated by the results of this investigation. *Note.* Shortly after this manuscript was completed, the author was informed by R. Hofstadter that his measurements on the He^3 cross section were completed and that experiments on H^3 were under way. The author hopes to utilize these results in a detailed calculation in the near future.

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[12] Reference 3, p. 238.

[13] G. R. Burleson, Phys. Rev. **121**, 624 (1961).