Effect of Nonlocal Potential on the Barrier Penetrability in Alpha Decay

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In the present paper, the penetrability factor for alpha emission has been calculated from the following standpoint. The barrier for alpha emission is the usual Coulomb field assumed here to be superposed by a nonlocal alpha-nucleus interaction. The static part of the barrier is taken to be the Igo potential and its nonlocal part is assumed to be represented by an approximate delta function.

Calculations for the ground-state transitions of all the even-even nuclei have been completed. The results show that the current values of penetrability are too low, and that the inclusion of the nonlocal correction factor increases these local values by more than 50%.

I. INTRODUCTION

CONSIDERABLE interest has been aroused[1-6] recently in the problem of the barrier penetrability in alpha decay. The need for the recalculation of the penetrability factor arose from the consideration that the traditional hypothesis of a pure Coulomb barrier for alpha emission with an abrupt cutoff at the nuclear boundary is unrealistic. It was early pointed out[6] that an appropriate change in the shape of the barrier should be made by superposing on the usual Coulomb field a short-range nuclear potential that operates between the emitted alpha particle and the residual nucleus. But an accurate theoretical calculation of the transmission factor has long been handicapped due to the uncertainties regarding the knowledge of the alpha-nucleus interaction involved.

Recently, the oscillator potential has been used[7-9] in the nuclear energy calculations. On the other hand, the Woods-Saxon form[10] or the function given by Igo[11] from the optical-model analysis of the alpha-scattering data has been considered reliable. Hence, different authors have superposed either one or the other form of the alpha-nucleus potential on the Coulomb barrier in their calculations of the penetrability factor.

It was further pointed out in a recent note[12] that all these calculations require re-examination, because the alpha-nucleus potential has been assumed in them to be completely static. On the other hand, it is known that the scattering experiments from which the Igo potential has been derived, indicate a velocity-dependent character for the assumed alpha-nucleus interaction. There are other evidences also. It has been shown by Brueckner[13] that the discrepancy between the theoretical values of the potential radius and the values experimentally determined can be resolved by taking the nuclear potential to be nonlocal. Besides, the average potential-energy calculations and the study of the nuclear photoeffect with the characteristic "giant resonances" also strongly suggest the momentum dependence of the nuclear potential, as has already been discussed by Weisskopf.[14]

In view of the above discussion it is reasonable that we should find an expression for the alpha-penetrability factor on the hypotheses: (1) that the barrier consists in the usual Coulomb potential superposed by a realistic alpha-nucleus interaction, (2) that the static part of the barrier is plausibly given by the Igo potential (some authors have used a different form), and (3) that the nonlocal part is assumed to be an approximate delta function represented by a Gaussian function employed by Frahn[15,16] in a different context.

In Sec. II of the present paper we give the penetrability factor determined on the above hypotheses and a method of straightforward calculation is indicated. The results for the ground-state transitions of all the even-even nuclei are listed in Table I. For comparison purposes we also list there the values of the penetrability for the corresponding static potential. It may be seen from the table that the nonlocal part introduces substantial changes in the current values of the penetrability factor.

It should be mentioned here that we have not included in our calculations other well-known corrections, such as (i) that due to noncentral electrostatic interaction given by Preston,[17] (ii) that due to the surface well potential model of Winslow,[2] and (iii) the correction due to the statistical many-body model (Blatt and Weisskopf[18]). Of course, the exceedingly

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TABLE I. Penetrability factors for nonlocal barrier and for static potential, for the ground-state transitions of even-even nuclei.

Charge and mass number	α-particle energy with recoil correction E (MeV)	Nuclear radii r_i (10^{-13} cm)	Penetrability factor for nonlocal barrier [Eq. (11)] P_N	Penetrability factor for static potential [Eq. (11) with $\epsilon(r)=1$] P_S
$_{60}Nd^{144}$	1.975	8.44	8.621(−43)[a]	6.211(−43)[a]
$_{62}Sm^{146}$	2.642	8.470	6.271(−35)	4.389(−35)
$_{64}Gd^{148}$	3.268	8.50	3.981(−30)	2.749(−30)
$_{72}Hf^{174}$	2.589	8.77	2.719(−43)	1.862(−43)
$_{78}Pt^{190}$	3.402	8.95	7.981(−38)	5.090(−38)
Pt^{192}	2.686	8.95	1.647(−46)	1.062(−46)
$_{84}Po^{202}$	5.722	9.11	7.055(−25)	4.748(−25)
Po^{204}	5.512	9.13	6.813(−26)	4.322(−26)
Po^{206}	5.354	9.15	1.068(−26)	6.752(−27)
Po^{208}	5.241	9.17	2.739(−27)	1.729(−27)
Po^{210}	5.435	9.20	3.510(−26)	2.231(−26)
Po^{212}	8.978	9.35	1.461(−13)	1.044(−13)
Po^{214}	7.859	9.33	1.784(−16)	1.228(−16)
Po^{216}	6.935	9.32	1.810(−19)	1.211(−19)
Po^{218}	6.143	9.32	1.350(−22)	8.832(−23)
$_{86}Em^{228}$	6.294	9.19	4.570(−23)	2.922(−23)
Em^{210}	6.188	9.21	1.706(−23)	1.089(−23)
Em^{212}	6.417	9.24	1.835(−22)	1.182(−22)
Em^{218}	7.294	9.34	5.173(−19)	3.444(−19)
Em^{220}	6.432	9.34	3.019(−22)	1.963(−22)
Em^{222}	5.621	9.33	5.389(−26)	3.417(−26)
$_{88}Ra^{222}$	6.706	9.35	5.443(−22)	3.503(−22)
Ra^{224}	5.819	9.34	6.120(−26)	3.725(−26)
Ra^{226}	4.898	9.34	3.196(−31)	1.964(−31)
$_{90}Th^{226}$	6.480	9.37	8.994(−24)	5.660(−24)
Th^{228}	5.554	9.36	2.695(−28)	1.651(−28)
Th^{230}	4.801	9.37	6.944(−34)	4.207(−34)
Th^{232}	4.110	9.37	2.374(−38)	1.503(−38)
$_{92}U^{230}$	6.026	9.38	8.918(−27)	5.451(−27)
U^{232}	5.448	9.39	7.252(−30)	4.373(−30)
U^{234}	4.883	9.40	1.969(−33)	1.177(−33)
U^{236}	4.613	9.41	2.332(−35)	1.387(−35)
U^{238}	4.290	9.42	6.500(−38)	3.851(−38)
$_{94}Pu^{236}$	5.901	9.43	2.723(−28)	1.636(−28)
Pu^{238}	5.628	9.44	3.067(−29)	1.829(−29)
Pu^{240}	5.289	9.46	9.604(−32)	5.712(−32)
Pu^{242}	5.109	9.47	1.793(−33)	1.051(−33)
$_{96}Cm^{240}$	6.416	9.47	5.291(−26)	3.213(−26)
Cm^{242}	6.253	9.49	2.348(−27)	1.405(−27)
Cm^{244}	5.935	9.50	5.856(−29)	3.477(−29)
$_{98}Cf^{246}$	6.906	9.53	3.430(−25)	2.056(−25)
Cf^{248}	6.404	9.54	1.855(−27)	1.099(−27)
Cf^{250}	6.160	9.56	1.206(−28)	7.124(−29)
Cf^{252}	6.252	9.58	3.879(−28)	2.298(−28)
$_{100}Fm^{254}$	7.378	9.62	5.848(−24)	3.524(−24)

[a] The numbers in the brackets in the columns 4 and 5 are the exponents of 10 multiplying the adjacent number.

tedious numerical calculations involved are probably somewhat responsible for avoiding further complications. However, in the present case, this can fortunately be rendered plausible by noting that the order of corrections introduced therein is certainly less than 2% for ground-state transitions, whereas the nonlocal corrections exceed 50%, in general.

II. PENETRABILITY FACTOR

The momentum dependence of the alpha-nucleus interaction discussed in Sec. I can be taken into account by representing it as usual in the form of an integral operator

$$V_{12}\psi(\mathbf{r})=\int J(\mathbf{r},\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}', \tag{1}$$

where $\psi(\mathbf{r})$ is the wave function for the alpha particle and $\mathbf{r}\neq\mathbf{r}'$ in the integral. On expanding $J(\mathbf{r},\mathbf{r}')$ in terms of spherical harmonics and $\chi_l(r,r')$ and by separation of the variables, the equation for the radial part of $\psi(\mathbf{r})$ can be reduced to

$$(\hbar^2/2\mu)[d^2u_l/dr^2-l(l+1)u_l/r^2]+[E-2(Z-2)e^2/r]u_l =\int_0^\infty \chi_l(r,r')u_l(r')dr', \tag{2}$$

for the region just outside the nuclear surface, where μ is the reduced mass of the alpha particle, Z is the charge number of the parent nucleus, E is the total alpha-decay energy, and $\chi_l(r,r')$ is related to the interaction kernel by

$$\chi_l(r,r')=2\pi rr'\int_{-1}^{+1} J(\mathbf{r},\mathbf{r}')P_l(\zeta)d\zeta \tag{3}$$

in which $\zeta=\cos\gamma$ and γ is the angle between the position vector $\mathbf{r}$ of the α particle and some other position $\mathbf{r}'$. Now by our hypothesis

$$J(\mathbf{r},\mathbf{r}')=-V_0 f\left(\left|\frac{\mathbf{r}+\mathbf{r}'}{2}\right|\right)\delta_b(\mathbf{r}-\mathbf{r}'), \tag{4}$$

in which V_0 and $f(r)$ are taken from the Igo potential[11] so that

$$|V_0|=1100\text{ MeV},\quad f(r)=\exp[-(r-1.17A^{1/3})/0.574]; \tag{5}$$

r is in fermis and the nonlocal δ_b function is assumed to be given by a Gaussian exponential,

$$\delta_b=\pi^{-3/2}b^{-3}\exp[-(r-r'/b)^2]. \tag{6}$$

Now the extent of nonlocality, b, chosen by Frahn[15,16] for neutrons or protons is 0.902×10^{-13} cm. This is to match Brueckner's result that the effective nucleon mass is $m^*=0.6\,m$. On the other hand, the value of the extent of nonlocality for the alpha particle has not yet been determined. But it may be noted that a Gaussian function has also been used here as was previously used for the nonlocal potential for the nucleon, and that both the internal nucleon and an alpha particle (with average decay energy) have large wavelengths compared to the range. Therefore, it seems plausible that the nucleon and the alpha particle are equivalent (to a good approximation) from the point of view of the effective range. Thus, the above value of the extent for nonlocality for the nucleon will be assumed by us to be the same as that for the alpha-nucleus potential used.

However, that this assumption is valid to a good approximation needs to be tested. This may be done either by a method similar to that for the nucleon or indirectly by applying the nonlocal results obtained to experiments such as the relative intensities of favored alpha transitions discussed in the last section.

Since the nonlocal deviation is small, on putting (3) in (2) and neglecting terms of order higher than b^2 we have

$$\int_0^\infty \chi_l(r,r')u_l(r')dr' = -V_0[u_l(r)f(r)+(b^2/4)\{f(r)d^2u_l/dr^2+f'(r)du_l/dr + [\tfrac{1}{4}f''(r)-\tfrac{1}{2}rf'(r)-l(l+1)f(r)/r^2]u_l\}] \quad (7)$$

and Eq. (2) is finally reduced to

$$\frac{d^2u_l}{dr^2}+\frac{2\mu}{\hbar^2}\left[\epsilon(r)E-\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} - \frac{2(Z-2)e^2}{r}\epsilon(r)+V_0f(r)\epsilon(r)\right]u_l + \eta\epsilon(r)\left\{\frac{f''(r)}{4}-\frac{f'(r)}{2r}+\frac{f'(r)u_l'}{u_l}\right\}u_l=0, \quad (8)$$

where

$$\epsilon(r)=[1+\eta f(r)]^{-1} \quad (9)$$

and

$$\eta=(\mu b^2/2\hbar^2)V_0. \quad (10)$$

Neglecting the small derivatives of $f(r)$ in (8) and using the WKB method of solution, we have finally for the penetrability factor, P_N, for a nonlocal barrier

$$P_N=\exp\left\{-2\frac{(2\mu)^{1/2}}{\hbar}\int_{r_i}^{r_0}\left[\frac{2(Z-2)e^2}{r}\epsilon(r)-V_0f(r)\epsilon(r) + \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}-E\epsilon(r)\right]^{1/2}dr\right\}. \quad (11)$$

III. RESULTS

We have calculated the integral in (11) for the ground-state transitions of all the even-even nuclei by employing Simpson's rule and taking 120 strips. The results are listed in Table I. The nuclear radii r_i and the classical turning points r_0 remain the same as for the static potential, since for $l=0$ the correction factor $\epsilon(r)$ in the integral is common to all the terms. The nuclear radius, r_i, where the alpha particle enters the barrier, and corrections due to recoil of the nuclei added to the alpha-particle energies, including screening corrections are taken from Perlman and Rasmussen.[19] Calculations have been done with a Remington Rand calculator. Since energies and radii are given to three significant figures, we find that the final numerical quantities for each strip, if entered in the calculator to six significant digits, will yield the same values for the integral to three significant figures as for higher digits entered for them. For comparison purposes, instead of giving the previous values of the penetrability factor, we have calculated from the formula (11) the corresponding values of the penetrability factor for a static potential, P_S, by putting the correction factor $\epsilon(r)=1$.

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IV. DISCUSSION

Let us compare the order of correction introduced by the factor $\epsilon(r)$ in (11). It is seen that in the rare earth region the correction is relatively small, being 39% at the minimum and gradually rising up to about 70% in the transuranic region. It is interesting to note that this monotonic percentage increase is interrupted only at the closure of shells at neutron number $N=82$ or 126 and where both N and proton number, P, correspond to 82. It may be seen that in the rare-earth region, for $_{78}$Pt, the correction is 56% which is the average value in this region. But for Nd, Sm, and Gd the correction drops to $\sim$42% on the average. Similarly for Po^{212} and Po^{214} also, the value abruptly drops to 45% from its neighboring value of 58%. This shows a very close correspondence with the trend of variation of E with neutron number for a given Z. At the closure of shells there is an abrupt change in decay energy resulting in a change of $\epsilon(r_0)$. This is reasonably expected, since E is connected with the strip width Δr affecting f, which ultimately is associated with r_i and the classical radii, r_0.

V. CONCLUSION

Since knowledge about the internal mechanism of alpha formation is yet lacking, it has been noted by many authors that an accurate estimate of the penetrability factor is important not only for its own sake but because it is also likely to be the basis for further probing into the mechanism of alpha decay. With this point in view we have presented these results.

It may be noted that until now there has been no way of comparing with experiment the theoretical values of the penetrability so as to give a final conclusion on the form of the alpha-nucleus interaction to be used. But it appears that some evidence is available from the studies of the relative intensities of the spectral lines for the so-called "favored alpha transitions." The theoretical values of intensities given by Bohr, Fröman, and Mottelson[20] for odd nuclei are consistently lower than the experimental values. But it may be pointed that in their calculations the penetrability factor, P_S for a static potential has been used. A simple replacement of this by the corresponding values P_N for a nonlocal potential should give considerable improvement in their results.

A quantitative study of this problem is likely to yield convincing results for the present problem.

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