

## Enhanced Excitation of $T=0$ , $S=0$ States of the Ground-State Configuration in $C^{12}-C^{12}$ Scattering at 126 MeV\*

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A detailed study was made on the scattering of  $C^{12}$  from  $C^{12}$  at 126 MeV in order to better understand the inelastic scattering mechanisms involved in these reactions. Earlier experiments noted four prominent peaks in the energy spectrum of the scattered particles at forward angles. One of these peaks is due to the elastic scattering and another to the excitation of the first excited state in  $C^{12}$  ( $Q = -4.43$  MeV). The two remaining peaks correspond to  $Q$  values of  $-9.0 \pm 0.7$  MeV and  $-14.0 \pm 1$  MeV. Using coincidence techniques it is shown that the principal contribution to the  $Q = -9$  MeV peak over the range of angles studied is due to the excitation of both  $C^{12}$  nuclei to their first excited state. An angular distribution for this process is obtained and compared to a Born approximation calculation in which all parameters are determined by previous experiments. The  $Q = -14$  MeV peak is shown to be due to the excitation of an  $\alpha$ -particle unstable level in  $C^{12}$  at  $14.0 \pm 0.5$  MeV. Measurement of the angular distribution of the  $\alpha$  particles that decay from this level to the  $Be^8$  ground state indicates that this level has spin and parity  $4+$ . Thus, the levels most strongly excited ( $4.43$  MeV,  $2+$ ;  $14.0 \pm 0.5$  MeV,  $4+$ ) in these reactions bear a strong similarity to the level sequence expected for a ground-state rotational band in  $C^{12}$ . However, the shell model in  $L \cdot S$  coupling also accounts for the observed results as the only states in  $C^{12}$  with  $T=0$  and  $S=0$  with the  $(p)^8$  configuration have  $L=0, 2$ , and  $4$  with relative predicted energies in agreement with our observations.

### INTRODUCTION

OVER the past three years considerable progress has been made in the understanding of direct interactions. The best understood of these processes, at present, are those in which the projectile and the observed inelastically scattered particle are the same. In the distorted-wave Born approximation,<sup>1-3</sup> which is the most general theory employed at present to account for these inelastic scattering events, the form of the transition probability between initial and final states is relatively independent of projectile type and further, the transition probability is closely related to the reduced transition probability for the radiative decay of the excited state to the ground state.<sup>4</sup> In nucleon-nucleus scattering the interaction potential responsible for the inelastic scattering is a sum of two-body forces taken throughout the nuclear volume. The spin-independent part of this interaction has the same form as does the electric multipole operator between the same states.<sup>4</sup> Complex projectiles such as deuterons and alpha particles are strongly absorbed in nuclear matter as is evidenced by the strongly oscillatory nature of the resulting angular distributions which are diffraction-like in

nature. These projectiles are expected to cause excitation of the normal modes of the surface of the target and, thus, their interaction is expressed in terms of the Bohr-Mottelson collective model. Again target states showing enhanced electric multipole radiation widths to the ground state are the states expected to be the most strongly excited.

10-MeV/amu heavy ions clearly fall into the category of strongly absorbed projectiles. They differ from simpler projectiles because larger amounts of energy and momentum can be brought into the reaction without increasing the mean free path for absorption of the projectile. There is, of course, an additional complexity in heavy-ion reactions as excitation of the projectile as well as the target must be considered.

The heavy-ion system which has been the most extensively studied at this laboratory is  $C^{12}$  on  $C^{12}$ . The initial experiments<sup>5,6</sup> measured the angular distributions for the elastic scattering and the inelastic scattering where one of the  $C^{12}$  nuclei was excited to the first excited state. The scattering to the first excited state in  $C^{12}$  was the strongest inelastic group observed, as expected for reasons presented above. However, there were two other prominent peaks in the energy spectrum of the scattered  $C^{12}$  nuclei over the range of angles studied corresponding to  $Q$  values of  $-9.0 \pm 0.7$  MeV and  $-14.0 \pm 1$  MeV. It was felt that in order to properly understand heavy-ion reactions that these groups be identified and interpreted within the existing framework of direct interactions.

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<sup>1</sup> E. Rost and N. Austern, *Phys. Rev.* **120**, 1375 (1960).

<sup>2</sup> E. Rost, Ph.D. thesis University of Pittsburgh, 1961 (unpublished).

<sup>3</sup> R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost (to be published).

<sup>4</sup> G. T. Pinkston, G. R. Satchler, and N. Austern, in *Proceedings of the Kingston Conference*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), pp. 394 and 323.

<sup>5</sup> S. D. Baker, K. H. Wang, and J. A. McIntyre, in *Proceedings of the Kingston Conference*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1962).

<sup>6</sup> D. J. Williams and F. E. Steigert, *Nucl. Phys.* **30**, 373 (1962).

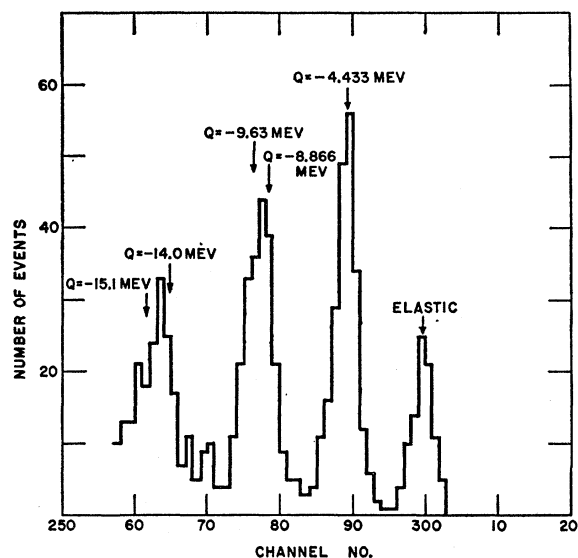


FIG. 1. Energy spectrum of the scattered particles at  $19.5^\circ$  in the laboratory with a 126-MeV  $C^{12}$  beam incident on a thin  $C^{12}$  target.

#### EXPERIMENTAL APPARATUS

The beam used in these experiments was the 126-MeV  $C^{12}$  beam from the Yale Heavy Ion Accelerator. The beam is magnetically analyzed and passed through a pair of 0.25-cm-diam slits 30 cm apart onto a thin  $C^{12}$  target. The energy spread in the beam after analysis is the order of 1%. The scattering chamber employed was one particularly designed for observing two-body heavy-ion reactions in kinematic coincidence. Two independently movable counters set in a plane containing the beam are operated in fast coincidence ( $\tau < 10^{-8}$  sec). One detector is positioned at a forward scattering angle while the other counter is set at an angle appropriate for the detection of recoils associated with the particular reaction under study. This is accomplished by inserting one detector through the upper lid and the other detector through the lower lid of the scattering chamber. As the scattering chamber must be evacuated and as the lids are some 20 in. in diameter, finding a vacuum seal that will permit rotation of the lids presents a serious problem. It was solved using a technique originated at this laboratory,<sup>7</sup> where Teflon strips are employed to take the load and a fabricated O-ring provides the vacuum seal. The coefficient of friction for Teflon under loaded conditions is small enough to permit the lids to be turned by reasonably sized electric motors.

The detectors used in all phases of the work reported herein were silicon junctions with phosphorus diffused into *p*-type silicon. The scattered particle detector is 0.25 cm in diameter while the recoil counter is 0.5 cm. The detectors with the associated electronics gave 30-

keV resolution for 8.78-MeV Th C'  $\alpha$  particles and thus have far better resolution than the 1-MeV spread in the beam energy. It was important that the dead layer on the recoil detector be as small as possible because the recoil nuclei have low energy ( $\sim 1$  MeV/amu) and suffer relatively large energy loss in matter. Detectors with the least dead layer were selected by obtaining, for each detector, the spectrum of  $Cf^{252}$  fission fragments using a calibrated charge-sensitive preamplifier. Comparison to the spectrum obtained by Milton and Frazer<sup>8</sup> show that the best detector had an ionization defect of only 7% on the most energetic group of fragments. It was not possible to ascertain if this loss was due to dead layer or recombination effects.

The electronics employed are quite conventional. The detector preamplifiers are a voltage sensitive, fast rise time ( $\sim 25$  nsec) type designed by E. R. Beringer of this laboratory. The fast-coincidence circuit used for determining kinematic coincidence is a Petch, Graham, and Bell type<sup>9</sup> with a Hoffman unitunnel diode used as the discriminator element. The circuit operated reliably with  $10^{-8}$  to  $5 \times 10^{-9}$  sec resolving times. Gates were used that could selectively require, in addition to kinematic coincidence, that the pulse height from the scattered particle detector satisfied certain conditions. In all runs two four-hundred channel analyzers were used and were gated appropriately to allow the observation of the desired phenomena.

#### Experiment on the 9-MeV Group

Figure 1 shows the energy spectrum obtained with a solid-state detector at  $19\frac{1}{2}^\circ$  in the laboratory with 126-MeV  $C^{12}$  ions incident on a thin ( $\sim 100 \mu\text{g}/\text{cm}^2$ )  $C^{12}$  target. Figure 2 shows the presently known energy levels in  $C^{12}$ .<sup>10</sup> Note the selectivity of the excitation process as only 4 large peaks are observed corresponding to excitations of 15 MeV or less in  $C^{12}$ . The first two peaks in Fig. 1 are attributable to the elastic scattering and scattering to the first excited state in  $C^{12}$ . The third group however is difficult to assign and is found to be a composite of two processes. They involve the excitation of a single nucleus to the 9.63-MeV level or the mutual excitation of both nuclei to their first excited state with a resulting  $Q$  value of  $-8.86$  MeV. It is impossible to separate the relative contributions of these two events with energy resolution because of the energy spread in the incident beam. The separation may be effected using kinematic coincidence and some results have been previously reported<sup>11</sup> but the lack of space did not permit an

<sup>8</sup> J. C. Milton and J. S. Frazer, *Phys. Rev.* **111**, 877 (1958).

<sup>9</sup> I. A. D. Lewis and F. H. Wells, *Millimicrosecond Pulse Techniques* (Pergamon Press Inc., New York, 1954).

<sup>10</sup> F. Ajzenberg-Selove and T. Lauritsen, *Nucl. Phys.* **11**, 1 (1958); *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.), Sets 546, 1962.

<sup>11</sup> G. T. Garvey, A. M. Smith, J. C. Hiebert, and F. E. Steigert, *Phys. Rev. Letters* **8**, 25 (1962).

<sup>7</sup> C. E. Anderson, A. R. Quinton, W. J. Knox, and Robert Long, *Nucl. Instr. Methods* **7**, 1 (1960).

adequate description of the experimental technique.

For a fixed forward scattering angle, the associated recoil angles for the center of mass of the recoil nucleus differ by less than a degree for the two processes contributing to the  $Q = -9.0$  MeV peak. However, the recoils associated with the group attributable to the excitation of the 9.6-MeV level are  $\alpha$ -particle unstable and decay to the ground state<sup>10</sup> of  $Be^8$ , which itself is unstable against the decay into two  $\alpha$  particles. Because of this  $\alpha$ -particle instability the contribution of the 9.6-MeV level to the coincidence counting rate is greatly attenuated relative to its contribution to the noncoincidence counting rate at the forward detector. To exploit the  $\alpha$ -particle instability of the 9.63-MeV level the following procedure is used. The recoil detector is set at the kinematically determined angles for the detection of  $C^{12}$  recoil nuclei that result from reactions with a  $Q$  value of  $\sim -9$  MeV. In one analyzer (No. 1) an ungated pulse-height spectrum of the scattered particle detector is stored while simultaneously in another analyzer (No. 2) only the pulses from the scattered particle detector that have an associated kinematic coincidence are stored. The ratio of the number of counts in the 9.0-MeV group in analyzer No. 2 to the corresponding number of counts in analyzer No. 1 will be a function of the relative amounts of the  $Q = -8.86$  MeV and  $Q = -9.63$  MeV contributions. As the accidental coincidence counting rate is very low the only counts that appear in analyzer No. 2 are in the  $Q = -9$  MeV group. To correct for losses in the coincidence counting rate owing to the fact that the recoil counter may not detect all recoils because of multiple scattering or solid angle effects a comparison measurement is made on the  $Q = -4.43$ -MeV group. This group contains no particle unstable recoils, so the ratio of counts in analyzer No. 2 to the number in analyzer No. 1, when the recoil counter is properly positioned for the detection of recoils associated with the  $Q = -4.43$  MeV process, represents the effective efficiency for the detection of two-body reactions for the forward scattering angle being studied. Thus, when suitable corrections are applied, the ratio

$$\frac{N_{\text{gated}}(9.0 \text{ MeV})}{N_{\text{ungated}}(9.0 \text{ MeV})} \bigg/ \frac{N_{\text{gated}}(4.43 \text{ MeV})}{N_{\text{ungated}}(4.43 \text{ MeV})}$$

yields the relative amount of  $Q = -8.86$  MeV contribution to the composite peak provided that the cross section for the 9.6-MeV process is not considerably larger, say by a factor of 10, than the mutual excitation process; a condition well satisfied over the range of angles studied. In the above expression,  $N_{\text{gated}}(9.0 \text{ MeV})$  refers to the number of counts in the gated analyzer (No. 2) in the 9.0-MeV peak, etc.

Estimates were made regarding the relative contribution of the 9.6-MeV level to the coincidence counting rate. The most elaborate of these calculations used an IBM 709 computer and considered: finite source, finite

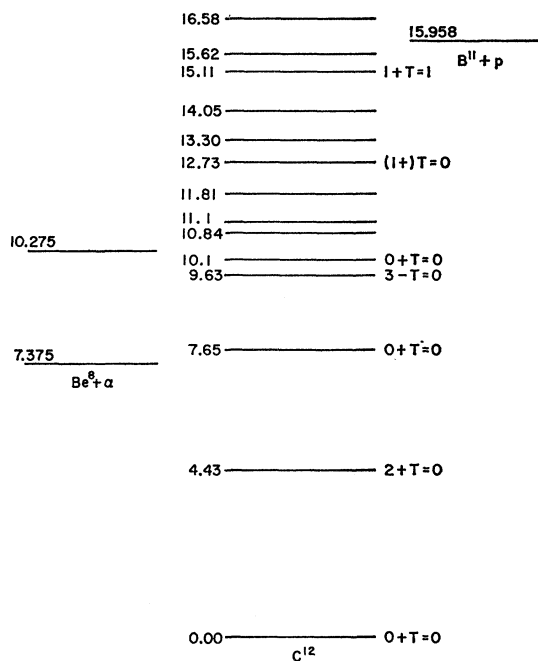


FIG. 2. Energy levels in  $C^{12}$ . See reference 10.

solid angle, nonparallel beam trajectories, and the correlation of the emitted  $\alpha$  particles to the recoil direction. This calculation predicted that the contribution of the  $\alpha$ -breakup process to the coincidence counting rate would be small; the order of 1% of the noncoincidence counting rate. This estimate was further checked experimentally by observing the spectrum of the recoil counter in kinematic coincidence when it was set over the range of angles suitable for the detection of  $Q = -8.86$  MeV,  $C^{12}$  two-body reactions. In order not to introduce undue spread in the kinetic energy of the detected recoiling  $C^{12}$  nuclei a thin ( $100 \mu\text{g}/\text{cm}^2$ ), self-supporting evaporated  $C^{12}$  target was used in place of the 1-mg/ $\text{cm}^2$  polyethylene target normally employed. Apart from a few accidental counts, all of the counts in the recoil spectrum were attributable to recoil  $C^{12}$  nuclei resulting from mutual excitation events rather than to  $\alpha$  particles resulting from the breakup of  $C^{12}$  nuclei excited to 9.6 MeV. These tests showed our estimates to be essentially correct.

Thus, the experimental procedure for determining the angular distribution for the mutual excitation process consisted of obtaining at each laboratory angle the gated and ungated spectrum for the  $Q = -9$  MeV groups and the  $Q = -4.43$  MeV group. After suitable corrections are applied to these data a comparison of the two ratios for fixed forward scattering angle yields the relative amount of the mutual excitation process at that particular angle. The distribution so obtained is shown in Fig. 3 along with other measured distributions for  $C^{12} - C^{12}$  scattering. The relative and absolute differential cross sections for the mutual excitation ( $Q = -8.86$

MeV) are accurate to 10 and 20%, respectively. The distributions shown are a compilation of these data and those of Wang *et al.*<sup>12</sup> The factor limiting the measurement of the  $Q = -8.86$  MeV angular distribution at small angles is the very low energy for recoil particles associated with projectiles scattered to small forward angles. The other distributions shown do not require detection of recoils to make an unambiguous identification of the process involved and hence are not subject to the above limitation. Three things should be noted regarding the  $Q = -8.86$ -MeV differential cross section. It oscillates out of phase with the distribution for the excitation of a single nucleus to the 4.43-MeV state, the envelope for the mutual excitation ( $Q = -8.86$  MeV) falls off less steeply with angle than that for the single excitation ( $Q = -4.43$  MeV), and finally the magnitude of the  $Q = -8.86$ -MeV process is larger than one might have expected. All of these features can be accounted for in terms of a calculation presented below.

### Born Approximation Analysis of the Angular Distributions

The Born approximation is used to calculate the angular distributions in a manner similar to other authors.<sup>13</sup> While it is admitted that the use of the Born approximation is diametrically opposed to our assumption of a strongly absorbing target nucleus, Blair<sup>14</sup> has

$$U = V(R) + R_0(4\pi)^{1/2} \sum_{l,m} \frac{\beta_{l,m}}{(2l+1)^{1/2}} Y_{l,m}^*(\theta,\phi) Y_{l,m}(\Omega) \frac{\partial V}{\partial R} \\ + \frac{1}{2} R_0^2 (4\pi) \sum_{l,l',m,m'} \frac{\beta_{l,m} \beta_{l',m'}}{(2l+1)^{1/2} (2l'+1)^{1/2}} Y_{l',m'}^*(\theta,\phi) Y_{l',m'}(\Omega) Y_{l,m}(\theta,\phi) Y_{l,m}(\Omega) \frac{\partial^2 V}{\partial R^2}. \quad (1)$$

The first term in the potential is a function of  $R$ , the separation distance, only and is responsible for the elastic scattering, while the higher order terms are responsible for the inelastic scattering. Assuming a square-well form for  $V(R)$  of depth  $V_0$  and extent  $R_0$  yields an elastic cross section

$$\frac{d\sigma}{d\Omega} = \left[ \frac{2\mu V_0 R_0^3}{\hbar^2} \right]^2 \left[ \frac{\mathbf{j}_1(K_T R_0)}{K_T R_0} \right]^2, \quad (2)$$

where  $K_T$  is the transferred momentum over to,  $K_T = |K_0 - K_f|$ , and  $V_0$  is an "effective potential" of sufficient depth to account for the magnitude of the elastic scattering. Assuming strong coupling wave functions for the target, the inelastic scattering to lowest

shown the similarity between the plane-wave results and the results of the more logically consistent adiabatic, Fraunhofer approximation. The best agreement between theory and experiment is obtained when the distorted-wave Born approximation<sup>1-4</sup> is employed; however, these calculations require extensive use of high-speed computers. Many of the important features of the angular distributions are evident in the Born approximation.

As the projectiles are expected to interact with the surface of the target nucleus, the scattering is parameterized in terms of the condition of the surface. Consider a nucleus whose surface can be characterized in terms of the Bohr-Mottelson<sup>15</sup> model:

$$r = r_0 \left[ 1 + \sum_{l,m} \alpha_{l,m} Y_{l,m}(\theta',\phi') \right].$$

The interaction between the target and the projectile is depicted as taking place about some radius  $R_0$ , which should be very nearly  $r_{0 \text{ target}} + r_{0 \text{ projectile}}$ . The interaction potential is expanded in a power series in the deformation parameter about  $R_0$ . Using strong coupling parameterization and assuming the nucleus to be spheroidally deformed, the interaction potential assumes the following approximate form to second order in the deformation:

order in  $\beta$  for an even-even nucleus excited to a level of spin  $I$  is given by the expression.

$$\frac{d\sigma}{d\Omega} = \frac{\beta_I^2}{4\pi} \left[ \frac{2\mu V_0 R_0^3}{\hbar^2} \right]^2 \frac{K_f}{K_0} \mathbf{j}_I^2(K_T R_0). \quad (3)$$

$\beta_I$  is the surface deformation of order  $I$ . Using the term in the potential of order  $\beta^2$  yields distributions that have an anomalous phase.<sup>18</sup>

To fit expression (2) to the experimental distribution,  $R_0$  is first selected so as to give the proper period of oscillation to the angular distribution.  $V_0$  is then selected so as to make the expression agree in magnitude with the observed cross section. Figure 4 shows the comparison between theory and experiment with  $R_0 = 6.5$  F and  $V_0 = 3.5$  MeV. A better fit to the periodicity can be obtained by choosing  $R_0$  to be slightly smaller but this

<sup>12</sup> K. H. Wang, S. D. Baker, and J. A. McIntyre, Phys. Rev. **12**, 187 (1962).

<sup>13</sup> R. H. Lemmer, A. de-Shalit, and N. S. Wall, Phys. Rev. **124**, 1155 (1961).

<sup>14</sup> J. S. Blair, Phys. Rev. **115**, 928 (1959).

<sup>15</sup> A. Bohr, Kgl. Danske Videnskab Mat.-Fys. Medd. **26**, No. 14 (1952); **27**, No. 16 (1953).

FIG. 3. Measured differential cross sections vs center-of-mass angles for the scattering of 126-MeV  $C^{12}$  ions from  $C^{12}$ . The  $Q = -8.86$  MeV distribution is the cross section for the excitation of both target and projectile to their first excited state.

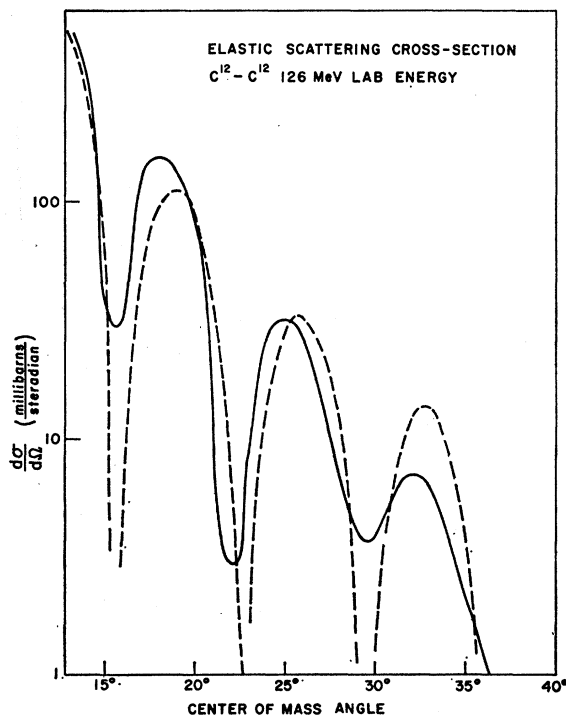
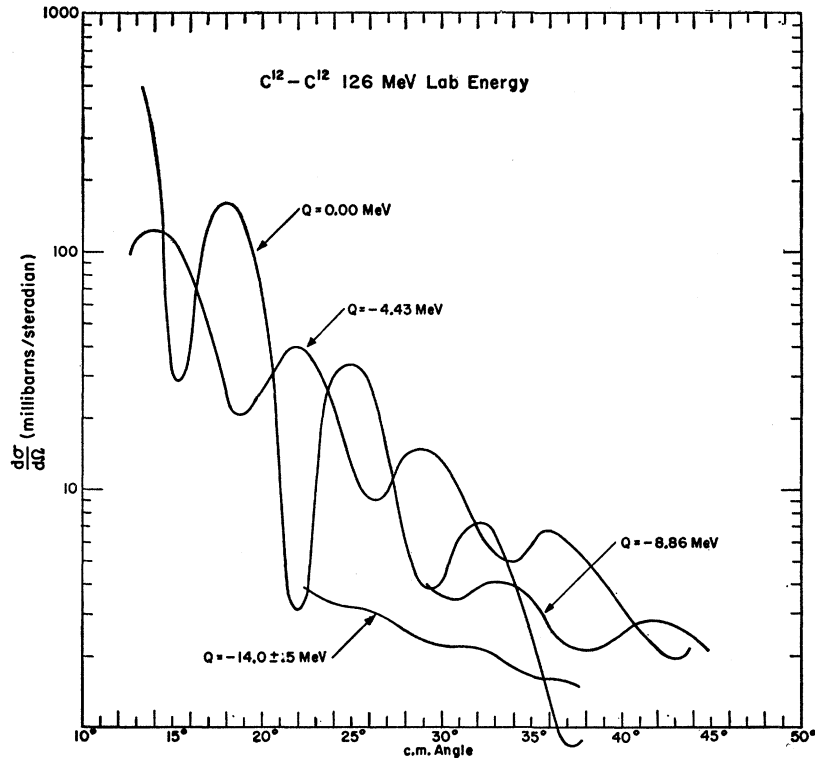


FIG. 4. Comparison of the measured elastic differential cross section (solid curve) to the Born-approximation prediction [Eq. (2)] with  $V_0 = 3.5$  MeV and  $R_0 = 6.5$  F.

value is chosen because it is more consistent with the data to follow. This gives  $r_0 = 1.39$  F, a reasonable value. The small effective well depth is completely consistent with the values obtained by the University of Washington group<sup>16</sup> using a similar analysis of  $\alpha$ -particle scattering.

The inelastic angular distribution from the 4.43-MeV  $2+$  state is characterized by  $j_2(K_T R_0)$  and, therefore, should oscillate out of phase with the elastic scattering distribution. This phenomenon, clearly shown in Fig. 3 is known as the Blair phase rule.<sup>14</sup> It states that for inelastic scattering from even-even nuclei, the angular distribution of the scattering from negative-parity states will oscillate in phase with the elastic scattering while the scattering from positive-parity states will be out of phase. This rule holds for all inelastic scattering that can be accounted for in lowest order in the deformation. Figure 5 compares Eq. (3) to the observed distribution, using  $R_0$  and  $V_0$  as determined from the elastic distribution, with  $\beta_2 = 0.16$ . The fact that either nucleus may be excited has been taken into account in determining this value of  $\beta_2$ . One sees that the phase is properly predicted but that the theoretical cross section tends to be too large at larger angles as is also the case in the elastic scattering.

For the case of the mutual excitation it is necessary to consider that two deformed nuclei are involved in the reaction. Thus, the expression for the interaction

<sup>16</sup> A. I. Yavin and G. W. Farwell, Nucl. Phys. 12, 1 (1959).

potential should consider both surfaces to be deformed or deformable. Employing this notion and expanding the potential to second order in the deformation param-

eter, a term is obtained that can cause excitation in both nuclei. Using this term, the following cross section is obtained:

$$\frac{d\sigma_{I,I'}}{dr} = \frac{\beta_I^2 \beta_{I'}^2 \left[ \frac{2\mu V_0 R_0^3}{\hbar^2} \right]^2 \frac{K_f}{K_0} \sum_m \left| \sum_l (-i)^l C(II'l, m-m_0) \right. \\ \left. \times C(II'l, 000) \left\{ \begin{array}{l} K_T R_0 \mathbf{j}_{l-1}(K_T R_0) - (l-1) \mathbf{j}_l(K_T R_0), \quad l \neq 0 \\ -K_T R_0 \mathbf{j}_1(K_T R_0) + 2 \mathbf{j}_0(K_T R_0), \quad l = 0 \end{array} \right\} \right|^2, \quad (4)$$

where  $I$  and  $I'$  refer to the spins of the states excited in the reaction. In the region of large momentum transfer the contribution to the cross section from the term to lowest order of  $\beta$  in the potential calculated, to second order in the Born approximation is very small so that the above expression should account for most of the cross section. Note that all parameters for the mutual excitation of the first excited state are determined as  $\beta_I^2 \beta_{I'}^2 = \beta_2^4$ . The  $C(II'l, MMO)$  are the vector addition coefficients as defined by Rose.<sup>17</sup> Figure 6 compares Eq. (4) to the observed cross section using for  $V_0$ ,  $R_0$ , and  $\beta_2$  the previously determined values. The agreement is satisfactory regarding the phase, the flatness of the envelope, and the magnitude of the maxima of the cross section.

Authors who employ the more exact distorted-wave Born approximation feel that the plane-wave calculations have been in error regarding second-order scat-

tering processes, and that the fits obtained are fortuitous.<sup>18</sup> The same criticism may well apply to the calculation presented above.

The peak at approximately 14 MeV in Fig. 1 has been shown by Bromley *et al.*<sup>19</sup> and Wang *et al.*<sup>12</sup> to be due primarily to excitation of  $C^{12}$  nuclei and not to single- or multi-nucleon transfer events. Wang *et al.* have further measured the angular distribution which is shown in Fig. 3 and labeled  $Q = -14$  MeV. Though the errors shown in the above paper are large the cross section is certainly known in magnitude to within a factor of 2. The slope of this distribution agrees perfectly with the slope of the envelope of the mutual excitation angular distribution ( $Q = -8.87$ ) and is less steep than the envelope of the distribution for the excitation of one nucleus to the first excited state ( $Q = -4.43$ ) by a factor of 1.9. The observed magnitude of the  $Q = -14$  MeV distribution agrees well with the predicted magnitude for a second-order scattering process taking place through the quadrupole deformation. This statement would also hold true in the more rigorous distorted wave theory. Using a relationship derived by Austern *et al.*<sup>18</sup> for the relative magnitude of second-order cross sections to associated first-order cross sections, good agreement is had between the observed  $Q = -4.43$  MeV and  $Q = -14.0$  MeV distributions.<sup>19a</sup> Any second-order process, exciting a level in  $C^{12}$ , involving the octupole deformation (vibration) would have to be smaller by a factor of more than 5 than the observed  $Q = -14$  MeV cross section due to the weak excitation observed for the 9.63-MeV 3- state. An alternate excitation mode for this  $Q$  value might have been the mutual excitation of the 4.43- and 9.63-MeV states but results reported herein show that this is not the case. Second-order quadrupole processes would be expected to excite a level of spin 0, 2, or 4; all with positive parity. Second-order quadrupole processes resulting in the excitation of unnatural parity

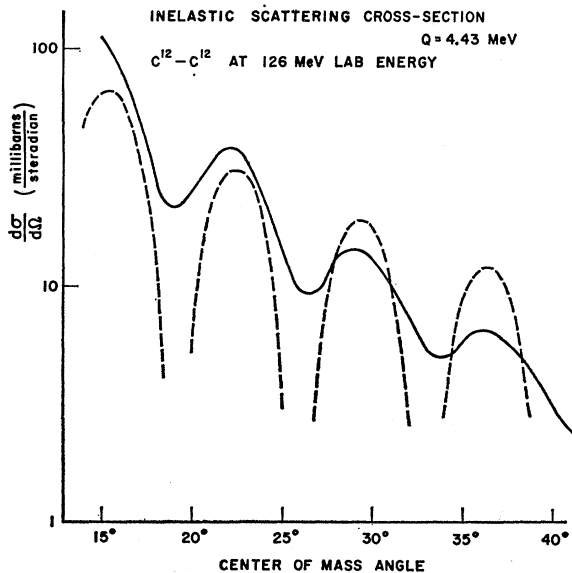


FIG. 5. Comparison of the measured inelastic differential cross section for the excitation of the 4.43-MeV level in  $C^{12}$  (solid curve) to the Born approximation prediction with  $V_0 = 3.5$  MeV,  $R_0 = 6.6$  F, and  $\beta_2 = 0.16$ .

<sup>17</sup> M. E. Rose, *Elementary Theory of Angular Momentum* (J. Wiley & Sons, Inc., New York, 1957).

<sup>18</sup> Brian Buck, *Phys. Rev.* **127**, 940 (1962); N. Austern, R. M. Drisko, E. Rost, and G. R. Satchler, *ibid.* **128**, 733 (1962).

<sup>19</sup> D. A. Bromley, M. Sachs, K. Nagatani, and C. E. Anderson, *Proceedings of the Rutherford Jubilee International Conference*, edited by J. B. Birks (Heywood and Company, Ltd., London, 1962).

<sup>19a</sup> Note added in proof. R. H. Bassel, G. R. Satchler, and R. M. Drisko, have actually carried out D.W.B.A. calculations and found good agreement with this hypothesis (private communication).

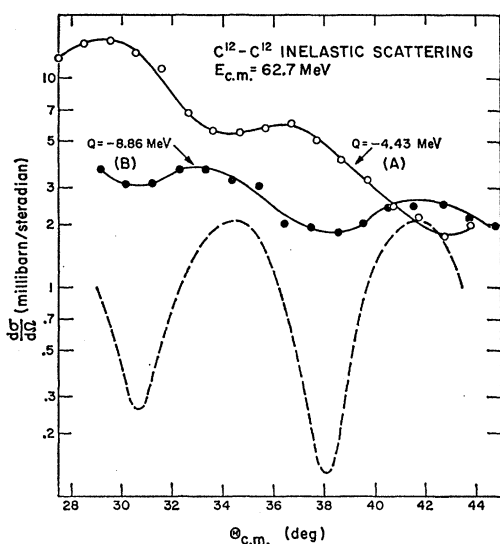


FIG. 6. Comparison of the measured angular distribution for the excitation of both target and projectile to their first excited state ( $Q = -8.86$  MeV) to the Born approximation prediction [Eq. (4)] using the parameters already obtained;  $V_0 = 3.5$  MeV,  $R_0 = 6.5$  F, and  $\beta_2 = 0.16$ . The measured distribution for the process where just one of the nuclei is excited to its first excited state is also shown.

levels are observed to be greatly inhibited at these energies<sup>20,21</sup>; further it is shown that the level excited corresponding to  $Q = -14$  MeV has natural parity.

### Investigation of the $Q = -14$ MeV Scattering Event

The first experiment examining the character of the 14-MeV event tested for the stability of the recoil nucleus against particle decay. First, for comparison purposes the detectors are set at proper relative angles for the detection of the  $Q = -4.43$  MeV scattering process. The pulses from the forward detector are analyzed in a single-channel analyzer where a "window" is set about the  $Q = -4.43$  MeV group. The pulses within this "window" are then stored in a multichannel analyzer. In another multichannel analyzer the pulses from the recoil counter, that satisfy the kinematic coincidence requirement in addition to being in coincidence with pulses from the forward detector that are within the window are stored. The observed results are shown in Figs. 7(a) and (b), respectively. The events in the recoil spectrum are due to the recoiling  $C^{12}$  nuclei. Next the recoil counter is moved over the range of angles corresponding to  $Q$  values of  $-13$  to  $-15$  MeV. The "window" on the forward counter spectrum is moved down to bracket the  $Q = -14$  MeV group. Figures 7(c) and (d) show a sample of the results obtained; (c) is the forward counter gated spectrum, while (d) is the

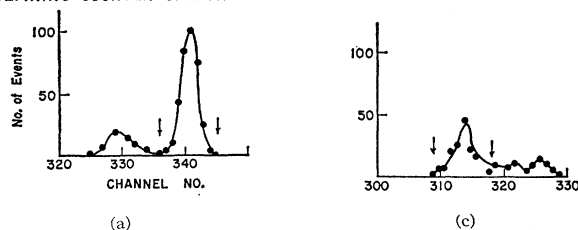
<sup>20</sup> Cyclotron Research Progress Report (University of Washington, 1962), p. 3.

<sup>21</sup> W. W. Eidson and J. G. Cramer, Jr. (to be published).

gated recoil spectrum. Figure 7(d) shows effectively no counts at the energy expected for  $C^{12}$  recoils. Thus, two conclusions may be drawn. The recoil nucleus associated with the  $Q = -14$  MeV event is particle unstable and further the contribution of the 15.1-MeV ( $1+ T=1$ ) level, which is known to have a  $\gamma$ -ray branching ratio greater than 0.5, is at most 3% of the observed peak.

The only particle instability available to  $C^{12}$  levels excited to less than 16 MeV is  $\alpha$ -particle decay<sup>10</sup> to  $Be^8$ . An excited level in  $C^{12}$  can undergo  $\alpha$  decay to the ground state of  $Be^8$  only if it has natural spin and parity as the  $Be^8$  ground state is  $0+$ . When it is energetically possible for the decay to proceed through the first excited state of  $Be^8$ , the decay to the  $Be^8$  ground state is inhibited relative to the decay to the  $2+$  first excited state of  $Be^8$  by a statistical factor of 5, further, if the decaying  $C^{12}$  level has a spin of 2 or greater the angular momentum barrier would favor decay to the  $Be^8$  excited state. However, if the decay mode to the  $Be^8$  ground state can be isolated and its angular distribution obtained one should be able to infer the spin of the decaying level in  $C^{12}$ . Within the Born approximation it can easily be shown that the angular distribution of the  $\alpha$  particles that proceed to the  $Be^8$  ground state, in the center of mass of the recoiling  $C^{12}$  nucleus, has the form  $[Y_l^0(\psi, 0)]^2$ , where  $l$  is the spin of the excited  $C^{12}$  level and  $\psi$  is measured relative to the recoil direction. This simple result is obtained in the Born approximation<sup>22</sup> because only the  $M=0$

### DEFINING COUNTER SPECTRA



### RECOIL COUNTER SPECTRA

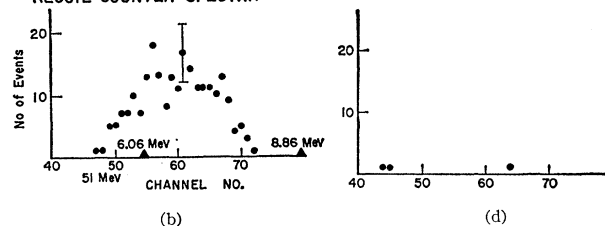


FIG. 7. (a) and (b) are gated energy spectra for the forward detector and recoil detector, respectively, when they are set appropriately for the detection of  $Q = -4.43$  MeV inelastic scattering events. The forward counter spectrum is gated by a single-channel analyzer set about the pulse height corresponding to the  $Q = -4.43$  MeV group. The recoil counter spectrum is gated by the fast kinematic coincidence circuit in addition to the gate set about the forward detector. (c) and (d) are a sample of the same spectra obtained with the gate set about the 14-MeV group in the forward detector with the recoil counter moved over the range of angles appropriate for the detection of  $C^{12}$  recoils associated with  $Q$  values of  $-13$  to  $-15$  MeV.

<sup>22</sup> G. R. Satchler, Proc. Phys. Soc. (London) A68, 1037 (1955).

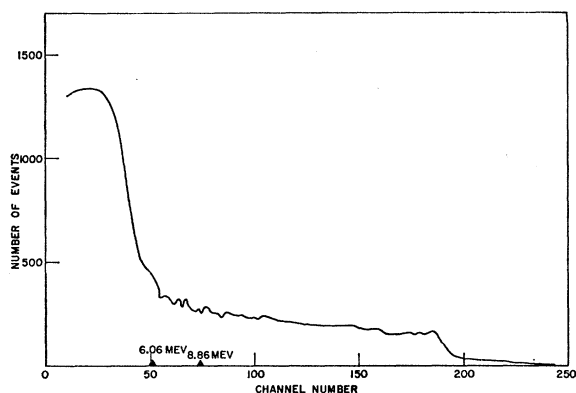


FIG. 8. The ungated spectrum of the recoil counter at  $61^\circ$  to the incident beam with a 0.0008-in. Al foil placed over the detector to exclude low-energy heavy particles. The sharp breaks in the energy spectra around channels 40 and 190 are due to the maximum energy that can be lost by protons and  $\alpha$  particles in the depletion layer of the detector.

magnetic substate is populated relative to the recoil axis. Previous experiments by Sherr and Hornyak,<sup>23</sup> Shook,<sup>24</sup> and Crut and Wall<sup>25</sup> show this approximation to be sufficiently good for spin determinations, with the principal disagreement from the theory being a shift in the correlation direction away from the recoil axis due to distortion effects. More recent work by McDaniels *et al.*<sup>26</sup> shows that more serious discrepancies may occur if one studies the correlation of the decay radiation in coincidence with particles scattered into minima of the relevant angular distribution.

To obtain the spectrum of the breakup  $\alpha$  particles, the scattered particle detector was set at  $14^\circ$  in the laboratory and the recoil counter was moved to within 2 cm of the target in order to be more efficient in the detection of decaying  $\alpha$  particles. The recoil detector was calibrated previous to every run using  $\alpha$  particles from the ThC' decay. A 0.0008-in. Al foil was placed over the recoil detector to keep out heavy fragments and low-energy background. This foil was also effective in removing a spurious signal which appeared on the recoil detector that was attributable to luminescence of the target when it was struck by the beam. Figure 8 shows an ungated spectrum of the recoil counter. The sharp breaks that occur in this spectrum at channels 40 and 190 correspond to the maximum energy that can be lost in the depletion region of this detector for protons and  $\alpha$  particles, respectively.

Figure 9 shows sample spectra from the recoil counter in coincidence with the 14-MeV group in the forward detector. The most energetic group of  $\alpha$  particles that can be observed in the recoil counter in this geometry, that

are in coincidence with the detection of a particle in the 14-MeV group at the forward detector, correspond to the  $\alpha$  particles emitted from the excited  $C^{12}$  level to the ground state of  $Be^8$ . The highest energy group observed in the gated recoil spectra corresponds to  $\alpha$  particles emitted to the ground state of  $Be^8$  from a  $C^{12}$  nucleus excited to  $14.2 \pm 0.3$  MeV. The measurement of the excitation of the  $C^{12}$  level is in excellent agreement with the direct measurement of  $14.0 \pm 0.5$  made by Wang *et al.*<sup>12</sup> The approximately 0.7-MeV width of the observed group is directly attributable to the distribution in kinetic energy of the recoiling  $C^{12}$  nuclei due to the finite angular acceptance of the forward detector. The other counts in the spectra are due to the alternate modes of  $\alpha$  decay of the  $C^{12}$  and to  $\alpha$  particles resulting from the breakup of  $Be^8$ . There is no evidence in the spectra for a group of  $\alpha$  particles that would result if the recoiling nucleus was in the 9.63-MeV state, thus, showing that the mutual excitation of the 9.63- and the 4.43-MeV levels is not the principal mechanism responsible for the  $Q = -14.0$  MeV peak. The decay to the ground state of  $Be^8$  shows the  $C^{12}$  level to have natural spin and parity. The true to accidental ratio in the spectra shown is the order of 30:1.

Next the angular distribution of the  $\alpha$  particles emitted to the  $Be^8$  ground state was obtained over a range of angles permitted by kinematic considerations and background problems. This angular distribution

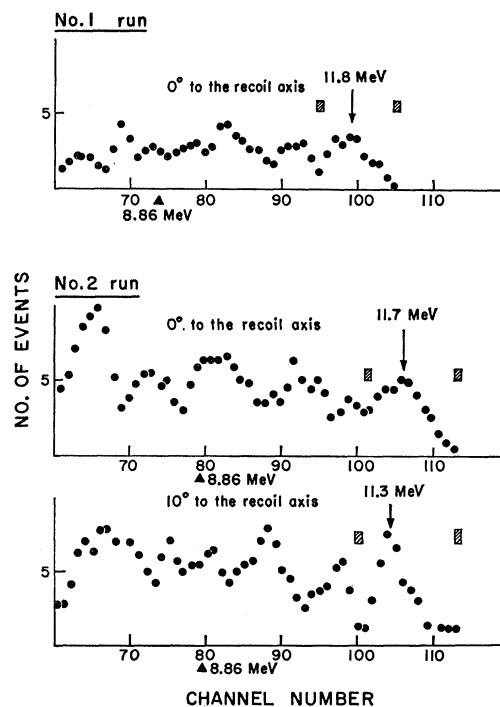


FIG. 9. Sample energy spectra of the recoil counter in slow-fast coincidence with the 14.0-MeV group in the forward detector. The forward detector is at  $13^\circ$  in the laboratory system while the angles assigned to the recoil counter are measured relative to the recoil direction ( $61.25^\circ$ ) in the plane of the scattering event.

<sup>23</sup> R. Sherr and W. F. Hornyak, *Bull. Am. Phys. Soc.* **1**, 197 (1956).

<sup>24</sup> G. B. Shook, *Phys. Rev.* **114**, 310 (1959).

<sup>25</sup> M. Crut and N. S. Wall, *Phys. Rev. Letters* **3**, 521 (1959).

<sup>26</sup> D. K. MacDaniels, D. L. Hendrie, R. H. Bassel, and G. R. Satchler, *Phys. Letters* **1**, 295 (1962).



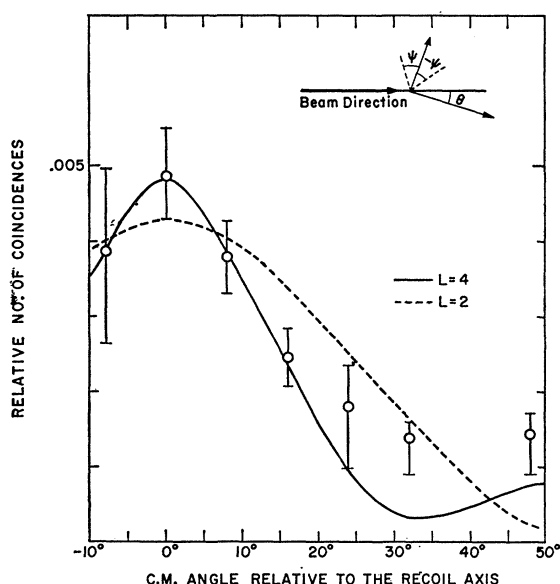


FIG. 10. The points shown represent the angular distribution of  $\alpha$  particles emitted from the  $C^{12}$  level at  $17.0 \pm 5$  MeV to the  $Be^8$  ground state in the center of mass of the recoiling  $C^{12}$  nucleus. Zero degrees is chosen as the recoil direction. The forward detector is fixed at  $13^\circ$  in the lab system. The distributions for  $L=2$  and 4, using the Born approximation, corrected for finite geometry are shown.

taken in the plane of the scattering event is shown in Fig. 10.  $\theta$  refers to the scattering angle, the other arrow indicates the recoil direction while  $\psi$  refers to the laboratory angles at which the distribution was measured. This distribution is then converted to the center of mass of the recoiling excited  $C^{12}$  nucleus, taking the recoil direction as  $\psi=0$ . Due to the finite solid angle subtended by the detectors, kinematic effects spread the energy of the detected  $\alpha$  particles associated with the possible decay modes until at  $\psi > 15^\circ$  some processes can overlap somewhat in energy with the decay to the  $Be^8$  ground state. This means that the points shown in Fig. 10 for center-of-mass angles greater than  $20^\circ$  are probably overestimated because in the analysis of the data all counts that fell within the calculated acceptable energy for the decay to the  $Be^8$  ground state were included at that particular angle, as no reliable estimate could be made as to the actual contribution of other processes at these larger angles. The distributions predicted by the Born approximation for  $L=2$  and 4 are shown in Fig. 10, each is corrected for finite solid angle.

The fit to  $L=4$  must be considered to be best as the points for angles less than  $20^\circ$  are the most reliable and further the population of sublevels other than  $m=0$  would make the predicted distributions shown for  $L=2$  and 4 more isotropic than they are shown. The angular distribution shows a good correlation to the recoil direction at this scattering angle ( $14^\circ$  laboratory), but it certainly would be desirable to measure it for other scattering angles. Due to the 1% duty cycle of the ac-

celerator, the relatively small cross section for the formation of the 14-MeV level, and the small branching ratio for the subsequent decay to the  $Be^8$  ground state coupled with the fact that thin targets ( $\sim 400 \mu\text{g}/\text{cm}^2$ ) must be employed to retain sufficient energy resolution each point takes an average of 30 h of running time. Thus, further experiments measuring these distributions would be impractical at this time. Instead a measurement was made on the angular distribution of the  $\alpha$  particles decaying from the 9.63-MeV level to the  $Be^8$  ground state. The cross section for the excitation of this level is small also, but all of the  $\alpha$  particles emitted from this level go to the ground state of  $Be^8$ . The forward detector was placed at  $12^\circ$  and sample coincidence spectra obtained in the  $\alpha$  particle detector are shown in Fig. 11. The interpretation of these spectra are much simpler than those in Fig. 9 as the only particles satisfying the coincidence requirements are  $\alpha$  particles from the 9.63-MeV level to the  $Be^8$  ground state and the subsequent  $\alpha$  decay of  $Be^8$  ground state. The spectra shown are in complete accord with what was expected, the  $\alpha$  peak having the proper energy and a width of approximately 1 MeV due to the finite angular acceptance of the forward detector. The spin of the 9.63-MeV level was believed to be  $1-^{10}$ ; experiments within the past year and a half<sup>27,28</sup> have shown it to be  $3-$ . The angular distribution for the decay  $\alpha$  particles is shown in Fig. 12 along with the expected fits for  $L=3$  and 1. The  $L=3$

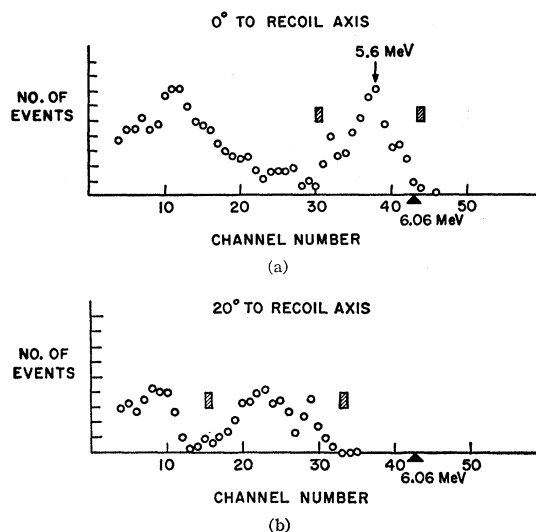


FIG. 11. Sample energy spectra from the recoil counter in slow-fast coincidence with the 9.0-MeV group in the forward detector. The forward detector is at  $12^\circ$  in laboratory while the angles assigned to the recoil counter are measured relative to the recoil direction ( $66.8^\circ$ ) in the plane of the scattering event. A 0.0004-in. Al foil excludes coincidence due to mutual excitation events which otherwise would contribute to the coincidence counting rate near  $0^\circ$  to the recoil axis.

<sup>27</sup> F. C. Barker, E. Bradford, and L. J. Tassie, Nucl. Phys. **19**, 101 (1960).

<sup>28</sup> R. R. Carlson, Nucl. Phys. **28**, 443 (1961).

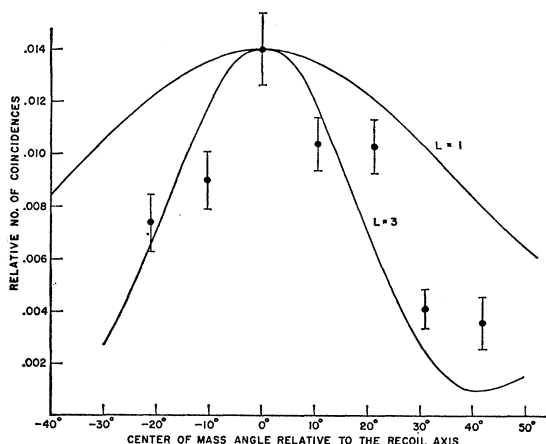


FIG. 12. The points shown represent the angular distribution of  $\alpha$  particles emitted from the  $C^{12}$  level at 9.63 MeV to the  $Be^8$  ground state in the center of mass of the recoiling  $C^{12}$  nucleus. Zero degrees is chosen as the recoil direction. The forward detector is at  $12^\circ$  in the laboratory system. The distributions for  $L=3$  and 1, in the Born approximation, corrected for finite geometry, are shown.

assignment is definitely favored in accord with the more recent experiments and again the approximate correlation to the recoil axis is observed. The error flags shown in Figs. 10 and 12 reflect the errors due to counting statistics which are large compared to other uncertainties in the experiment.

#### DISCUSSION OF RESULTS

As no evidence to the contrary can be found, it is felt that the level observed as a  $4+$  corresponds to a level observed in  $C^{12}$  at 14.05 MeV.<sup>10</sup> It is interesting to note that the energy of the  $4+$  state observed in this work, if it is considered to be the  $4+$  member of a ground-state rotational band is in substantial agreement with an  $L(L+1)$  energy dependence expected in a rotational band [ $(10/3)4.43$  MeV = 14.8 MeV].  $E2$  decay to the first excited state from the  $4+$  level at 14 MeV competes poorly ( $\Gamma_\gamma/\Gamma \sim 10^{-4}$ ) with the  $\alpha$  decay so that the possibility of observing enhanced  $E2$  radiation to the first excited state which would be a further consequence of a rotational model is poor. However, one need not invoke a collective model to predict the existence of a  $4+$  level at about 14-MeV excitation. In 1937 Feenberg and Phillips<sup>29</sup> showed for  $L \cdot S$  coupling in the  $p$  shell, the class of levels characterized by  $T=0$  and  $S=0$  have an  $L(L+1)$  energy dependence. Further for  $C^{12}$  the only states in the ground-state configuration  $(p)^8$  that have  $T=0$  and  $S=0$  have spin 0, 2, and 4 with positive parity. The  $0+$  state is identified with the ground state, the  $2+$  state with the first excited state, and, thus, the  $4+$  state would be expected at 14.8 MeV in this coupling scheme. The predicted energy for this level is not much altered when the effects of configuration mixing are

<sup>29</sup> E. Feenberg and M. Phillips, Phys. Rev. **51**, 597 (1937).

included<sup>30</sup>; choosing  $a/K=6.3$  with  $L/K=6.8$  from Kurath's paper fixes the energy of first excited state correctly and puts the  $4+$  level at 14 MeV, in better agreement with our observations. Virtually every model<sup>31</sup> of the  $C^{12}$  nucleus predicts a  $4+$  level at 13 to 15 MeV about the ground state and in every case it is believed to have a structure similar to the ground state.

This level is no doubt rather strongly excited in the inelastic scattering of medium energy  $\alpha$  particles from  $C^{12}$  but it is difficult to observe because of the background due to  $\alpha$ -particle breakup of  $C^{12}$ . The fact that this level shows up so clearly in heavy-ion scattering but seems not to have been observed in other inelastic scattering experiments indicates that high-energy heavy projectiles may be very useful for uncovering high-lying states that have structure not unlike the ground state.

As has been already indicated the strongest inelastic processes observed were the excitation of the first excited state, both singly and mutually, and the excitation of a  $4+$  level at 14.0 MeV. Both of these states have a high degree of spatial symmetry and this is probably the fundamental reason why they dominate the inelastic scattering. All these final states are members of the same family, that is they are just rearrangements of the  $(p)^8$  configuration with  $S$  and  $T=0$ . Therefore, any proper attempt to calculate their cross section should treat these states as strongly coupled.

The cross section for exciting the 9.63-MeV level over the range of angles studied is smaller than the mutual excitation of first excited state by a factor of at least 2. However, at smaller angles ( $<14^\circ$  lab) the steeper increase in the slope of the envelope for the combined group ( $Q=-8.86$  MeV,  $Q=-9.63$  MeV) indicates that in this region the excitation of the 9.63-MeV level is dominant. It is still, however, a factor of 6 to 10 times smaller than the cross section for the first excited state.

There also seems to be little evidence for the excitation of the 7.66-MeV state in  $C^{12}$  which is somewhat surprising in lieu of Alburger's measurement<sup>32</sup> which showed the  $E2$  decay from the 7.66-MeV state to the first excited state to be enhanced. Thus, this state could be excited by a second-order quadrupole process with the spin coupled to 0 total angular momentum. In fact in  $(e,e')$  inelastic scattering the 7.66-MeV state is clearly observed and with a relatively large cross section. The most recent calculations<sup>33</sup> accounting for the large magnitude of this cross section invoke a vibrational model and consider the scattering interaction to second order in the deformability. Reasonable agreement with experiment is obtained. It is realized, however, with regard to the  $C^{12}$ ,  $C^{12}$  scattering that owing to the 0 spin

<sup>30</sup> D. Kurath, Phys. Rev. **101**, 216 (1956).

<sup>31</sup> A. E. Glassgold and A. Galonsky, Phys. Rev. **103**, 701 (1956); K. Wildermuth and Th. Kanellopoulos, *The Application of the "Cluster" Model to Nuclear Physics* (CERN, Geneva, 1961).

<sup>32</sup> D. Alburger, Phys. Rev. **124**, 193 (1961).

<sup>33</sup> J. D. Walecka, Phys. Rev. **126**, 653, 663 (1962).

of the level, statistical factors will mitigate against its observation.

#### ACKNOWLEDGMENTS

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## Pion Capture in Complex Nuclei\*

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The effect of the finite lifetime of pion-nucleon resonances (so-called "isobars") on intranuclear cascades is considered. The existence of these isobars inside the nucleus must be postulated if the impulse approximation is valid for intranuclear cascades involving pions. These isobars may interact with neighboring nucleons before decaying. One such interaction provides a mechanism for the capture of high-energy pions. Assuming the impulse approximation to be valid for isobar-nucleon interactions, the cross section for the capture of a high-energy pion by two nucleons inside the nucleus is estimated. Since this is a pion-nucleon cross section, an intranuclear cascade calculation must be made before the pion-nucleus capture cross section can be obtained and compared with experiment. However, the same mechanism should also hold in the capture of high-energy pions in deuterium. Here the calculated cross section is in satisfactory agreement with the experiment.

### I. INTRODUCTION

IN the calculation of the interaction of high energy particles with complex nuclei, it is generally assumed that the incoming particle in traversing the nucleus makes collisions with individual nucleons of the nucleus. These collisions are assumed to be essentially two-particle interactions and except for the Pauli principle which forbids certain final states, the cross sections of the process are the same as in free space.<sup>1,2</sup> The trajectory of the particle while entering and leaving the nucleus and also between collisions is, in addition, governed by the interaction between the particle and a real potential which constitutes the interaction of the particle with the nucleus as a whole as compared to interactions with individual nucleons of the nucleus. This potential scattering is always elastic in the sense that the particle does not transfer excitation energy to the nucleus, whereas the particle-nucleon interaction constitutes the inelastic part of the particle-nucleus interaction. Another type of scattering which also must be taken into account in the comparison of cascade particle spectra with experiment is diffraction scattering. This type of scattering is particularly important at very high bombarding energies when potential scattering may be neglected. However, in the above semiclassical

picture, diffraction scattering does not affect the path of the cascade particle within the nucleus.

In the past calculations of this process have been made for nucleons as bombarding particles. These calculations were mostly of the Monte Carlo type.<sup>2-9</sup> Recently,<sup>9</sup> a calculation was made for pions as well as nucleons as bombarding particles. This calculation also took into account the production of pions inside the nucleus due to the inelastic interactions of high energy cascade particles with the nucleons of the nucleus. Pion production and scattering cross sections within the nucleus were again assumed to be identical with the respective free space cross sections except for the restrictions posed by the Pauli principle. Pion capture within the nucleus was calculated assuming it to proceed through the quasideuteron capture process.<sup>10</sup> In this process the pion is captured by two "close" nucleons in the nucleus similar to the absorption of pions in deuterium.<sup>11</sup> The capture cross section in a complex nucleus

<sup>3</sup> G. Bernadini, E. T. Booth, and S. J. Lindenbaum, *Phys. Rev.* **85**, 826 (1952).

<sup>4</sup> H. McManus, W. T. Sharp, and H. Gellman, *Phys. Rev.* **93**, 924 (1954).

<sup>5</sup> J. W. Meadows, *Phys. Rev.* **98**, 744 (1955).

<sup>6</sup> N. S. Ivanova and I. I. Pianov, *Zh. Eksperim. i Teor. Fiz.* **31**, 416 (1956) [translation: *Soviet Phys.—JETP* **4**, 367 (1957)].

<sup>7</sup> J. Combe, *Ann. Phys. (N. Y.)* **3**, 468 (1958).

<sup>8</sup> N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, *Phys. Rev.* **110**, 185 (1958).

<sup>9</sup> N. Metropolis, R. Bivins, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevich, *Phys. Rev.* **110**, 204 (1958).

<sup>10</sup> K. A. Brueckner, R. Serber, and K. M. Watson, *Phys. Rev.* **81**, 575 (1951).

<sup>11</sup> K. A. Brueckner, R. Serber, and K. M. Watson, *Phys. Rev.* **84**, 258 (1951).

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<sup>1</sup> R. Serber, *Phys. Rev.* **72**, 1114 (1947).

<sup>2</sup> M. L. Goldberger, *Phys. Rev.* **74**, 1269 (1948).