of the level, statistical factors will mitigate against its observation.

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Pion Capture in Complex Nuclei*

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The effect of the finite lifetime of pion-nucleon resonances (so-called "isobars") on intranuclear cascades is considered. The existence of these isobars inside the nucleus must be postulated if the impulse approximation is valid for intranuclear cascades involving pions. These isobars may interact with neighboring nucleons before decaying. One such interaction provides a mechanism for the capture of high-energy pions. Assuming the impulse approximation to be valid for isobar-nucleon interactions, the cross section for the capture of a high-energy pion by two nucleons inside the nucleus is estimated. Since this is a pion-*nucleon* cross section, an intranuclear cascade calculation must be made before the pion-*nucleus* capture cross section can be obtained and compared with experiment. However, the same mechanism should also hold in the capture of high-energy pions in deuterium. Here the calculated cross section is in satisfactory agreement with the experiment.

I. INTRODUCTION

IN the calculation of the interaction of high energy particles with complex nuclei, it is generally assumed that the incoming particle in traversing the nucleus makes collisions with individual nucleons of the nucleus. These collisions are assumed to be essentially two-particle interactions and except for the Pauli principle which forbids certain final states, the cross sections of the process are the same as in free space.[1,2] The trajectory of the particle while entering and leaving the nucleus and also between collisions is, in addition, governed by the interaction between the particle and a real potential which constitutes the interaction of the particle with the nucleus as a whole as compared to interactions with individual nucleons of the nucleus. This potential scattering is always elastic in the sense that the particle does not transfer excitation energy to the nucleus, whereas the particle-nucleon interaction constitutes the inelastic part of the particle-nucleus interaction. Another type of scattering which also must be taken into account in the comparison of cascade particle spectra with experiment is diffraction scattering. This type of scattering is particularly important at very high bombarding energies when potential scattering may be neglected. However, in the above semiclassical picture, diffraction scattering does not affect the path of the cascade particle within the nucleus.

In the past calculations of this process have been made for nucleons as bombarding particles. These calculations were mostly of the Monte Carlo type.[2–9] Recently,[9] a calculation was made for pions as well as nucleons as bombarding particles. This calculation also took into account the production of pions inside the nucleus due to the inelastic interactions of high energy cascade particles with the nucleons of the nucleus. Pion production and scattering cross sections within the nucleus were again assumed to be identical with the respective free space cross sections except for the restrictions posed by the Pauli principle. Pion capture within the nucleus was calculated assuming it to proceed through the quasideuteron capture process.[10] In this process the pion is captured by two "close" nucleons in the nucleus similar to the absorption of pions in deuterium.[11] The capture cross section in a complex nucleus

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is obtained from the deuteron capture cross section through an estimate of the probability of finding a nucleon pair in close vicinity in a complex nucleus as compared to this probability for a deuteron. The characteristic distance between the two nucleons is of the order of $\hbar/p$ or less, where p is the momentum transfered to the nucleons by capturing the pion. $p\simeq(M\mu)^{1/2}c$ if the three particles had little relative momentum before the capture. (M and μ are the nucleon and pion masses, respectively.)

While the above model of intranuclear cascades seems to be in essential agreement with experimental results,[9] it is interesting to examine whether besides interactions which are also observed in two-particle collisions in free space, there occur interactions within the complex nucleus which will not occur in other circumstances. This possibility arises from the fact that all unstable particles or "resonances" which decay through strong interactions have a lifetime which is normally too short for the particle to make an interaction before its decay (except for final state interactions in its creation). However, if this particle is produced within the nucleus by an intranuclear cascade, it has, in general, a fair chance to make interactions with nucleons of the complex nucleus before it decays either within or outside the nucleus. This is due to the fact that the time interval between two interactions of a cascade particle in the nucleus is of the same order of magnitude as the lifetime of these particles. Moreover, the lifetime of the unstable particle inside a complex nucleus is increased at *low* kinetic energies of the particle if one of its decay products is a nucleon due to the Pauli principle. For *high* kinetic energies the lifetime of the particle is increased by a factor of $\gamma=E/M$ (E and M are the total energy and mass, respectively, of the unstable particle) due to relativistic time dilation.

Among the short lived particles or resonances which are of particular interest for the investigation of pion interactions inside complex nuclei are the pion-nucleon resonances or "isobars" at approximately 200 MeV (3–3 resonance), 600 and 890 MeV ($T=\frac{1}{2}$ resonances) pion bombarding energy. Assuming these resonances to constitute single levels whose width is given by their lifetime through the uncertainty principle, a mean lifetime in free space of approximately 0.75×10^{-23} sec (in their own barycentric system) is obtained. This lifetime is not short enough to rule out interactions between these particles and the nucleons of the nucleus before the former decay.

II. THE EXISTENCE OF ISOBARS IN COMPLEX NUCLEI

The question which arises in this connection is whether "excited nucleons" or isobars display in a complex nucleus the same properties which are observed in two-particle interactions in free space or whether the high density of nuclear matter affects their characteristics to a large degree. This question is important not only with respect to the interactions between nucleons and isobars but also has direct bearing on the interaction of pions within the nucleus. If these resonances do not have the same characteristics in free space and inside a nucleus, the interaction between the incoming pions and the nucleons of the nucleus must necessarily differ substantially from pion-nucleon interactions in free space and the impulse approximation[12,13] does not hold with respect to the interactions of pions inside a complex nucleus.

The question of the existence of isobars in a complex nucleus is difficult to answer, both experimentally and theoretically. No thorough experimental examination has yet been attempted. (Even if it is found that a substantial number of pions produced in a high energy nuclear reaction leave the nucleus in a resonance state with an accompanying nucleon, this would not constitute a proof for the existence of such resonances in the inner region of the nucleus, since these resonances may have been produced in the outer edge of the nucleus.) The intranuclear cascade calculations which have been carried out in the past assumed the impulse approximation to hold for pions as well as for nucleons but ignored the possible effect of the finite lifetime of the resonances on the process. If the agreement of the calculations with experimental results were sufficiently good, one could conclude that the pion-nucleon resonances are not affected by the high nucleon density inside the nucleus (i.e., the impulse approximation is valid) but that the interactions of these isobars do not affect the cascade interactions to any substantial degree. Unfortunately, the agreement with experimental results is rather poor with respect to outgoing pions.[9] Since the discrepancy may be due to other causes, no conclusions can be drawn at this time with respect to the properties of isobars inside the nucleus.

III. ISOBAR INTERACTIONS INSIDE THE NUCLEUS

One possible way of answering the question of the properties of the pion-nucleon resonances in the nucleus, is to calculate the effect of their interactions with other nucleons on the intranuclear cascade process. Such a calculation would assume that the properties of the isobars are not affected by the high density of nuclear matter. Good agreement between such a calculation and the experimental results for high energy nuclear reactions could then be considered as supporting the view that isobars indeed exist inside complex nuclei with the same poperties as in free space.

The interactions of pion nucleon isobars with other particles are, in general, not known. In their calculation of very high energy nuclear cascades Maor and

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Yekutieli[14] assumed that the isobars have the same cross-sections in nuclear matter as nucleons. The general problem of the interactions of isobars with nucleons will not be discussed here. Instead, we deal with one particular interaction which can be calculated on the basis of existing theory of pion nucleon interactions and which may have a substantial effect on the intranuclear cascade process. This is the interaction in which the isobar, in colliding with a nucleon, decays without the emission of a pion. It is the inverse reaction to the inelastic nucleon-nucleon collision in which the final state consists of a pion-nucleon isobar and the "spectator" nucleon.

IV. PION CAPTURE THROUGH ISOBAR DECAY

The inelastic nucleon-nucleon collision in which a pion is produced in a resonance state with one of the nucleons is well known from experiment.[15] The isobar models of Lindenbaum and Sternheimer[16] and of Mandelstam[17] are based on this experimental fact. Recently, the extrapolation method of Chew and Low[18] has been used to calculate the cross section of this process.[19] Based on the principle of microscopic reversibility the existence of the inverse process, in which an isobar decays in an interaction with a nucleon without the emission of a pion, must be postulated. We conjecture that this process is *the basic mode of high energy pion capture inside a complex nucleus*. This capture mode consists of two consecutive stages. In the first stage a pion-nucleon isobar is formed in an elastic pion-nucleon interaction. In the second stage the isobar decays in an interaction with another nucleon without emitting a pion. Schematically the two stages of the capture process may be written in the following way:

$$\pi + N_1 \rightarrow N_1^*, \tag{1}$$

$$N_1^* + N_2 \rightarrow N_1' + N_2'. \tag{2}$$

Here N and N' denote nucleons and N^* an isobar. $N_1' + N_2'$ may or may not be the same type of nucleon (i.e., proton or neutron) as N_1 and N_2, respectively.

Obviously the isobars in the isobar capture process (2) do not have to originate from an elastic pion-nucleon collision (1) but may be isobars produced in an inelastic nucleon-nucleon collision, the inverse reaction to process (2).

V. CALCULATION OF THE ISOBAR CAPTURE PROCESS

The process of isobar production, whether by an elastic pion-nucleon interaction or by an inelastic nucleon-nucleon process, is relatively well known and will not be discussed here. We will try to calculate the second stage of the capture process, namely, isobar capture through an isobar-nucleon interaction. We calculate this process from its inverse, isobar production in a nucleon-nucleon interaction, with the aid of the principle of detailed balance.[20] It would have been best to use experimental data for this purpose. Unfortunately, there are not sufficient experimental data available on the isobar production process to make this possible. Hence, we calculate the capture process from a theoretical model for the isobar production. This is the "peripheral interaction" or one-pion-exchange (OPE) model. We, therefore, assume that isobar production in a nucleon-nucleon collision is an OPE interaction. The extrapolation method of Chew and Low[18] is used to calculate the cross-section of this process. This method has been tested for isobar production in nucleon-nucleon collisions[19,21] and good agreement was found. The great advantage of using this method is that the reaction

$$N_1 + N_2 \rightarrow N_1^* + N_2' \tag{3}$$

is then completely determined (i.e., the total cross section as well as the differential cross sections and partial cross sections as a function of the "mass" of the isobar N^* are known). As a result the inverse process, namely, the isobar capture reaction (2) is also completely determined. We, hence, tacitly assume that the pion-nucleon isobar produced in an inelastic nucleon-nucleon collision is identical in all respects with the isobar formed in pion-nucleon elastic scattering. This pertains particularly to the polarization of the isobar. (The "elastic" isobar is polarized, as evidenced from its decay properties.) This assumption is consistent with the OPE-model which assumes that the pion-nucleon isobar in the pion production process is created by the resonant scattering of the "virtual" exchange pion against the incoming nucleon, and the scattering matrix element for this process may be taken to a first approximation to be identical with the matrix element for the scattering of a real pion.[19] In this approximation the following expression is obtained for the partial cross section for isobar production (i.e., single pion production) in an inelastic nucleon-nucleon interaction with the exchange neutral pion[18,19] as shown in Fig. 1 (we use units in which $c=\hbar=1$):

$$\frac{\partial^2\sigma}{\partial(\Delta^2)\partial V} = \frac{1}{(p_u U)^2}\frac{f^2 M^2}{\pi\mu^2}\sigma_{\mathrm{res}}(V) p_v V^2 \frac{\Delta^2}{(\Delta^2+\mu^2)^2}; \tag{4}$$

here Δ^2 is the invariant square of the exchange four-momentum, V is the total energy of the pion-nucleon isobar in its own barycentric system, i.e., the "mass" of the isobar, $p_v = |\mathbf{p}_v|$ is the absolute value of the in-

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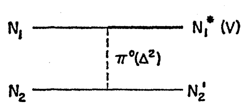

FIG. 1. One-pion-exchange (OPE) diagram for the production of a pion-nucleon isobar in a nucleon-nucleon collision.

ternal momentum of the isobar (i.e., the momentum of the pion and the nucleon in the barycentric system of the isobar, once the isobar decays)

$$p_v=\frac{1}{2V}[(V^2-M^2-\mu^2)^2-4M^2\mu^2]^{1/2}. \quad (5)$$

M and μ are the masses of nucleon and pion, respectively, U is the total energy in the over-all barycentric system, $p_u=|\mathbf{p}_u|$ is the absolute value of the momentum of M_1 and M_2 in the over-all barycentric system (U system):

$$p_u=\frac{1}{2U}(U^2-4M^2)^{1/2}. \quad (6)$$

f^2 is the square of the renormalized pion-nucleon coupling constant: $f^2\approx 0.08$, $\sigma_{\rm res}(V)$ is the total pion-nucleon resonance scattering cross section for total energy V in their barycentric system. Assuming N_1 to be a proton and the exchange pion to be neutral and N_1^* to be a $T=\frac{3}{2}$ isobar, we have $\sigma_{\rm res}(V)=\frac{2}{3}\sigma_{\rm res}(\frac{3}{2},V)$, where $\sigma_{\rm res}(\frac{3}{2},V)$ is the total resonance scattering cross section in the $T=\frac{3}{2}$ state at total barycentric energy V.

In calculating the total cross section for the production of a $T=\frac{3}{2}$ isobar, four diagrams of the type shown in Fig. 1 must be considered. Figure 2 shows the diagrams for p-p interactions and Fig. 3 the diagrams for p-n interactions. The charge state (T_z) of the outgoing isobar is indicated in the figures as well as the relative amplitude associated with each diagram. It is seen from the comparison of Figs. 2 and 3 that the total cross section for isobar production in p-p (or n-n) interactions in twice that of the p-n interactions. This is, of course, due to the fact that the reaction goes solely through the $T=1$ state. It is also seen that the amplitude for the production of an isobar of given charge state consists of the contributions of two diagrams. In calculating the cross section for a given process the amplitudes of the two diagrams as well as the interference term between them must be taken into account.

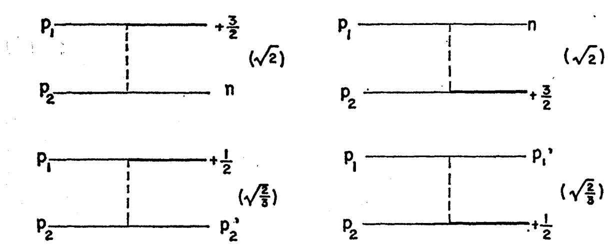

FIG. 2. OPE diagrams for $T=\frac{3}{2}$ isobar production in proton-proton collisions. The charge state (T_z) of the isobar is indicated. The relative amplitude of each diagram is shown in parentheses.

In Eq. (4), there are two independent variables, Δ^2 and V. If we integrate with respect to V we obtain

$$\frac{\partial\sigma}{\partial(\Delta^2)}=\frac{1}{(p_uU)^2}\frac{f^2M^2}{\pi\mu^2}\frac{\Delta^2}{(\Delta^2+\mu^2)^2}\int_{V_1}^{V_2}\sigma_{\rm res}(V)p_vV^2dV. \quad (7)$$

The integration limits V_1 and V_2 are the lower and upper limits (in the total barycentric energy) of the pion-nucleon resonance in question. Thus, if we wish to compute the cross section for the production of the $T=\frac{3}{2}$ isobar for a given invariant square of exchange momentum Δ^2, we may choose the limits: $V_1=1.08$ BeV. $V_2=1.40$ BeV, corresponding to a pion kinetic energy range in the nucleon rest system of 0–430 MeV.

The inverse process to isobar production, namely, isobar decay by pion capture (isobar capture) is described in Fig. 4. *We assume that the matrix element for this process is independent of the isobar mass V.* This assumption is based on the proposition that the isobar is a single excitation level of the nucleon which is broad-

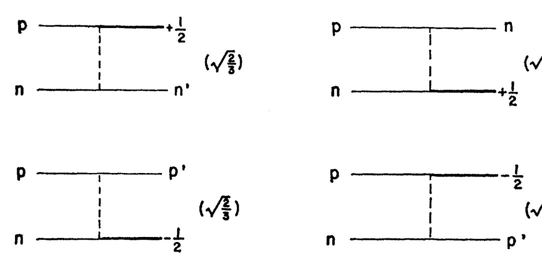

FIG. 3. OPE diagrams for $T=\frac{3}{2}$ isobar production in proton-neutron collisions. The charge state (T_z) of the isobar is indicated. The relative amplitude of each diagram is shown in parentheses.

ened only due to its short lifetime. Equation (7) gives the partial cross section for the *production* of an isobar (of any mass V) for a given Δ^2. The partial cross section for the isobar *capture* reaction (of any mass V) for a given invariant square of the exchange momentum Δ'^2 is thus obtained from Eq. (7) by the principle of detailed balance (see Appendix A). The result is given in Eq. (8):

$$\frac{d\sigma_{\rm capt}}{d(\Delta'^2)}=\frac{1}{(p_u'U)^2}\frac{f^2M^2}{\pi\mu^2}\frac{(\Delta')^2}{[(\Delta')^2+\mu^2]^2}\int_{V_1}^{V_2}\sigma_{\rm res}(V)p_vV^2dV. \quad (8)$$

It is seen that for the same square of the exchange momentum Δ^2 this equation is identical with Eq. (7) except that p_u is replaced by p_u'—the absolute value of the momentum of the incoming isobar and target nucleon in the barycentric system. It should also be noted that although the matrix element for the isobar capture was assumed to be independent of the isobar mass V, this is not so for the cross section [Eq. (8)] since the invariant flux $p_u'U$ is a function of V.[22]

[22] The V-dependence of the cross section [Eq. (8)] is a result of the requirement of momentum and energy conservation in each intranuclear interaction. It involves an inconsistency since we assume the width of the resonance $V(E)$ to be due to its shor

As in the case of isobar production, the total cross section for the capture of an isobar of given charge state T_z by a proton or a neutron consists of the contribution of two diagrams of the type shown in Fig. 4, and an interference term. These diagrams are shown in Fig. 5 for $(T=\frac{3}{2})$ isobar capture reactions which result in two protons (the reactions which result in two neutrons are obtained from Fig. 5 by charge symmetry) and in Fig. 6 for reactions in which the final particles are one proton and one neutron. Also shown are the relative amplitudes associated with each diagram. Comparing Fig. 5 with Fig. 2 we see that the relative amplitudes of the capture reactions differ by a factor of $1/\sqrt{2}$ from those of the production reactions. This is due to the requirement of antisymmetry in the wave function of reactions in which the final state consists of two identical particles.

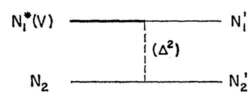

FIG. 4. OPE diagram for the decay of an isobar by pion capture (isobar capture reaction).

The interference terms depend on the phase shifts of the pion-nucleon scattering amplitude. Assuming only the $P_{3/2}$ phase shift to be important, the effect of the interference term is a reduction of the cross section as obtained from the contribution of the two diagrams by 10–20%, the interference term becoming relatively smaller as the kinetic energy of the incoming isobar increases. In view of the approximate nature of the calculations in the subsequent sections we shall neglect the interference terms. In this approximation the total cross section for isobar capture is obtained by integrating Eq. (8) with respect to Δ'^2. The integration yields

$$\sigma_{\text{capt}}(V,E_v)=\frac{1}{(p_u'U)^2}\frac{f^2M^2}{\pi\mu^2}\left(\int_{V_1}^{V_2}\sigma_{\text{res}}(V)p_vV^2dV\right)\times\left[\ln\left(\frac{\Delta_2'^2+\mu^2}{\Delta_1'^2+\mu^2}\right)-\frac{\mu^2(\Delta_2'^2-\Delta_1'^2)}{(\Delta_2'^2+\mu^2)(\Delta_1'^2+\mu^2)}\right], \quad (9)$$

where $\Delta_1'^2$ is the minimum value of the invariant square of the exchange momentum (corresponding to a barycentric scattering angle $\theta_u'=0$) and $\Delta_2'^2$ is the maximum value (corresponding to $\theta_u'=\pi$). In order to obtain the cross section for the capture of a $T=\frac{3}{2}$ isobar of given charge state T_z by a proton or a neutron, the cross section as given by Eq. (9) must be multiplied by the factor associated with the appropriate diagrams, e.g., for the reaction $N^*(+\frac{1}{2})+n\rightarrow p+p$ the factor as obtained from Fig. 5 is: $2\times(\sqrt{\frac{2}{3}})^2=\frac{4}{3}$. In Eq. (9) $\sigma_{\text{res}}(V)$ is the total $T=\frac{3}{2}$ pion-nucleon resonance scattering cross section, and V_1 and V_2 are the limits of the $T=\frac{3}{2}$, $J=\frac{3}{2}$ pion-nucleon resonance.

lifetime, whereas we assign an exact mass V to the isobar. This inconsistency is common to all intranuclear cascade calculations of the Monte Carlo type. Calculations of this type also assign exact values to both the momentum and the position of intranuclear cascade particles although this also violates the uncertainty principle (reference 2). However, these semiclassical approximations are not expected to affect the results of these calculations appreciably.

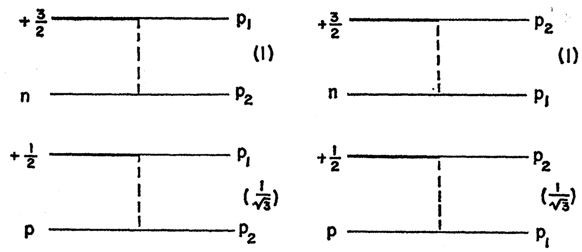

FIG. 5. OPE diagrams for $T=\frac{3}{2}$ isobar capture resulting in two protons in the final state. The charge state T_z of the isobar is indicated. The relative amplitude of each diagram is shown in parentheses.

We have considered so far only the capture of $T=\frac{3}{2}$ isobars. However, Eqs. (8) and (9) are, of course, equally valid for the capture of the higher $(T=\frac{1}{2})$ isobars at 600 and 890 MeV pion bombarding energy. For these isobars $\sigma_{\text{res}}(V)$ is the total $(T=\frac{1}{2})$ resonance cross section of the isobar in question and V_1, V_2 are the lower and upper limits (in the total barycentric energy) of the resonance curve. Despite the much smaller resonance cross section $\sigma_{\text{res}}(V)$ of these higher isobars, the integral

$$I=\int_{V_1}^{V_2}\sigma_{\text{res}}(V)p_vV^2dV \quad (10)$$

is of the same order of magnitude for the three resonances and, hence, the capture cross sections for the three isobars are comparable. The diagrams for the capture of a positive $T=\frac{1}{2}$ isobar $(T_z=+\frac{1}{2})$ are shown in Fig. 7. The diagrams for the capture of negative $T=\frac{1}{2}$ isobars $(T_z=-\frac{1}{2})$ can be inferred through charge symmetry.

The total isobar capture cross section as a function of the isobar kinetic energy in the rest system of the target nucleon is shown in Fig. 8 for a typical capture reaction of a $T=\frac{3}{2}$ isobar: $N_1^*(+\frac{3}{2})+n\rightarrow p+p$. The mass of the N_1^* isobar was taken $V(N_1^*)=1.24$ BeV. The kinetic energy region of Fig. 8 is that of "elastic" isobars produced in the nucleus by a resonance elastic scattering of a pion and one of the nucleons of the nucleus. (Since the nucleons inside the nucleus are *not* at rest, the kinetic energy of the "elastic" isobar is not uniquely determined by its mass V.) Figure 9 shows the total isobar capture cross section as a function of the isobar kinetic energy in the rest system of the target nucleon for typical capture reactions of the three pion-nucleon isobars. For the first $(T=\frac{3}{2}, J=\frac{3}{2})$ isobar the reaction is again: $N_1^*(+\frac{3}{2})+n\rightarrow p+p$, whereas for the second and third isobars ($T=\frac{1}{2}$ isobars) the reactions are: $N^*(+\frac{1}{2})+n\rightarrow p+n$. The masses of the three isobars were chosen $V(N_1^*)=1.24$ BeV, $V(N_2^*)=1.52$ BeV, and $V(N_3^*)$

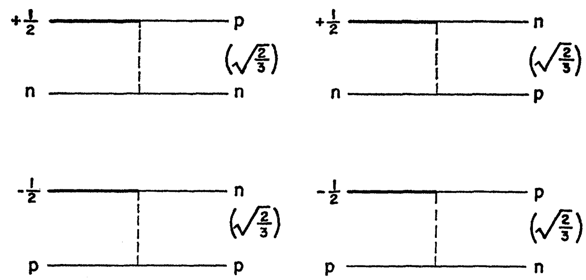

FIG. 6. OPE diagrams for $T=\frac{3}{2}$ isobar capture resulting in one proton and one neutron in the final state. The charge state T_z of the isobar is indicated. The relative amplitude of each diagram is shown in parentheses.

$=1.68$ BeV. The energy range is that of interest for reactions of the higher isobars and of "inelastic" N_1^* isobars (i.e., N_1^* isobars produced in inelastic nucleon-nucleon interactions). The integral $I=\int_{V_1}^{V_2}\sigma_{\text{res}}(V)p_vV^2dV$ in Eq. (9) was assumed to have the following values for the three isobars: $I(N_1^*)=8.74$ mb BeV^4, $I(N_2^*)=6.63$ mb BeV^4, and $I(N_3^*)=11.4$ mb BeV^4. These values were obtained by integrating over the experimentally observed resonance scattering cross sections.[23–25] As already mentioned, the interference terms in the cross section were neglected.

As a first approximation for the isobar capture probability in a complex nucleus, we may calculate the capture probability of an isobar of given mass V in a static nucleon gas of density ρ, in terms of the mean free paths for free decay λ_{decay} and capture λ_{capt}:

$$P_{\text{capt}}=\frac{\lambda_{\text{decay}}}{\lambda_{\text{decay}}+\lambda_{\text{capt}}}. \tag{11}$$

A more refined calculation must take into account the motion of the nucleons in the nucleus and the possibility of isobar-nucleon interactions other than isobar capture. However, if we assume the cross section for the other isobar-nucleon interactions to be similar to nucleon-nucleon cross sections, the probability of these interactions is small compared to that of free decay for isobar kinetic energies below 1 BeV. In the rest system of the nucleons

$$\lambda_{\text{capt}}=[\sigma_{\text{capt}}(E_v)\rho]^{-1}, \tag{12}$$

$$\lambda_{\text{decay}}=\beta\gamma\tau, \tag{13}$$

where β is the velocity of the isobar, $\gamma=(1-\beta^2)^{-1/2}$ and τ is the mean life of the isobar. Equations (12) and (13) do not include the effect of the Pauli principle. This effect will be discussed below. For the isobar N_1^* we will assume $\tau=0.73\times10^{-23}$ sec corresponding to a resonance width of $\Gamma=90$ MeV.[26] Figure 10 shows the capture probability P_{capt} as a function of the isobar kinetic energy for an isobar N_1^* of mass $V=1.24$ BeV in a static nucleon gas of density $\rho=1.087\times10^{38}$ cm^{-3} (corresponding to a nucleus of constant density and of radius $r=1.30\times A^{1/3}$ F). The nucleon gas is assumed to consist of an equal number of protons and neutrons. The capture probability will thus be the same for all charge states of N_1^*. This is, of course, again a result of the fact that the isobar capture process can proceed only in the $T=1$ state and, hence, a single cross section governs the reaction for all possible charge states. Figure 10 shows the capture probability P_{capt} to be a slowly decreasing function of the isobar kinetic energy, amounting to approximately 0.4 at the limit $E_v=0$. Similar graphs are obtained for the higher isobars N_2^* and N_3^*.

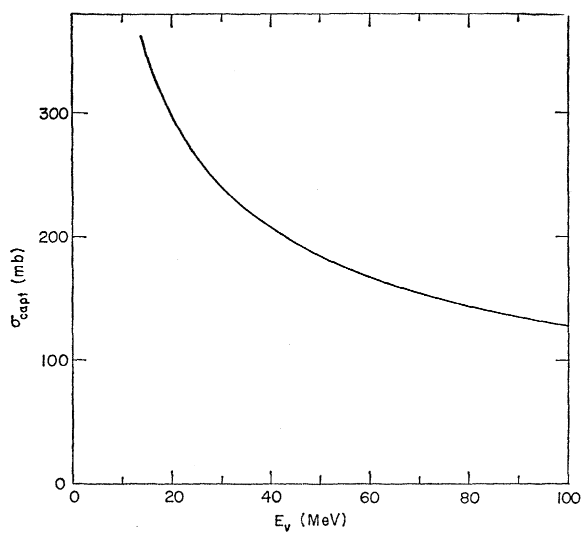

FIG. 8. Total cross section for the reaction $N_1^*(+\frac{3}{2})+n\rightarrow p+p$ as a function of the isobar kinetic energy in the rest system of the target neutron. N_1^* is the $J=\frac{3}{2}$, $T=\frac{3}{2}$ pion-nucleon isobar of mass $V_1=1.24$ BeV. The kinetic energy region is that of "elastic" isobar capture. The interference term in the expression for the cross section has been neglected.

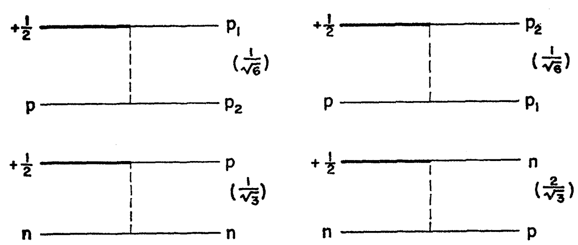

FIG. 7. OPE diagram for the capture of positive $T=\frac{1}{2}$ isobars ($T_z=+\frac{1}{2}$). The relative amplitude of each diagram is shown in parentheses.

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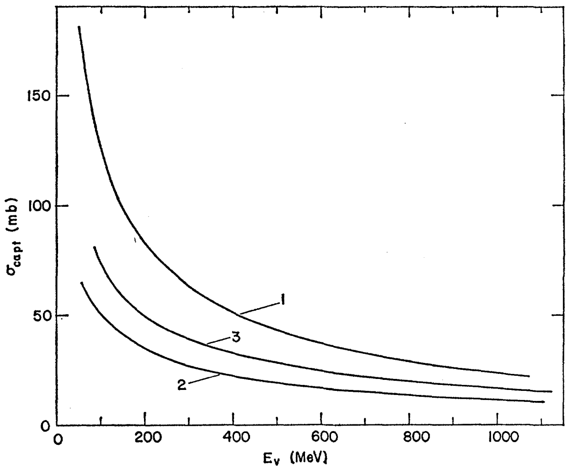

FIG. 9. Total isobar capture cross section as a function of the isobar kinetic energy in the rest system of the target nucleon, for the following reactions: (1) $N_1^*(+\frac{3}{2})+n \rightarrow p+p$ (kinetic energy region of "inelastic" isobar capture); (2) $N_2^*(+\frac{1}{2})+n \rightarrow p+n$; (3) $N_3^*(+\frac{1}{2})+n \rightarrow p+n$. N_1^* is the first ($J=\frac{3}{2}$, $T=\frac{3}{2}$) pion-nucleon isobar of mass $V(N_1^*)=1.24$ BeV; N_2^* is the second ($T=\frac{1}{2}$) isobar of mass $V(N_2^*)=1.52$ BeV; and N_3^* is the third (also $T=\frac{1}{2}$) isobar of mass $V(N_3^*)=1.68$ BeV. The interference terms in the expressions for the cross section have been neglected.

In computing P_{capt} we have neglected the effect of the Pauli principle which forbids final nucleon states of momentum smaller than the Fermi momentum ($p_F \approx 0.23$ BeV/c for a nucleon density of $\rho=10^{38}$ cm^{-3}). Although the Pauli principle affects both λ_{decay} and λ_{capt}, the actual effect on λ_{capt} is negligible. The effect on λ_{decay} is an increase by approximately 10% of λ_{decay} for an "elastic" isobar, produced by the elastic interaction of a 200 MeV pion with a stationary nucleon ($V \approx 1.24$ MeV, $E_v \approx 37$ MeV). The resulting increase in P_{capt} is approximately 5%. The effect decreases for higher isobar kinetic energies and, for $E_v > 80$ MeV, λ_{decay} is not affected by the Pauli principle. However, for light isobars ($V<1.25$ BeV) the importance of the Pauli principle increases as the isobar kinetic energy E_v decreases, λ_{decay} becoming infinite at very small energies. This, however, does not imply that the capture probability becomes unity at these energies since isobar-nucleon scattering reactions, which have been neglected here, will compete with the capture process.

We may now estimate *the cross section for a pion of energy E to form an isobar* (through an elastic pion-nucleon interaction) *inside a complex nucleus and this isobar subsequently being captured by a second nucleon of that nucleus*. We will again substitute a static nucleon gas of constant density and equal number of protons and neutrons for the complex nucleus and we restrict ourselves to pion kinetic energies below 500 MeV. In this energy region only N_1^* formation is possible and the isobar kinetic energy is less than 130 MeV. Thus we find from Fig. 10 that for the pion kinetic energy range of interest $P_{capt} \approx 0.4$. Hence, the pion capture cross section as defined above is equal to approximately $0.4 \times \frac{2}{3}\sigma_{el} \approx \frac{1}{4}\sigma_{el}$ where σ_{el} is the total $T=\frac{3}{2}$ pion nucleon cross section at the pion bombarding energy E. ($\frac{2}{3}\sigma_{el}$ is the average pion-nucleon cross section in a nucleon gas of equal number of protons and neutrons assuming the $T=\frac{1}{2}$ cross section to be negligible.)

Similarly we find that for pions of kinetic energy in the range 500–1000 MeV (i.e., the range of N_2^* and N_3^*) the capture cross section is approximately $0.15 \times \frac{1}{3}\sigma_{res}$ where σ_{res} is the total $T=\frac{1}{2}$ pion-nucleon resonant cross section.[24] This value again pertains to the static nucleon gas and is based on the assumption that N_2^* and N_3^* have the same mean life τ as the N_1^* isobar. Thus we find that the pion capture cross section above 500 MeV is small compared to the total pion-nucleon cross section.

It must be emphasized that the pion capture cross section which we discussed here is *not* the pion capture cross section by a complex nucleus as it is normally defined. It is a pion-*nucleon* rather than a pion-*nucleus* cross section. In order to evaluate the pion-*nucleus* cross section for our model we must take into account the probability that the incoming pion may make several interactions within the nucleus before being captured. Moreover the momentum distribution of the nucleons in the nucleus and the effect of the Pauli principle must also be taken into account. An exact calculation of the pion absorption cross section is most readily performed by a Monte Carlo technique. Such a calculation is in progress.

VI. PION CAPTURE BY DEUTERONS

The mechanism for pion capture which has been described above is not restricted to complex nuclei and it should apply to high energy pion capture in deuterons as well. Since the pion-capture cross section of deuterons has been measured directly[27–32] and also can be inferred from the $N+N \rightarrow \pi+d$ reaction by the principle of detailed balance,[33] this cross section provides a convenient test for the proposed mechanism of pion capture. An additional advantage of using the deuteron capture process lies in the fact that no Monte Carlo calculation is necessary for the comparison of the calculated cross section with the experimental results.

While the same basic mechanism should hold for pion capture in complex nuclei and in the deuteron, our method of calculating the pion capture cross section in

[27] R. Durbin, H. Low, and J. Steinberger, Phys. Rev. **84**, 581 (1951).
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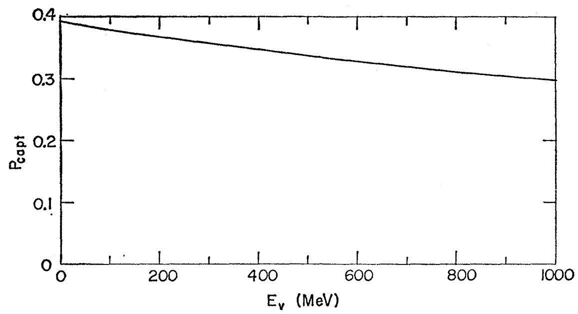

FIG. 10. The capture probability P_{capt} of an isobar N_1^* of mass $V=1.24$ BeV and kinetic energy E_v in a static nucleon gas of density $\rho=1.087\times10^{38}$ cm^{-3} consisting of an equal number of protons and neutrons. The effect of the Pauli principle has been neglected.

complex nuclei, i.e., Eq. (9) based on the OPE model is not particularly suitable for the calculation of the deuteron capture cross section since it is based on some approximations which become rather poor for deuteron capture. This is, in particular, true for the angular distribution which in the deuteron capture is determined by the very few angular momentum states available for deuteron-pion system. However, since the deuteron is a loosely coupled system the OPE approximation should give a reasonably good estimate of the *total* pion-capture cross section particularly at energies above ~100 MeV where the impulse approximation is certainly valid. It should, nevertheless, be clear that our purpose in calculating the pion-capture cross section for deuterons is to test our method rather than to present an accurate calculation for this particular reaction. Calculations which deal specifically with the deuteron capture reaction or its inverse, the $N+N\rightarrow\pi+d$ reaction, based on similar ideas have been published by several authors.[17,34,35]

The experimental deuteron capture cross section is dominated by a peak resembling the (3-3) resonance in pion-nucleon scattering except that the peak position is shifted to a somewhat lower pion energy. The (3-3) resonance peak will be clearly reproduced by our mechanism since it assumes that the first stage of the capture process is a pion nucleon scattering (isobar production) reaction. The shift to lower energies may be due to the onset of competing reactions such as isobar-nucleon scattering processes. The latter processes will again be neglected in this calculation.

We assume the average "nucleon density" of the second nucleon with respect to the pion nucleon isobar to have the form

$$\rho=\frac{3}{4\pi r_1^3}\int_0^{x_1}u^2(x)dx;\ x=\frac{r}{r_0};\ r_0=4.315\times10^{-13}\text{ cm.}\quad(14)$$

[34] S. Matsuyawa and H. Miyazawa, Progr. Theoret. Phys. (Kyoto) **9**, 492 (1953).
[35] N. Austern, Phys. Rev. **100**, 1522 (1955).

$u(r)/r$ is the radial part of the S-wave deuteron wavefunction (we neglect the D-wave part of the wavefunction) normalized to

$$\int_0^\infty u^2(x)dx=1.\quad(15)$$

We may approximate $u(x)$ as calculated by Hulthén and Sugawara[36] in the region $0\leq x\leq 0.3$ corresponding to $0\leq r\leq \lambda\!\!\!^{-}_\pi$ ($\lambda\!\!\!^{-}_\pi$-pion wavelength$=1.4\times10^{-13}$ cm) by the expression

$$u(x)\approx 2.4x.\quad(16)$$

Equation (14) was chosen for ρ so as to make it fairly insensitive to the detailed shape of the wavefunction near the origin which is not known very well. It is readily seen that within the range for which the linear expression [Eq. (16)] is valid, ρ will be independent of the choice of the upper integration limit of Eq. (14), namely, r_1.

As in the case of pion capture in complex nuclei, the pion capture cross section for deuterons is assumed to have the form

$$\sigma(\pi+d\rightarrow N+N)=\sigma_{\pi d}(E_\pi)\frac{\lambda_{decay}}{\lambda_{decay}+\lambda_{capt}},\quad(17)$$

where $\sigma_{\pi d}$ is the total pion-deuteron cross section and $\lambda_{decay}/(\lambda_{decay}+\lambda_{capt})$ is the probability that the isobar which was produced by the interaction of the pion with the first nucleon, will be captured by the second nucleon. Assuming the $T=\frac{3}{2}$ state to dominate in the energy range of interest we have $\sigma_{\pi d}=\frac{4}{3}(1-\Delta)\sigma_{\pi N}(T=\frac{3}{2})$. The factor Δ includes the shadow effect in the deuteron[35] and the interference effect in the scattering amplitudes of the two nucleons. For pion-deuteron scattering in the energy range of interest $\Delta\approx0.15$.[23] We have for the mean free path for the capture of the isobar (see Appendix B)

$$\lambda_{capt}=[\tfrac{4}{3}\rho\sigma_{capt}(V,E_v)]^{-1},\quad(18)$$

where ρ is given by Eq. (14) and $\sigma_{capt}(V,E_v)$ is the capture cross section of a $(T_z=+\frac{3}{2})$ isobar of mass V and energy E_v by a neutron.

The cross section for the reaction $\pi+d\rightarrow N+N$ as obtained from Eq. (17) is shown in Fig. 11. In calculating the capture cross section $\sigma_{capt}(V,E_v)$ we assumed that the destructive interference between the two amplitudes of the isobar capture process reduces the isobar capture cross section as obtained from the two diagrams alone by 20%. We also assumed the factor Δ in the expression for $\sigma_{\pi d}$ to have the value $\Delta=0.15$. The same cross section is obtained for the reaction $\pi+d\rightarrow N+N$ for all charge states of the pion. Also shown in Fig. 11 are the experimental values obtained for this cross section. At the lowest pion energies the impulse approximation is not valid and hence our calculation is not

[36] L. Hulthén and M. Sugawara, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 1.

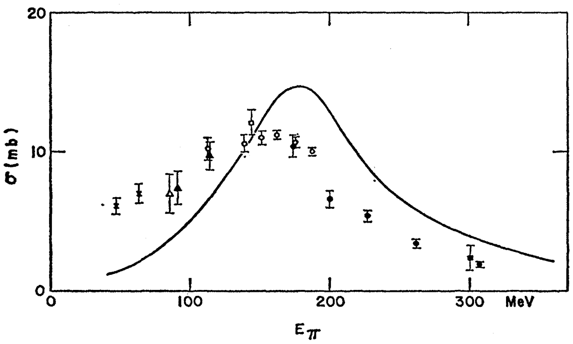

FIG. 11. The cross section of the reaction $\pi^+ + d \rightarrow p + p$ as a function of the pion laboratory kinetic energy. The calculated values are shown by the solid curve. The experimental values were taken from: ×-reference 27, ▲-reference 28, △-reference 29, □-reference 30, ●-reference 31, ■-reference 32, ○-reference 33. The experimental values of reference 33 refer to the inverse reaction $(p+p \rightarrow \pi^+ + d)$. The corresponding values shown here were obtained by using the principle of detailed balance.

applicable there. The somewhat higher calculated values for the higher pion energies may be due to our neglect of the competing reactions to the isobar capture reaction. However, in view of the uncertainty of the deuteron wavefunction near the origin and the resultant uncertainty of the calculated values the importance of competing reactions cannot be ascertained from Fig. 11.

VII. CONCLUSIONS

The agreement between the calculated values for the pion capture cross section in deuterium and the experimental values is considered satisfactory in view of the approximations which were used. Most of these approximations are not encountered in the calculation of the capture of high energy pions in complex nuclei in which the interference terms in the cross section, the internal motion of the nucleons and the effect of the Pauli principle are taken into account in a rigorous fashion. It is, hence, to be expected that such a calculation should give good agreement with experiment. Unfortunately, this cross section cannot be measured directly in most cases and, in general, the comparison between theory and experiment can only be carried out by comparing the results of a high energy nuclear reaction with a complete calculation of the fast cascade process. In order to carry out such a calculation the other isobar-nucleon interactions must be known. One of the difficulties in the calculation of the other processes, which are mainly isobar-nucleon scattering interactions, is the fact that unlike the isobar capture reaction the inverse process for these reactions is, in general, also unknown.

It has already been mentioned that our considerations of the finite lifetime of the multiparticle resonant states in complex nuclei are not restricted to pion-nucleon isobars. The significance of the latter is due to their large cross section and the importance of the pion-nucleon interactions in all high energy nuclear reactions. On the other hand, the relatively short lifetime of the pion nucleon isobars limits the number of interactions in which the isobars participate before decaying, and and approximately one half of the pion-nucleon isobars produced in complex nuclei decay before making even one such interaction. This is not true for some of the other resonances such as the ω meson or the Y_0^{**} (the width of these particles is $\Gamma \leq 15$ MeV), whereas the mean life of the η meson is probably very much larger than its transit time in the nucleus. Thus there seems to be no doubt that these particles interact many times with the nucleons of the nucleus before decaying or escaping from the nucleus. Some of the interactions of these particles inside a complex nucleus will be discussed elsewhere.

ACKNOWLEDGMENTS

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APPENDIX A. THE PRINCIPLE OF DETAILED BALANCE

The principle of detailed balance yields the following relation between the differential cross sections of inverse reactions (Fig. 12) evaluated in the barycentric system[37]

$$|\mathbf{p}|^2 g\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{I}\rightarrow\mathrm{II}} = |\mathbf{p}'|^2 g'\left(\frac{d\sigma'}{d\Omega'}\right)_{\mathrm{II}\rightarrow\mathrm{I}}, \qquad \text{(A1)}$$

where $|\mathbf{p}|$ and $|\mathbf{p}'|$ are the absolute values of the momenta of the particles in states (I) and (II), respectively, g and g' the total spin statistical factors of the particles in the respective states and $(d\sigma/d\Omega)_{i\rightarrow j}$ the differential cross section for the reaction $i \rightarrow j$. We have in the barycentric system

$$d\Omega = 2\pi d(\cos\theta), \qquad \text{(A2)}$$

where θ is the scattering angle in the barycentric system. The invariant square of the exchange momentum Δ^2 is given by

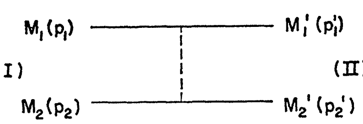
$$\Delta^2 = (p_1 - p_1')^2 = -(M_1^2 + M_1'^2) + 2(E_1 E_1' - \mathbf{p}_1 \cdot \mathbf{p}_1'), \qquad \text{(A3)}$$

or when evaluated in the barycentric system

$$\Delta^2 = (p - p')^2 = -(M_1^2 + M_1'^2) + 2(E_1 E_1' - \mathbf{p} \cdot \mathbf{p}'), \qquad \text{(A4)}$$

where M_1 and M_1' are the masses of the particles connected by a common vertex (see Fig. 12) and E_1 and E_1'

FIG. 12. General one-pion-exchange diagram.



[37] See, for example, W. S. C. Williams, *An Introduction to Elementary Particles* (Academic Press Inc., New York, 1961), p. 89.

their respective total energies. A similar expression can be written for Δ^2 when evaluated for the other vertex. For a given total energy $U=E_1+E_2=E_1'+E_2'$ and given masses M_1, M_2, M_1', and M_2' we have

$$d(\Delta^2)=-|\mathbf{p}|\cdot|\mathbf{p}'|d(\cos\theta). \tag{A5}$$

Similarly, we have for the inverse reaction

$$d\Omega'=2\pi d(\cos\theta'), \tag{A2'}$$

$$d(\Delta'^2)=-|\mathbf{p}|\cdot|\mathbf{p}'|d(\cos\theta'). \tag{A5'}$$

Assuming M_1, M_2, and M_2' to be nucleons and M_1' to be a $(T=\frac{3}{2}, J=\frac{3}{2})$ isobar we have $g=4$ whereas g' depends on our assumptions with respect to the decay properties of the isobar. If we assume the isobar to decay isotropically in its own barycentric system, then the spin statistical factor for M_1' is $(2J+1)=4$ and $g'=8$. However, it is well known that the "elastic" isobar (the isobar produced by an elastic pion-nucleon interaction) is polarized and we assume the "inelastic" isobar (i.e., the isobar created in inelastic nucleon-nucleon collisions) to be identical in all respects with "elastic" isobars of the same mass. Hence, we assume the spin statistical factor of M_1' to be 2 and as a result $g'=g=4$.

Denoting $|\mathbf{p}|=p_u$ and $|\mathbf{p}'|=p_u'$ we have

$$\left(\frac{d\sigma'}{d(\Delta'^2)}\right)_{\mathrm{II}\to\mathrm{I}}=\frac{p_u^2}{p_u'^2}\left(\frac{d\sigma}{d(\Delta^2)}\right)_{\mathrm{I}\to\mathrm{II}}. \tag{A6}$$

Equation (8) follows directly from Eqs. (7) and (A6).

APPENDIX B. PION CAPTURE CROSS SECTION IN DEUTERIUM

The capture of a positive pion by a deuteron may proceed through two channels: (1) The pion interacts first with the proton thereby producing a $(\frac{3}{2}, +\frac{3}{2})$ isobar (which is subsequently "captured" by the neutron), or (2) the pion interacts first with the neutron thereby producing a $(\frac{3}{2}, +\frac{1}{2})$ isobar (which is subsequently captured by the proton).

The normalized isobar wave function is given by

$$\psi=\frac{\sqrt{3}}{2}\psi(T_z=+\tfrac{3}{2})+\tfrac{1}{2}\psi(T_z=+\tfrac{1}{2}). \tag{B1}$$

For the capture of the isobar by the second nucleon we have

$$\langle p,p|(\tfrac{3}{2},+\tfrac{1}{2}),p\rangle=+\frac{1}{\sqrt{3}}\langle p,p|(\tfrac{3}{2},+\tfrac{3}{2}),n\rangle. \tag{B2}$$

The amplitudes (1) and (2) add coherently. The total amplitude is given by

$$A=A(1)+A(2)=\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\frac{1}{\sqrt{3}}\right)\langle p,p|(\tfrac{3}{2},+\tfrac{3}{2}),n\rangle, \tag{B3}$$

and the total cross section for the isobar capture by the second nucleon is

$$\sigma_{\mathrm{capt}}=\tfrac{4}{3}\sigma(V,E_v) \tag{B4}$$

where $\sigma(V,E_v)$ is the cross section for the capture of a $(\frac{3}{2}, +\frac{3}{2})$ isobar of mass V and kinetic energy E_v by a stationary neutron. The mean free path for isobar capture is, thus, in our approximation

$$\lambda_{\mathrm{capt}}=[\tfrac{4}{3}\rho\sigma(V,E_v)]^{-1}.$$

It is readily verified that the same expression is obtained for the other charge states of the pion.