Internal Pairs from Aligned Nuclei*

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The problem of pair emission from light nuclei where the emitting state is oriented is examined. More specifically, as long as the polarization of neither member of the pair is observed, only nuclear alignment plays a role. Detailed plane wave calculations presented for the pure magnetic multipole show that if the dihedral angle between the planes of the alignment axis and recoil momentum and the plane of the pair is summed over, the distribution of pairs is the product of the unaligned distribution and the angular distribution of spin-one particles (photons). The ratio of pairs with fixed angle between the two members and fixed angle between the alignment and recoil directions to the number of photons with the latter angle also fixed is independent of alignment. For electric transitions the corresponding pair distribution is an incoherent sum of transverse and longitudinal parts. The first part, with nuclear orientation, is again multiplied by the spin-one angular distribution function but the longitudinal part is multiplied by a spin-zero angular distribution function. Hence, alignment will influence the ratio of pair and photon angular distributions because of the presence of the longitudinal field in the electron-nucleus interaction.

I. INTRODUCTION

THE measurement of the branching ratio of internal pairs to γ rays in radiative transitions has become a useful tool in nuclear spectroscopy. This branching ratio, sometimes referred to as the internal pair conversion coefficient, has been discussed for emission from unoriented nuclei several years ago.[1] Somewhat later the extension to the case of oriented nuclei was considered by Goldring.[2]

For axially symmetric alignment, which is the case of greatest interest, the transition probability for pair emission, N_π, depends on three angles and the positron (or electron) energy, if the measurements are made with energy discrimination. The angles are: Θ, the angle between electron and positron momenta, $\mathbf{p}_+$ and $\mathbf{p}_-$; θ the angle between the alignment axis $\hat{n}$ and the recoil direction[3] $\mathbf{k}=\mathbf{p}_++\mathbf{p}_-$ and δ the dihedral angle between the $\mathbf{p}_+$, $\mathbf{p}_-$ plane and the $\mathbf{k}$, $\hat{n}$ plane. In addition, the angular momentum transfer L, the energy transfer k_0 and the nuclear parity change enter parametrically.[4] For our considerations only pure multipole transitions need be discussed as will be clear from the remarks made below. In any event, whether or not the observations include a summation over energy, the distribution of N_π in the three angles will depend on the statistical tensors which describe the orientation.[5] When the orientation is produced by low-temperature techniques these statistical tensors are (usually) unknown parameters which depend on coupling constants in the spin Hamiltonian. In the more practical case that the orientation is produced by defining the direction of a radiation feeding the pair-emitting state, in which case $\hat{n}$ is just this direction, the statistical tensors depend on the dynamics of the reaction leading to this state and are essentially unknown from a theoretical standpoint. Of course, these tensors can be determined by measuring the distribution in θ of the emitted γ rays. But the central question is: How do these tensors enter into the pair distribution function?

The purpose of this paper is to provide the answer to this question for the case in which the measurements sum over the dihedral angle δ. It is not difficult to provide an answer for the case in which all three angles are present but, if the summation cited above is carried out, there emerges a particularly simple result which provides for a clear insight into the physical aspects of the problem. It is our intention to present results for the more detailed problem in a subsequent publication. It should be recognized that in practice summation over δ is equivalent to summing over all directions of alignment around a cone with half-angle θ and with $\mathbf{k}$ as axis. This is certainly a feasible operation if $\hat{n}$ is defined either by a crystal axis in the target or by the direction of an emitted radiation, e.g., a β-particle transition feeding the pair-emitting state.

II. MAGNETIC MULTIPOLE CALCULATION

In this section we present the details of the pure magnetic multipole case as an illustration of the method used. We also present the results for electric transitions. For the situation envisaged there is no interference term in mixed magnetic-electric transitions.

The notation used below is as follows: The number of pairs emitted per unit time in the range $dW_+ \sin\Theta d\Theta \times \sin\theta d\theta d\delta$ is

$$N_\pi(\Theta,\theta,\delta,W_+)dW_+ \sin\Theta d\Theta \sin\theta d\theta d\delta.$$

The number per unit time, after summation over δ, per unit range of the remaining variables, is

$$N_\pi(\Theta,\theta,W_+)=\int_0^{2\pi} d\delta\, N(\Theta,\theta,\delta,W_+). \quad (1a)$$

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[1] M. E. Rose, Phys. Rev. **76**, 678 (1949); **78**, 184 (1950).
[2] G. Goldring, Proc. Phys. Soc. (London) **A66**, 341 (1953).
[3] For alignment there is no need to distinguish between $\hat{n}$ and $-\hat{n}$ or between $\hat{k}$ and $-\hat{k}$.
[4] In the paper cited in Ref. 1 the notation used was q for k, k for k_0 and l for L. In all our considerations, we take $m=c=\hbar=1$.
[5] See, for example, M. E. Rose, Phys. Rev. **108**, 362 (1957).

One can then define, in a corresponding way,

$$N_\pi(\Theta,W_+)=\int_0^\pi \sin\theta d\theta N_\pi(\Theta,\theta,W_+) \quad \text{(1b)}$$

and

$$N_\pi(W_+)=\int_0^\pi \sin\Theta d\Theta N_\pi(\Theta,W_+). \quad \text{(1c)}$$

Finally, the total pair rate is

$$N_\pi=\int_1^{k_0-1} dW_+ N_\pi(W_+). \quad \text{(1d)}$$

Experimentally, it is possible to observe the distributions integrated over W_+ but not over the angles.

For the competing γ ray the number of photons emitted per unit time in the range $\sin\theta d\theta$ is

$$N_\gamma(\theta)\sin\theta d\theta,$$

and the total number of photons per unit time is

$$N_\gamma=\int_0^\pi \sin\theta d\theta N_\gamma(\theta). \quad \text{(2a)}$$

The total pair formation coefficient is

$$\Gamma_L=N_\pi/N_\gamma, \quad \text{(2b)}$$

but one can define partial pair formation coefficients as follows[6]:

$$\gamma_L(\Theta,W_+)=N_\pi(\Theta,W_+)/N_\gamma \quad \text{(3a)}$$

and

$$\Gamma_L(W_+)=N_\pi(W_+)/N_\gamma. \quad \text{(3b)}$$

We shall also be interested in the branching ratio wherein both pairs and γ rays are observed with $\mathbf{k}$ in the direction making an angle θ with $\hat{n}$. Here $\mathbf{k}$ is also used as the photon propagation vector. This branching ratio is denoted by

$$\gamma_L(\Theta,\theta,W_+)=N_\pi(\Theta,\theta,W_+)/N_\gamma(\theta). \quad \text{(3c)}$$

The starting point of the calculation[7] is

$$N_\pi(\Theta,\theta,\delta,W_+)=(2\pi)^{-4}\alpha p_+p_-W_+W_-S|\mathfrak{M}|^2. \quad \text{(4)}$$

Note that an integration over an irrelevant azimuth angle has been performed. Here $p_\pm$ are the magnitudes of the positron (+) and electron (−) momenta and $W_\pm$ give the corresponding energies including rest energy. The matrix element $\mathfrak{M}$ is given by

$$\mathfrak{M}=-\int\int J_\mu(\mathbf{r}')\frac{e^{ik_0R}}{R}j_\mu(\mathbf{r})d\mathbf{r}'d\mathbf{r}, \quad \text{(5)}$$

[6] In Ref. 1, $\gamma_L(\Theta,W_+)$ was denoted by $\gamma_l(\Theta)$.

[7] Our procedure here is different in form from but equivalent to that used in Ref. 1.

where $R=|\mathbf{r}-\mathbf{r}'|$, J_μ is the 4-vector which describes the dynamic nuclear four-current density and

$$j_\mu=\bar{\psi}_f\gamma_\mu\psi_i$$

is the corresponding quantity for the electrons. For these particles plane waves are used as usual. In (4) S comprises: summation over electron and positron spin directions, summation over magnetic quantum numbers of the final nuclear state and averaging over the initial nuclear state with appropriate elements of the density matrix describing that state. Specifically, the last operation is carried out by applying the operation

$$\sum_{m_i} p_{m_i}\equiv\sum_{m_i}\sum_\nu \alpha_\nu(2\nu+1)^{1/2}C(j_i\nu j_i;m_i0). \quad \text{(6)}$$

Here, j_i and m_i refer to the angular momentum and its projection on the orientation axis in the initial nuclear state while ν, which gives the tensor rank of the orientation, runs from 0 to $2j_i$. The normalization is such that

$$\sum_{m_i} p_{m_i}=1,$$

which implies that

$$\alpha_0=(2j_i+1)^{-1}. \quad \text{(7)}$$

The nuclear alignment is given by

$$\frac{\mathrm{Tr}[3j_z^2-j_i(j_i+1)]D_i}{j_i(2j_i-1)}=\frac{2j_i+1}{\sqrt{5}}\left[\frac{(j_i+1)(2j_i+3)}{j_i(2j_i-1)}\right]^{1/2}\alpha_2, \quad \text{(8)}$$

where D_i, the initial state density matrix has diagonal elements p_{m_i} so that $\mathrm{Tr}D_i=1$. In general, the statistical tensor α_ν are related to the G_ν discussed elsewhere[5] by

$$\alpha_\nu=(2j_i+1)^{-1/2}G_\nu.$$

In Eq. (5) we write

$$J_\mu(e^{ik_0R}/R)j_\mu=\mathbf{J}_N\cdot\mathbf{G}\cdot\mathbf{j}-\rho_N G_0\rho_e, \quad \text{(9a)}$$

where $J_N, i\rho_N\equiv J_\mu$; $\mathbf{j}, i\rho_e\equiv j_\mu$. Also, $\mathbf{G}=\mathbf{I}[\exp(ik_0R)]/R$, $\mathbf{I}$ being the unit dyadic, and $G_0=[\exp(ik_0R)]/R$ are the dyadic and scalar Green's functions. We expand $\mathbf{G}$ and G_0 in the usual way into multipole fields and assume[8] $r\geqslant r'$. Then

$$G_0=4\pi ik_0\sum_{LM} j_L(k_0r')h_L(k_0r)Y_L^{M*}(\hat{r}')Y_L^M(\hat{r}) \quad \text{(9b)}$$

[8] One does not need to make this assumption since with plane waves the integration over the electron coordinates is easily carried out giving the Møller potentials. However, in calculating N_π/N_γ the nuclear matrix elements will then be slightly different. Our assumption which corresponds to saying that $k_0R_N\ll1$, where R_N is the nuclear radius, makes the matrix elements the same. We note that in all cases $k<k_0$. Obviously, in cases of practical interest, the condition $k_0R_N\ll1$ is extremely well fulfilled. The expansions of G_0 and $\mathbf{G}$ are given in M. E. Rose, *Multipole Fields* (John Wiley & Sons, Inc., New York, 1955). A different definition of $\mathbf{T}_{LL}^M$ and the radial functions was used there.

and

$$\mathbf{G}=4\pi i k_0 \sum_{LM\tau} \mathbf{A}_L{}^{M*}(\mathbf{r}';\tau)\mathbf{B}_L{}^M(\mathbf{r};\tau)\,, \qquad (9c)$$

with $\tau=e,\ m,\ l$ meaning transverse electric, magnetic, and longitudinal, respectively. The three fields for the standing waves are

$$\mathbf{A}_L{}^M(m)=j_L\mathbf{T}_{LL}{}^M, \qquad (10a)$$

$$\mathbf{A}_L{}^M(e)=\left(\frac{L+1}{2L+1}\right)^{1/2} j_{L-1}\mathbf{T}_{LL-1}{}^M - \left(\frac{L}{2L+1}\right)^{1/2} j_{L+1}\mathbf{T}_{LL+1}{}^M, \qquad (10b)$$

$$\mathbf{A}_L{}^M(l)=\left(\frac{L}{2L+1}\right)^{1/2} j_{L-1}\mathbf{T}_{LL-1}{}^M + \left(\frac{L+1}{2L+1}\right)^{1/2} j_{L+1}\mathbf{T}_{LL+1}{}^M, \qquad (10c)$$

$$=\nabla j_L Y_L{}^M/k_0\,,$$

with the spin-one angular functions

$$\mathbf{T}_{L\lambda}{}^M(\hat{r})=\sum_\mu C(\lambda 1 L;\, M-\mu,\, \mu) Y_\lambda{}^{M-\mu}(\hat{r})\boldsymbol{\xi}_\mu \qquad (10d)$$

in terms of the spherical basis vectors $\boldsymbol{\xi}_\mu$. The C symbols are Clebsch-Gordan coefficients. For the $\mathbf{B}$ fields the spherical Bessel functions j_λ are replaced by their Hankel counterparts h_λ.

For the magnetic multipole case we select one value of L and $\tau=m$ only. The M sum is also incoherent since $m_f=m_i+M$ where m_f refers to the final nuclear state. The electronic matrix element

$$\mathfrak{M}_e(m)\equiv\int \mathbf{j}\cdot\mathbf{B}_L{}^M(m)d\mathbf{r} = \mathbf{b}\cdot\int e^{-i\mathbf{k}\cdot\mathbf{r}} h_L(k_0 r)\mathbf{T}_{LL}{}^M d\mathbf{r} \qquad (11a)$$

with

$$\mathbf{b}=u_f{}^*\boldsymbol{\alpha} u_i \qquad (11b)$$

and $u_{f,i}$ are the Dirac amplitudes, is readily evaluated. We find

$$\mathfrak{M}_e(m)=\frac{4\pi(-i)^{L+1}}{k^2-k_0{}^2}\frac{k^L}{k_0{}^{L+1}}\mathbf{b}\cdot\mathbf{T}_{LL}{}^M(\hat{k}) \qquad (11c)$$

and $\mathbf{T}_{LL}{}^M(\hat{k})$ is defined as in (10d) with $\hat{k}$ replacing $\hat{r}$. The sum over the spins of electron and positron, after squaring, gives

$$W_+W_-S_e|\mathfrak{M}_e(m)|^2=\frac{(4\pi)^4}{(k_0{}^2-k^2)^2}\left(\frac{k}{k_0}\right)^{2L} \times\{(W_+W_-+1-\mathbf{p}_+\cdot\mathbf{p}_-)|\mathbf{T}_{LL}{}^M|^2 +2\,\mathrm{Re}\,\mathbf{p}_+\cdot\mathbf{T}_{LL}{}^M\mathbf{p}_-\cdot\mathbf{T}_{LL}{}^{M*}\} \qquad (12)$$

and

$$N_\pi(\Theta,\theta,\delta,W_+)=\frac{16\alpha p_+p_-}{(k_0{}^2-k^2)^2}\left(\frac{k}{k_0}\right)^{2L} S_N\{\cdots\}|\mathfrak{M}_N|^2, \qquad (13a)$$

where

$$\mathfrak{M}_N=\int d\mathbf{r}'\mathbf{J}_N\cdot\mathbf{A}_L{}^M(m)\,, \qquad (13b)$$

and S_N is the nuclear part of the S summation. The curly bracket in (13) is the quantity in braces in Eq. (12).

The rate of γ-ray emission is obtained by calculating the matrix element of $\mathbf{J}_N\cdot\mathbf{A}$ where $\mathbf{A}$ is the vector potential of a plane wave in the direction $\hat{k}$, normalized to a total photon energy of k_0 in a box of unit volume. This plane wave is expanded into multipoles $\mathbf{A}_L{}^M(\mathbf{r}',\tau)$ with $\tau=e,\ m$.[9] Then, after integrating over an irrelevant azimuth angle, we find

$$N_\gamma(\theta)=2\pi(2L+1)k_0\sum_P S_N|D_{MP}{}^L(\hat{k})|^2|\mathfrak{M}_N|^2, \qquad (14)$$

where $P=\pm1$ refers to right (left) circular polarization. In (14) $D_{MP}{}^L(\hat{k})$ is the rotation group matrix element. After carrying out the indicated operations, we obtain

$$N_\gamma(\theta)=4\pi k_0(2j_f+1)\sum_\nu \alpha_\nu F_\nu{}^{(1)}(Lj_fj_i)P_\nu(\hat{k}\cdot\hat{n})\,, \qquad (15)$$

and only even ν occurs in the sum. Here we have omitted the square of the reduced matrix element of $\mathfrak{M}_N$. The same omission will be made in the evaluation of (13). Also $F_\nu{}^{(1)}(Lj_fj_i)$ is the well-known angular distribution quantity which enters in γ-ray angular correlations.[10] For convenience we give its definition here

$$F_\nu{}^{(1)}(Lj_fj_i)=(-)^{j_f-j_i-1}(2j_i+1)^{1/2}(2L+1) \times C(LL\nu;\,1,\,-1)W(j_ij_iLL;\,\nu j_f)$$

and W is a Racah coefficient. For the total number of photons per unit time we have

$$N_\gamma=8\pi k_0(2j_f+1)/(2j_i+1)\,,$$

since $F_0{}^{(1)}(Lj_fj_i)=1$.

We now perform the δ integration in Eq. (13). It will be noted that only the second term in the curly bracket contains δ and that

$$\mathbf{p}_+\cdot\mathbf{T}_{LL}{}^M\mathbf{p}_-\cdot\mathbf{T}_{LL}{}^{M*}=-|\mathbf{p}_+\cdot\mathbf{T}_{LL}{}^M|^2,$$

since $\mathbf{k}\cdot\mathbf{T}_{LL}{}^M=0$. We decompose $\mathbf{p}_+$ according to

$$\mathbf{p}_+=\mathbf{p}_+\cdot\hat{k}\hat{k}+\mathbf{v}^+,\quad \mathbf{v}_+=\hat{k}\times(\mathbf{p}_+\times\hat{k})\,,$$

[9] In the form used here this expansion appears in M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), p. 137.

[10] This quantity was first introduced by L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. **25**, 729 (1953). It was there denoted by $F_\nu(Lj_fj_i)$.

and then

$$\int_0^{2\pi} d\delta |\mathbf{p}_+\cdot\mathbf{T}_{LL}{}^M|^2 = \int_0^{2\pi} d\delta |\mathbf{v}_+\cdot\mathbf{T}_{LL}{}^M|^2$$

$$=\pi(\mathbf{v}_+)^2|\mathbf{T}_{LL}{}^M|^2 = \frac{\pi p_+{}^2 p_-{}^2}{k^2}\sin^2\Theta|\mathbf{T}_{LL}{}^M|^2,$$

since $\mathbf{k}$ is constant in the integration.

It is clear now that after the δ integration $N_\pi(\Theta,\theta,W_+)$ appears as a product of two factors,

$$N_\pi(\Theta,\theta,W_+) = N(\Theta,W_+)F(\theta), \tag{16}$$

where

$$N(\Theta,W_+) = \frac{32\pi\alpha p_+ p_-}{(k_0{}^2-k^2)^2}\left(\frac{k}{k_0}\right)^{2L} \times\left\{W_+W_- + 1 - \mathbf{p}_+\cdot\mathbf{p}_- - \frac{(\mathbf{p}_+\times\mathbf{p}_-)^2}{k^2}\right\} \tag{16a}$$

which is independent of the nuclear orientation and

$$F(\theta) = S_N|\mathbf{T}_{LL}{}^M|^2|\mathfrak{M}_N|^2 \tag{16b}$$

which does depend on the orientation.

The calculation of $F(\theta)$ is straightforward though lengthy. The definition of $\mathbf{T}_{LL}{}^M$ in (10d) is used and the coupling rule of spherical harmonics applied. By standard Racah techniques we find

$$F(\theta) = \frac{1}{4\pi}(2j_f+1)\sum_\nu \alpha_\nu F_\nu{}^{(1)}(Lj_fj_i)P_\nu(\hat{k}\cdot\hat{n}) = \frac{1}{16\pi^2 k_0}N_\gamma(\theta). \tag{17}$$

Here again ν is even so that polarizing the nucleus has no effect. Of course, if the polarization of either electron or positron were also measured there would be an effect of nuclear polarization.

We observe that the angular distribution of the resultant momentum, or of the nuclear recoil momentum, is the same as for the competing γ ray. Moreover, for the aligned source,

$$N_\pi(\Theta,\theta,W_+) = \frac{2\alpha}{\pi}\frac{p_+p_-}{(k_0{}^2-k^2)^2}\frac{k^{2L}}{k_0{}^{2L+1}} \times\left[1+W_+W_- - \mathbf{p}_+\cdot\mathbf{p}_- - \frac{(\mathbf{p}_+\times\mathbf{p}_-)^2}{k^2}\right]N_\gamma(\theta) = \gamma_L(\Theta,W_+)N_\gamma(\theta). \tag{18}$$

Here $\gamma_L(\Theta,W_+)$ is the same as for no orientation as is trivially to be expected in view of preceding remarks. Hence, if the pair distribution is measured for fixed Θ and θ with the dihedral angle δ summed and the γ rays are measured with fixed θ, the pair to γ ratio, $\bar{\gamma}_L(\Theta,\theta,W_+)$, is *independent* of nuclear orientation, independent of θ, and is equal to $\gamma_L(\Theta W_+)$. The ratio of pairs with fixed Θ and θ to the total number of γ rays is

$$N_\pi(\Theta,\theta,W_+)/N_\gamma = \gamma_L(\Theta,W_+)[N_\gamma(\theta)/N_\gamma], \tag{19}$$

where obviously

$$f_1 \equiv \frac{N_\gamma(\theta)}{N_\gamma} = \frac{2j_i+1}{2}\sum_\nu \alpha_\nu F_\nu{}^{(1)}P_\nu(\hat{k}\cdot\hat{n}), \tag{20}$$

which is the γ-ray angular distribution normalized according to

$$\int_0^\pi f_1(\theta)\sin\theta d\theta = 1.$$

It is also clear that if one considers the angular distribution of pairs integrated over W_+ the same remarks apply. Alignment multiplies the distribution by the function $f_1(\theta)$ and for the ratio of such pairs to photons emitted in the same direction θ, there is no effect of alignment.

Turning to the electric transitions the procedure is very similar but the results are somewhat different. It is important in the multipole expansions in Eq. (9) to keep the transverse electric and longitudinal contributions separate. The distribution $N_\pi(\Theta,\theta,\delta,W_+)$ in all three angles shows an interference term in the transverse and longitudinal parts arising from the alignment but on summing over δ this disappears. It is also worthwhile noting that the nuclear matrix elements for the transverse electric and longitudinal potentials are related by

$$\int \rho_N j_L Y_L{}^M d\mathbf{r}' \Big/ \int \mathbf{J}_N\cdot\mathbf{A}_L{}^M(e)d\mathbf{r}' = i\left[\int \mathbf{J}_N\cdot\mathbf{A}_L{}^M(l)d\mathbf{r} \Big/ \int \mathbf{J}_N\cdot\mathbf{A}_L{}^M(e)d\mathbf{r}\right] = i\left(\frac{L}{L+1}\right)^{1/2}, \tag{21}$$

by virtue of the fact that $k_0R_N \ll 1$. In the matrix element obtained after integrating over the electron coordinates the $\mathbf{T}_{LL}{}^M$ is replaced by $\hat{k}\times\mathbf{T}_{LL}{}^M$ in the transverse contribution.

Then we can express the branching ratio without nuclear orientation in the following way:

$$\gamma_L(\Theta,W_+) = \frac{L}{L+1}\gamma_L{}^l(\Theta,W_+) + \gamma_L{}^e(\Theta,W_+), \tag{22a}$$

where the first term arises from the longitudinal part and is given by

$$\gamma_L{}^l = \frac{2\alpha}{\pi k_0}p_+p_-\left(\frac{k}{k_0}\right)^{2L-2}\frac{W_+W_- + \mathbf{p}_+\cdot\mathbf{p}_- - 1}{k^2k_0{}^2} \tag{22b}$$

and the second term arises from the transverse electric part and is given by

$$\gamma_L{}^e=\frac{2\alpha}{\pi k_0}p_+p_-\left(\frac{k}{k_0}\right)^{2L-2}\frac{W_+W_- - \mathbf{p}_+\cdot\hat{k}\mathbf{p}_-\cdot\hat{k}+1}{(k_0{}^2-k^2)^2}. \quad (22c)$$

This is identical with $\gamma_{L-1}(\Theta,W_+)$ for magnetic multipoles as obtained from Eq. (18).

When the nucleus is aligned we obtain

$$\frac{N_\pi(\Theta,\theta,W_+)}{N_\gamma}=\frac{L}{L+1}\gamma_L{}^l f_0(\theta)+\gamma_L{}^e f_1(\theta). \quad (23)$$

Here f_1 is as defined in Eq. (20) while f_0 is the angular distribution function, similarly normalized, for spin-zero particles.[11] Specifically,

$$f_0(\theta)=\frac{2j_i+1}{2}\sum_\nu \alpha_\nu F_\nu{}^{(0)}P_\nu(\hat{k}\cdot\hat{n}), \quad (24a)$$

and

$$F_\nu{}^{(0)}=(-)^{j_f-j_i}(2j_i+1)^{1/2}(2L+1)C(LL\nu;00) \times W(j_ij_iLL;\nu j_f) \quad (24b)$$

with $F_0{}^{(0)}=1$. We now recognize that the ratio

$$\frac{N^\pi(\Theta,\theta,W_+)}{N_\gamma(\theta)}=\frac{L}{L+1}\gamma_L{}^l\frac{f_0}{f_i}+\gamma_L{}^e, \quad (25)$$

and will show an effect of nuclear alignment. It is convenient to remember that

$$C(LL\nu;00)[\nu(\nu+1)-2L(L+1)] =2L(L+1)C(LL\nu;1,-1).$$

[11] See Ref. 8, pp. 176–179.

When one considers what is taking place here the interpretation to which one arrives is that by averaging or summing over the dihedral angle δ one effectively decouples the variables of the problem in such a way that the distribution of $\mathbf{p}_+$ and $\mathbf{p}_-$ relative to $\hat{k}$ is unaffected by the orientation but $\hat{k}$ is distributed relative to $\hat{n}$ in the same way as a "particle" of spin s would be in similar circumstances. For the transverse fields $s=1$ which is a way of saying that the distribution of radiation in the emission of such a particle from an aligned state is a geometrical property and has nothing to do with such physical questions as the dispersion relation between the energy and momentum of the particle. We have in mind the fact that the virtual quanta which are involved in the electron-nucleus interaction are not on the mass shell since $k\neq k_0$. For the longitudinal fields $s=0$ which is hardly surprising. It is to be noted that the particles in question are represented by plane waves and so no physical parameters such as those describing the electrostatic electron-nucleus interaction play a role. It may be conjectured that the present results would be modified if the Coulomb distortion of the electron wave functions were taken into account. However, as a purely formal question, it may be also conjectured that for high energies ($k_0\rightarrow\infty$) the results presented here would again be valid even if a Coulomb field were present.[12] Nevertheless, since the plane wave approximation has been demonstrated to be a good one for internal pair formation one may expect that for the light nuclei, where this phenomenon is usually observed, the results presented above would be applicable to experimental situations.

[12] Indications that this would be so appear in related problems: See M. E. Rose, L. C. Biedenharn, and G. B. Arfken, Phys. Rev. **85**, 5 (1952); and R. L. Becker and M. E. Rose, Nuovo Cimento **13**, 1182 (1959).