The width of the K^* is given by

$$\Gamma=\Gamma(K^{*+}\to K^0+\pi^+)+\Gamma(K^{*+}\to K^++\pi^0)$$
$$=2\left(\frac{f^2}{4\pi}\right)\frac{p^3}{M^2}, \quad (18)$$

where p is the center-of-mass momentum of the decay pion and M the mass of the K^*.

We obtain $f^2/4\pi=0.80$ which gives $g^2/4\pi=0.144$. This value is smaller than the estimate of Chan[13] obtained under the assumption that the K^* exchange term gives the total cross section at an incident pion kinetic energy of 960 MeV and with a width for the K^* decay of 23 MeV. Our product of coupling constants falls close to the value obtained by MacDowell *et al.*[2] from the experimental data at $T_\pi=1300$ MeV.

After this work was completed we learned of a related work by Feld and Layson[14] who analyzed the experimental data on the total $\pi^\pm p$ cross sections and the differential elastic $\pi^- p$ scattering cross section for energies between 0.3 and 1.3 BeV. They found that the best fitting of the angular distribution requires a $T=1/2, p_{1/2}$ resonance near 950 MeV ($W=1716$ MeV) in agreement with out results. Also Kuo[15] has fitted the low energy $\gamma+p\to\Lambda+K^+$ data (excitation function, angular distribution, and one experimental point in the polarization) using a model similar to ours which included a Kanazawa resonance at $W=1700$ MeV and obtained a slightly better fit in the $p_{1/2}$ case.

We should add a comment on a work by Gourdin and Rimpault[16] in which a model somewhat similar to ours was proposed. These authors added to the K^* exchange the contributions from the Σ and Y_1^* exchanges, the nucleon pole, and the resonances $N_{1/2}^*$ and $N_{1/2}^{**}$, but an agreement with experiment for total and differential cross sections was found only in the cases of odd $\Sigma\Lambda$ parity, spin of K^* equal to 1, and even $\Sigma\Lambda$ parity, spin of K^* equal to 0. It is well known at the present time that the spin of the K^* is one[17] and the $\Sigma\Lambda$ parity even,[18] so this model is no longer valid. Their value for the $K^*\Lambda N$ coupling constant $g^2/4\pi=1.8$ should, therefore, not be considered reliable.

ACKNOWLEDGMENTS

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A stimulating conversation with Leo Stodolsky was of great help.

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[14] B. T. Feld and W. M. Layson, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Scientific Information Service, Geneva, 1962), p. 147. See also W. M. Layson, Nuovo Cimento **27**, 718 (1963).

[15] T. K. Kuo, Phys. Rev. **129**, 2264 (1963).

[16] M. Gourdin and M. Rimpault, Nuovo Cimento **24**, 414, (1962).

[17] See W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962).

[18] See Robert D. Tripp, Mason W. Watson, and Massimiliano Ferro-Luzzi, Phys. Rev. Letters **8**, 175 (1962). More recently, see H. Courant, H. Filthouth, P. Franzini, R. G. Glasser, A. Minquzzi-Ranzi, A. Segar, W. Willis, R. A. Burnstein, T. B. Day, B. Kehoe, A. J. Herz, M. Sakitt, B. Sechi-Zorn, N. Seeman, and G. A. Snow, *ibid.* **10**, 409 (1963).

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Dynamical Model of the K^* Resonance†

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The K^* vector meson is regarded as a P-wave resonance in the coupled, isotopic spin $\frac{1}{2}$, $\pi+K$, and $\eta+K$ states. All forces other than the K^* and ρ exchange forces are neglected, and a modification of the self-consistency technique of Zachariasen and Zemach is used to calculate the K^* mass and two relations among the three coupling-constant products $\gamma_{K^*\pi K}{}^2$, $\gamma_{K^*\eta K}{}^2$, and $\gamma_{\rho\pi\pi}\gamma_{\rho KK}$. The calculated K^* mass agrees with experiment. The factors in the self-consistency equations that depend on the $\pi-\eta-K$ and $K^*-\rho$ mass differences are isolated, and the effects of these mass differences on the results are discussed. The relationship of the results to the predictions of unitary symmetry is discussed.

I. INTRODUCTION

MANY authors have speculated that the strong-interaction coupling constants and the relative masses of the strongly interacting particles may be determinable from some form of dispersion relations.[1] Recently, several different attempts have been made to determine the ρ-meson mass and width from dispersion relations for the pion-pion scattering amplitude.[2–4]

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[1] See, for example, G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **8**, 41 (1962); R. H. Capps, Phys. Rev. **128**, 2842 (1962).

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The agreement of the predictions with experiment has not been spectacular; the chief difficulty is that the predicted reduced width is too large.[2,3] At first glance it appears that a dispersion-theoretic prediction of the width of the K^* or ω meson would disagree with experiment even more, since these mesons are of mass comparable with the ρ, but are much narrower. In fact, the existence of many high-energy resonances of rather small widths is a serious obstacle to the point of view that all resonances and particles are of dynamical origin with all interaction constants calculable, since simple dynamical calculations generally lead to large widths.

There exists within the dispersion-theory framework a rather simple mechanism which may lead naturally to narrow resonances, however. This mechanism is the strong coupling of the resonances to states of higher rest mass. In this paper we consider the example of the K^* vector meson,[5] and regard it as a resonance in the coupled, isotopic spin $\frac{1}{2}$, $\pi+K$, and $\eta+K$ channels, using a modification of the "bootstrap" procedure of Zachariasen and Zemach.[4] In order to see how the coupling of the K^* to the high rest mass $\eta+K$ channel may lead to a small K^* width we consider the hypothetical situation in which the $\pi+K \rightarrow \eta+K$ inelastic amplitude is zero, so that the $\pi+K$ and $\eta+K$ states are eigenstates of the scattering. The K^* would then be either a pure $\pi+K$ resonance, with an appreciable width, or an $\eta+K$ bound state, with zero width. It is apparent that in a realistic calculation in which the $\pi+K \rightarrow \eta+K$ coupling is not zero, a predicted K^* width anywhere between zero and an appreciably large value is possible. The small observed width may mean that the K^* is coupled more strongly to the $\eta+K$ state, or another state of high rest mass, than to the $\pi+K$ state into which it is forced to decay for energetic reasons. We note that in a dispersion theory there is no difficulty in the concept of the coupling of a resonance to states of high rest mass, since amplitudes may be analytically continued into unphysical energy regions.

Another reason for considering the coupling of the $\eta+K$ channel to the $\pi+K$ channel is the recent successes of the octet model of unitary symmetry.[6] This model has successfully predicted the existence of the pseudoscalar η particle and the vector-meson octet,[7] has produced a mass formula that is satisfied very well by the pseudoscalar meson and baryon octets,[8] and has predicted successfully the $I=\frac{1}{2}$ cascade-pion resonance at $\sim$1532 MeV.[9] The dynamical prediction of the Ξ^* from unitary symmetry depends crucially on the assumption that the η and K interactions are of comparable importance with the π interactions.[10] However, the relative strengths of the π, K, and η interactions cannot be predicted from unitary symmetry as long as the origin of the mass differences of these particles is not understood. We shall treat these relative strengths as undetermined parameters, but adopt the principle that all members of the π, K, η octet must be considered whenever any one of them is.

In a previous paper by the author,[11] the values of the five PS-PS-V (pseudoscalar-pseudoscalar-vector) meson-coupling constants were calculated from approximate dispersion relations of the "bootstrap" type, with the mass differences among the PS mesons and among the V mesons neglected. The ratios of the calculated constants are in agreement with the predictions of the octet model of unitary symmetry, so that this symmetry is predicted by the dispersion relations. However, if future experiments do verify the validity of unitary symmetry, it is clear that this will not prove that the symmetry has anything to do with dispersion relations. In the author's opinion there are two types of methods that may provide tests for the hypothesis of a dispersion-theoretic origin of unitary symmetry. The first has to do with the predictive power of dispersion theory. If it becomes possible to start with a few simple dispersion-theoretic principles and give many correct answers to such questions as: (1) What are the masses and coupling constants of particles? (2) What are the basic interaction symmetries? and, (3) Why do so many particles exist?, then the theory will certainly be attractive even though the necessity of a dispersion-theoretic formulation will not have been "proved."

The second method of testing our hypothesis has to do with the PS-meson mass differences and V-meson mass differences; it is to this question that the present paper relates. Because of the existence of the mass differences we know that the basic interaction symmetry must be broken in some manner. It is hoped that the dispersion relations will predict a particular, testable relation between mass differences and the breaking of the interaction symmetry.

We cannot discover the reason for the mass differences by considering only the PS-PS meson scattering amplitudes and only the V-meson exchange forces in a bootstrap calculation. However, we can hope that these amplitudes and forces are the most important in determining the relations of the V-meson mass differences and V-PS-PS coupling constants to the PS-meson

[5] For experimental evidence of the vector nature of the K^*, see W. Chinowsky, G. Goldhaber, S. Goldhaber, W. Lee, and T. O'Halloran, Phys. Rev. Letters **9**, 330 (1962).

[6] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[7] For a summary of the evidence concerning the pseudoscalar nature of the η particle, see M. Chretien, F. Bulos, H. R. Crouch, Jr., R. E. Lanou, Jr., J. T. Massimo, *et al.*, Phys. Rev. Letters **9**, 127 (1962).

[8] S. Okubo, Prog. Theoret. Physics (Kyoto) **27**, 949 (1962); see also Ref. 6.

[9] The prediction of the Ξ^* is made by R. Behrends, J. Dreitlein, C. Fronsdal, and W. Lee, Rev. Mod. Phys. **34**, 1 (1962); the discovery of the Ξ^* is reported by G. M. Pjerrou, D. J. Prowse, P. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **9**, 114 (1962), and by L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra, *et al.*, *ibid.* **9**, 180 (1962).

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[11] R. H. Capps, Phys. Rev. Letters **10**, 312 (1963).

mass differences. Such relations may be investigated by means of a program of calculations in which the physical masses of the PS mesons are assumed, and the bootstrap technique is used to calculate the V-meson masses and interactions. This paper represents a part of such a program; we consider only the K^*—$\pi+K$—$\eta+K$ system.

In our calculation the K^* exchange force contributes to all three processes $\pi+K\rightarrow\pi+K$, $\eta+K\rightarrow\eta+K$, and $\pi+K\rightarrow\eta+K$. The only other force considered is ρ exchange, which contributes to $\pi+K$ elastic scattering. These forces are characterized by five parameters, the masses m_{K^*} and m_ρ, the coupling constants $\gamma_{K^*\pi K}{}^2$ and $\gamma_{K^*\eta K}{}^2$, and the product $\gamma_{\rho\pi\pi}\gamma_{\rho KK}$. We take the ρ mass from experiment, so there are four undetermined parameters. The N/D dispersion relations yield as output the three parameters m_{K^*}, $\gamma_{K^*\pi K}{}^2$, and $\gamma_{K^*\eta K}{}^2$. Thus, the requirement of self-consistency does not determine all the parameters, but leaves one unfixed. It is shown in Sec. III that the K^* mass is extremely insensitive to this arbitrariness and may be calculated in our model. The relationship of the results regarding the coupling constants to unitary symmetry is discussed in Sec. IV.

II. DERIVATION OF THE EQUATIONS

The general procedure we use is similar to that of Refs. 4 and 11. We assume that the π, η, and K are pseudoscalar particles and consider the P-wave amplitudes for the processes $\pi+K\rightarrow\pi+K$ (in the isotopic spin-$\frac{1}{2}$ state), $\eta+K\rightarrow\eta+K$, and $\pi+K\rightarrow\eta+K$. The input forces are assumed to result entirely from the K^* and ρ exchange graphs shown in Fig. 1. The first approximation to the matrix N/D method is used for the amplitudes; i.e., the Born-approximation amplitudes resulting from these forces are taken to be equal to the numerator matrix, and a once-subtracted dispersion relation is used for the denominator matrix. For suitable choices of the input parameters, the K^* resonance is generated by the dispersion relations, as shown in Fig. 2. We then apply the self-consistency

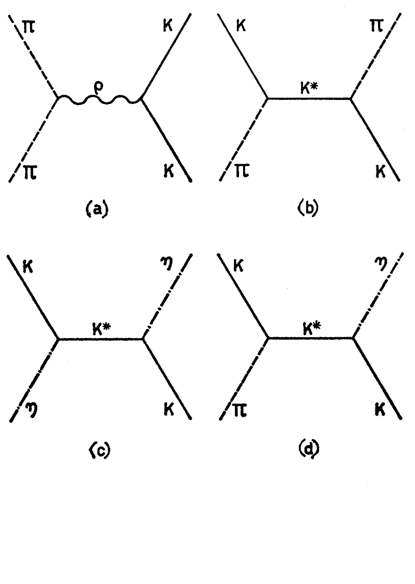

FIG. 1. Graphs of the input forces to the $\pi+K\rightarrow\pi+K$, $\eta+K\rightarrow\eta+K$, and $\pi+K\rightarrow\eta+K$ processes.

requirement that the values of the K^* mass and coupling constants resulting from the dispersion relations are equal to those used to specify the forces, and study the resulting implied relations among these constants.

A. The Input Forces

We denote the $\pi+K$ and $\eta+K$ states with the single indices π and η, and the $K^*\pi K$, $K^*\eta K$, $\rho\pi\pi$, and ρKK coupling constants by $\gamma_{K^*\pi}$, $\gamma_{K^*\eta}$, $\gamma_{\rho\pi}$, and $\gamma_{\rho K}$. We define the P-wave amplitudes T_{ij} for the three basic processes in terms of elements of the unitary scattering matrix U by the equation

$$T_{ij}=\frac{(U_{ij}-\delta_{ij})s^{1/2}}{2iq_i^{3/2}q_j^{3/2}},\qquad(1)$$

where s is the square of the total energy in the center-of-mass system, and q_i and q_j are the magnitudes of the initial and final particle momenta in the center-of-mass system. The constants $\hbar$ and c are taken as unity. The $(q_iq_j)^{-3/2}$ factor is included in the definition so that T_{ij} has no zeroes, poles, or branch points at the threshold energies.

Because of the $\pi-K$ and $\eta-K$ mass differences, the calculation of the Born-approximation amplitudes resulting from the input forces is not as straightforward as that in Refs. 4 and 11. We illustrate the calculation by considering the inelastic process $\pi+K\rightarrow\eta+K$. The force for this process may be determined from the amplitude for the crossed $\pi+K\rightarrow\eta+K$ process obtained by looking at Fig. 1(d) from the side. We consider only the contribution of the K^* to this crossed process, and denote the K^* mass by M. The invariant amplitude A for the crossed process is assumed to be of the form,[12,13]

$$A=4\left(\frac{\gamma_{K^*\pi}\gamma_{K^*\eta}}{4\pi}\right)\frac{q_{\pi,c}q_{\eta,c}\cos\theta_c}{M^2-s_c},\qquad(2)$$

where s_c, θ_c, $q_{\pi,c}$, and $q_{\eta,c}$ are the appropriate energy, angle, and momentum variables for the crossed process.

[12] Our normalization of the invariant amplitude is the same as that of Geoffrey F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin and Co., New York, 1961), Chap. 2. The calculation of Born-approximation amplitudes by using the "crossing" or "substitution" law is discussed in detail in this reference.

[13] In the pole approximation one must make a choice concerning the exact definition of the amplitude to be replaced by a simple pole. Our choice is the amplitude T_{ij} of Eq. (1). This choice is the customary one in the limit that the mass differences are neglected; it agrees with that of Ref. 4, for example. It is well known that for short-range forces the P-wave amplitude T_{ij} of Eq. (1) has no singularities at the channel thresholds unless a "bound-state" pole occurs accidently at a threshold energy. Therefore, one would be treating the nearby, threshold singularities incorrectly if he were to assume that $q_iq_jT_{ij}$, rather than T_{ij}, is represented by a simple pole. For this reason, the factors $q_{\pi,c}$ and $q_{\eta,c}$ must be included in Eq. (2); they cannot be replaced by their values at $s_c=M^2$. [The relation between $T_{ij,c}$ and A is given in Eq. (3).]

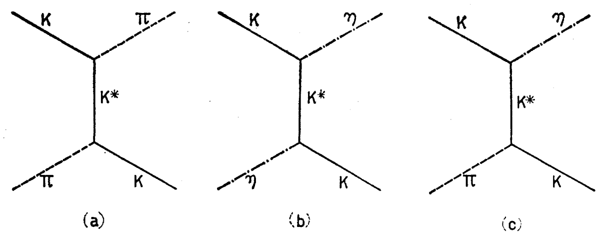

FIG. 2. The output; appearance of the K^* resonance in the $\pi+K$ and $\eta+K$ states.

If the K^* resonance were to occur above the $\pi+K$ and $\eta+K$ thresholds, the denominator M^2-s_c should have an imaginary part, but we consistently neglect this imaginary part in computing the Born-approximation amplitudes. The momentum $q_{\eta,c}$ is actually imaginary at the mass of the K^*, but this causes no difficulty.

The P-wave amplitude in the crossed process is related to the invariant amplitude by the equation,

$$T_{\pi\eta,c}=(q_{\pi,c}q_{\eta,c})^{-1}\int_{-1}^{1}A(s_c,\cos\theta_c)\cos\theta_c d(\cos\theta_c). \quad (3)$$

From this equation and Eq. (2) it follows that our definition of the coupling constants is[14]

$$\gamma_{K^*i}\gamma_{K^*j}/(4\pi)=\tfrac{3}{8}[(M^2-s)T_{ij}]_{s=M^2}, \quad (4)$$

where i and j each refer to either of the $\pi+K$ and $\eta+K$ states.

In order to compute the Born approximation for the s-channel amplitude, we need the relations between s_c, $q_{\pi,c}q_{\eta,c}\cos\theta_c$, and s-channel variables. These are

$$2q_{\pi,c}q_{\eta,c}\cos\theta_c=s+\frac{s_c}{2}-\frac{\Sigma_{\pi\eta}}{2}-\frac{\Delta_\pi\Delta_\eta}{2s_c}, \quad (5a)$$

$$s_c=\tfrac{1}{2}\Sigma_{\pi\eta}-\frac{s}{2}+\frac{\Delta_\pi\Delta_\eta}{2s}+2q_\pi q_\eta\cos\theta, \quad (5b)$$

where

$$\Sigma_{ij}=2m_K^2+m_i^2+m_j^2, \quad (5c)$$

$$\Delta_i=m_K^2-m_i^2, \quad (5d)$$

and the relation between q_i and s is

$$q_i=[\tfrac{1}{4}s-\tfrac{1}{2}(m_K^2+m_i^2)+\tfrac{1}{4}(\Delta_i^2/s)]^{1/2}. \quad (6)$$

If the expression for A [Eq. (2)] is written in terms of s-channel variables, the Born approximation for $T_{\pi\eta}$ may be obtained from the analog of Eq. (3) [Eq. (3) with the subscripts c removed]. We denote the contribution resulting from the exchange of the V meson l to the Born approximation for the amplitude T_i by the symbol $N_{ij,l}$.

Unfortunately, the above procedure leads to an unwanted singularity in $N_{\pi\eta,K^*}$ that arises because the masses of the PS mesons are not the same. The singularity in $N_{\pi\eta,K^*}$ results from the singularity at $s_c=0$ of the expression for $\cos\theta_c$, Eq. (5a). There is a branch cut in $N_{\pi\eta,K^*}$ at those values of s for which the integration over $\cos\theta$ involves integrating over the point $s_c=0$. In order to get rid of this singularity we replace the factor $\frac{1}{2}\Delta_\pi\Delta_\eta/s_c$ by $\frac{1}{2}\Delta_\pi\Delta_\eta/M^2$ in Eq. (5a), when substituting this equation into the expression for the invariant amplitude. [No change is made in Eq. (5b), however.] A similar procedure is used in computing $N_{\pi\pi,K^*}$ and $N_{\eta\eta,K^*}$. The Born-approximation amplitudes $N_{ij,l}$ resulting from this modified procedure may be written in the form

$$N_{ij,l}=C_{ij,l}\gamma_{li}\gamma_{lj}\mathfrak{N}_{ij,l}, \quad (7a)$$

$$\mathfrak{N}_{ij,l}=Q(z)X_{ij,l}/(8\pi Y_{ij,l}^2), \quad (7b)$$

$$z=q_iq_j/Y_{ij,l}, \quad (7c)$$

$$Q(z)=-\frac{4}{z^2}+\left(\frac{2}{z^2}+\frac{1}{z^3}\right)\ln(1+4z). \quad (7d)$$

The constants $C_{ij,l}$ are isotopic-spin-dependent factors that may be determined from the crossing matrices, and $X_{ij,l}$ and $Y_{ij,l}$ are functions of s. For the amplitude $N_{\pi\eta}$ (we drop the unnecessary index K^* on $N_{\pi\eta,K^*}$ and $N_{\eta\eta,K^*}$), in the energy region above $\eta+K$ threshold, these factors are

$$C_{\pi\eta}=1, \quad (8a)$$

$$X_{\pi\eta}=s+\tfrac{1}{2}M^2-\tfrac{1}{2}\Sigma_{\pi\eta}-\tfrac{1}{2}(\Delta_\pi\Delta_\eta/M^2), \quad (8b)$$

$$Y_{\pi\eta}=\tfrac{1}{2}s+M^2-\tfrac{1}{2}\Sigma_{\pi\eta}-\tfrac{1}{2}(\Delta_\pi\Delta_\eta/s)-2q_\pi q_\eta. \quad (8c)$$

In the region between the $\pi+K$ and $\eta+K$ thresholds, where q_η is imaginary, these functions may be analytically continued. The resulting formulas may be expressed simply, if one makes the simultaneous substitutions, $Q(z)\to Q'(z')$ and $Y_{\pi\eta}\to Y'$, where

$$z'=|q_\pi q_\eta|/Y',$$

$$Q'=\frac{4}{z'^2}-\frac{2}{z'^3}\arctan(2z'),$$

$$Y'=\tfrac{1}{2}s+M^2-\tfrac{1}{2}\Sigma_{\pi\eta}-\tfrac{1}{2}(\Delta_\pi\Delta_\eta/s).$$

The Born approximations for $\pi+K$ and $\eta+K$ elastic scattering resulting from the diagrams of Figs. 1(a), 1(b), and 1(c) may be determined in a similar manner. The $\pi+K\to\pi+K$ amplitude has two contributions, $N_{\pi\pi,K^*}$ and $N_{\pi\pi,\rho}$. The results for these three contributions may be expressed in the form of Eqs. (7), where the various C, X, and Y functions are

$$C_{\pi\pi,\rho}=\sqrt{2}, \quad (8d)$$

$$X_{\pi\pi,\rho}=s+\tfrac{1}{2}m_\rho^2-\tfrac{1}{2}\Sigma_{\pi\pi}, \quad (8e)$$

$$Y_{\pi\pi,\rho}=m_\rho^2, \quad (8f)$$

$$C_{\pi\pi,K^*}=-\tfrac{1}{3}, \quad (8g)$$

[14] Our normalization of the coupling constants is the same as that of Refs. 4 and 11.

$$X_{\pi\pi,K^*}=s+\tfrac{1}{2}M^2-\tfrac{1}{2}\Sigma_{\pi\pi}-\tfrac{1}{2}(\Delta_\pi^2/M^2), \quad (8h)$$

$$Y_{\pi\pi,K^*}=M^2-(\Delta_\pi^2/s), \quad (8i)$$

$$C_{\eta\eta}=1, \quad (8j)$$

$$X_{\eta\eta}=s+\tfrac{1}{2}M^2-\Sigma_{\eta\eta}-\tfrac{1}{2}(\Delta_\eta^2/M^2), \quad (8k)$$

$$Y_{\eta\eta}=M^2-(\Delta_\eta^2/s). \quad (8l)$$

The Σ_{ij} are defined in Eq. (5c). These formulas are valid both above and below the $\eta+K$ threshold.

The replacement procedure discussed above, that of substituting $\Delta_i\Delta_j/M^2$ for $\Delta_i\Delta_j/s_c$, is made for all the K^*-exchange contributions. This procedure is equivalent to assuming a nonresonating S-wave amplitude, proportional to $\Delta_i\Delta_j/(M^2 s_c)$, in addition to the resonant P-wave contribution, in the crossed channel. It is not clear whether this assumption is better or worse than the assumption of a zero S-wave amplitude. In fact, it is actually inconsistent to neglect forces resulting from the exchange of S-wave PS-meson pairs, since the V-meson exchange mechanism leads to forces in the "crossed" channels in the S waves as well as the P waves. One should write equations for the S waves, P waves, and other angular momenta, simultaneously. This would complicate the problem greatly, however, so we use only that small S-wave amplitude in the crossed channel necessary to remove the $1/s_c$ singularity, as described above. The early work of Chew and Mandelstam on the S- and P-wave $\pi-\pi$ scattering amplitudes partially justifies our approach (or any other approach in which the S-wave amplitudes are small), for in this work it was found that if the P wave is resonant, the forces on the P-wave amplitudes contributed by the exchange of S-wave meson pairs is relatively small.[15]

The Born-approximation amplitudes are often derived from perturbation theory involving the vertex function and propagator for the vector meson, so we shall discuss how Eqs. (7) and (8) may be derived in this manner. One can write the vertex factor for the $K^*\pi K$ interaction occurring in Figs. 1(b) and 1(d) in the form

$$\langle p_K|J_{K^*,\mu}|p_\pi\rangle=a(p_\pi+p_K)_\mu+b(p_\pi-p_K)_\mu,$$

where p_π and p_K are the four momenta of the π and K. The ratio b/a may be determined from the "current conservation" condition $J_{K^*}(p_K-p_\pi)=0$. The result is

$$\langle p_K|J_{K^*,\mu}|p_\pi\rangle=a[(p_\pi+p_K)+(\Delta_\pi/s_c)(p_\pi-p_K)]_\mu,$$

where $s_c=-(p_\pi-p_K)^2$. It is easy to show by considering the "crossed vertex" $\pi+K\rightarrow K^*$ that the Δ_π/s_c term prevents the occurrence of coupling of S-wave $\pi+K$ pairs to the fourth component of the K^* vector. Use of this vertex function would lead to the $1/s_c$ singularity discussed earlier. We may derive Eqs. (7) and (8) by neglecting this term of the vertex function and using the following K^*-meson propagator,

$$\frac{\delta_{\mu\nu}-(p_\pi-p_K)_\mu(p_\pi-p_K)_\nu/M^2}{s_c-M^2}.$$

The second term of the propagator is necessary; leaving it out would be equivalent to neglecting the $\Delta_\pi\Delta_\eta/s_c$ term in Eq. (5a) entirely, which would correspond to the assumption of a resonant S-wave amplitude in the crossed channel.

B. The N/D Dispersion Relations

In the matrix N/D method, one writes $T=ND^{-1}$, where T, N, and D are square matrices.[16] We follow the general procedure of Refs. 4 and 11 and choose N to be the Born-approximation matrix amplitude of Eqs. (7) and (8), i.e., $N_{\pi\pi}=N_{\pi\pi,\rho}+N_{\pi\pi,K^*}$, $N_{\eta\eta}=N_{\eta\eta,K^*}$, and $N_{\pi\eta}=N_{\pi\eta,K^*}$. We write a once-subtracted dispersion relation for D, setting D equal to the unit matrix $\mathbf{1}$ at the subtraction energy s_t. The dispersion relation is

$$D(s)=\mathbf{1}+\frac{s-s_t}{\pi}\int\frac{ds'\,\mathrm{Im}D(s')}{(s'-s_t)(s'-s-i\epsilon)}. \quad (9)$$

Only the physical $\pi+K$ and $\eta+K$ branch cuts are included in D. The unitarity relation is $\mathrm{Im}D=(\mathrm{Im}T^{-1})N$, where

$$(\mathrm{Im}T^{-1})_{ij}=-\delta_{ij}(q_i^3/s^{1/2})\theta_i(s). \quad (10)$$

The function $\theta_i(s)$ is defined to be one for $q_i^2>0$ and zero for $q_i^2<0$. These equations, together with the expressions for N_{ij} in Sec. II A, are the equations for the amplitudes. The integral in Eq. (9) is convergent, so that no cutoff or further subtraction is necessary.

This method is only approximate, as is discussed in Ref. 4. The amplitudes satisfy the unitarity condition exactly on the right-hand cut, but only approximately on the left-hand cut. Furthermore, further approximations to the N/D method diverge when vector particles (or other states of angular momentum $\geqq 1$) are exchanged.[17,3] It is widely hoped that if the asymptotic forms of the various crossed-channel amplitudes are taken to be that suggested by Regge; it will be possible to construct a consistent, convergent theory.[17,3] However, since our approximate equations are simple and convergent, we do not postulate the Regge behavior here.

It is pointed out in Ref. 4 that if the position of the derived resonance lies on the left-hand cut of one of the amplitudes, then this method is inconsistent. The inconsistency will manifest itself by the occurrence of a branch point in the logarithmic function of Eq. (7d) at some real value of s equal to or greater than M^2.

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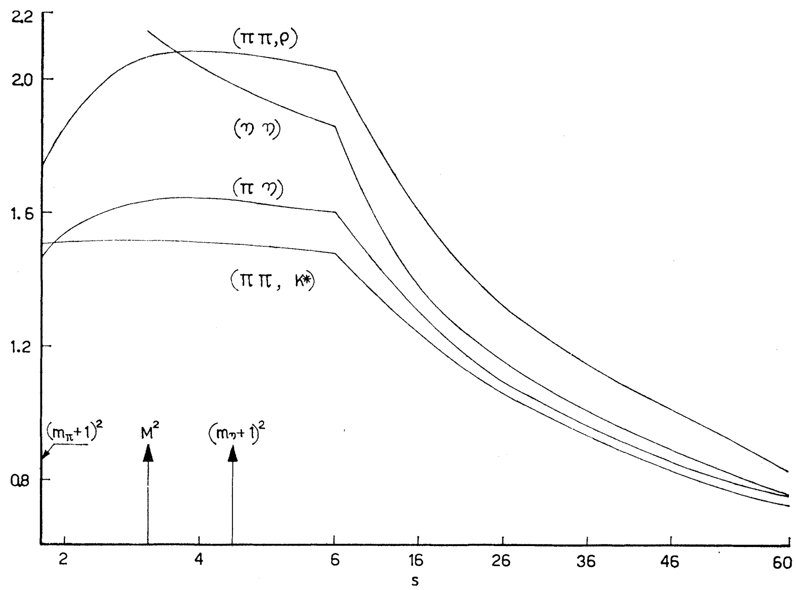

FIG. 3. The four force functions $\mathfrak{N}_{ij,l}$. The energy variable s is in units of m_K^2, and the $(8\pi\mathfrak{N})$ are in units of m_K^{-2}. The energies of the $\pi+K$ threshold, K^* mass, and $\eta+K$ threshold are shown. For convenience the energy scale is changed at $s=6$; the derivatives of the curves are actually continuous at this point.

It can be seen from Eqs. (6), (7), and (8) that no such inconsistency occurs in our case for values of M^2 equal to or greater than the physical value.

One well-known difficulty with the matrix N/D method is that approximate solutions are not in general symmetric, despite the fact that N is symmetric.[18] We illustrate this point by using Eqs. (9) and (10) to write the equation for $T_{\pi\eta}=(ND^{-1})_{\pi\eta}$ in the following form:

$$T_{\pi\eta}=|D|^{-1}\left[N_{\pi\eta}+\frac{s-s_t}{\pi}(N_{\pi\pi}I_{\pi\eta}-N_{\pi\eta}I_{\pi\pi})\right], \quad (11)$$

$$I_{ij}=\int_{(m_K+m_i)^2}^{\infty}\frac{\rho_i^2N_{ij}(s')ds'}{(s'-s_t)(s'-s-i\epsilon)s'^{1/2}}, \quad (12)$$

where $|D|$ is the determinant of D, and ρ_i is a kinematic factor which we chose equal to $q_i^{3/2}$ in our definition of T_{ij} in Eq. (1). The corresponding equation for $T_{\eta\pi}$ may be obtained by reversing the π and η subscripts in Eq. (11). It is seen that the coefficients of $(s-s_t)$ are not the same in the expressions for $T_{\pi\eta}$ and $T_{\eta\pi}$ so that, in general, $T_{\pi\eta}\neq T_{\eta\pi}$.

It is commonly believed that nothing can be done about this asymmetry without complicating the procedure greatly. Actually, however, the amount of asymmetry depends on the ratio of the kinematic factors ρ_π and ρ_η corresponding to the two channels. If $N_{\pi\pi}$ is proportional to $N_{\pi\eta}$ for all energies greater than the $\pi+K$ rest mass, and $N_{\eta\eta}$ is proportional to $N_{\pi\eta}$ for all energies greater than the $\eta+K$ rest mass, the $(s-s_t)$ terms of Eq. (11) and the corresponding equation for $T_{\eta\pi}$ vanish, and $T_{\pi\eta}=T_{\eta\pi}$. Hence, one should choose the ratio ρ_η/ρ_π so that the N_{ij} are as nearly proportional as possible. [A common function multiplying both ρ_π and ρ_η makes no difference in the results, since it leaves the products $\rho_i^2N_{ij}$ occurring in Eq. (12) unchanged.] Our choice of $\rho_i=q_i^{3/2}$, which eliminates the singularities at the $\pi+K$ and $\eta+K$ thresholds, leads to N_{ij} that are nearly proportional. This is shown in Fig. 3, where the functions $\mathfrak{N}_{ij,l}$ corresponding to the four V-meson exchange graphs of Fig. 1 (computed by using physical values for m_ρ^2 and M^2) are compared. In view of this approximate proportionality, we simplify the equations by making the approximation in which the $\mathfrak{N}_{ij,l}$ are all proportional to $\mathfrak{N}_{\pi\eta}$. We make the replacement

$$\mathfrak{N}_{ij,l}(s)\rightarrow\kappa_{i,l}\mathfrak{N}_{\pi\eta}(s), \quad (13)$$

where the constants $\kappa_{i,l}$ are defined by

$$\kappa_{i,l}=\frac{\mathfrak{N}_{ii,l}[(m_\eta+m_K)^2]}{\mathfrak{N}_{\pi\eta}[(m_\eta+m_K)^2]}.$$

The $\eta+K$ threshold is chosen for the definition of the κ because it is an intermediate energy for this problem. If the physical values of m_ρ^2 and M^2 are used, none of the actual $\mathfrak{N}_{ii,l}(s)/\mathfrak{N}_{\pi\eta}(s)$ ratios varies by more than 22% from its value at $\eta+K$ threshold.

The matrix amplitude T is symmetric in the approximation, and may be written in the form

$$T_{ij}=\mathfrak{N}_{\pi\eta}(s)R_{ij}(s)/|D(s)|, \quad (14)$$

$$R_{\pi\eta}=\gamma_{K^*\pi}\gamma_{K^*\eta}, \quad (15a)$$

$$R_{\pi\pi}=-\tfrac{1}{3}\kappa_\pi\gamma_{K^*\pi}^2+\sqrt{2}\kappa_\rho\gamma_{\rho\pi}\gamma_{\rho K}+\alpha_\eta(s)H, \quad (15b)$$

$$R_{\eta\eta}=\kappa_\eta\gamma_{K^*\eta}^2+\alpha_\pi(s)H, \quad (15c)$$

$$|D|=1-\alpha_\pi(s)[-\tfrac{1}{3}\kappa_\pi\gamma_{K^*\pi}^2+\sqrt{2}\kappa_\rho\gamma_{\rho\pi}\gamma_{\rho K}]-\alpha_\eta(s)\kappa_\eta\gamma_{K^*\eta}^2-\alpha_\pi(s)\alpha_\eta(s)H, \quad (15d)$$

$$\alpha_i=\frac{(s-s_t)}{\pi}\int_{(m_K+m_i)^2}^{\infty}\frac{ds'q_i'^3\mathfrak{N}_{\pi\eta}(s')}{(s'-s_t)(s'-s-i\epsilon)s'^{1/2}}, \quad (15e)$$

[18] The exact solutions are symmetric, however. See J. D. Bjørken and M. Nauenberg, Phys. Rev. 121, 1250 (1961).

$$H=\gamma_{K^*\pi}{}^2\gamma_{K^*\eta}{}^2(1+\tfrac{1}{3}\kappa_\pi\kappa_\eta)-\sqrt{2}\kappa_\eta\kappa_\rho\gamma_{K^*\eta}{}^2\gamma_{\rho\pi}\gamma_{\rho K}. \quad (15f)$$

The symbols $\kappa_{\pi,\rho}$, κ_{π,K^*}, and κ_{η,K^*} have been abbreviated to κ_ρ, κ_π, and κ_η, respectively.

This approximate form of the amplitude equations has several advantages. One is that some of the self-consistency equations (derived in Sec. II C) are simple algebraic equations. Another is that only the factors α and κ of Eqs. (14) and (15) depend on the PS and V-meson mass differences, so that the effects of these mass differences on the symmetry of the interaction constants may be studied conveniently.

C. The Self-Consistency Equations

The application of the bootstrap mechanism requires that a resonance occurs in the derived amplitudes, which is identified with the K^*. We assume the physical situation in which the resonance occurs at an energy above the $\pi+K$ threshold but below the $\eta+K$ threshold. The function $\alpha_\eta(s)$ is real for such an energy, so that the complex nature of the amplitudes in Eqs. (14) and (15) arises entirely through their dependence on the complex function $\alpha_\pi(s)$. It is convenient to define $\alpha_{\pi,r}$ to be the real part of α_π, and to add a subscript r on the T_{ij}, R_{ij}, and D to denote the real expressions that result if α is replaced by α_r. We define the resonance energy s_0 as the energy at which the $\pi+K$ phase shift increases through 90°. Hence, $\mathrm{Re}T_{\pi\pi}(s_0)=0$, and since $\mathfrak{N}_{\pi\eta}(s_0)R_{\pi\pi}(s_0)$ is real, this requires that $\mathrm{Re}|D(s_0)|=|D_r(s_0)|=0$. The coupling constants are defined by an equation analogous to (4), i.e.,

$$\gamma_{K^*i}\gamma_{K^*j}/(4\pi)=\tfrac{3}{8}[(s_0-s)T_{ij,r}]_{s=s_0}. \quad (16)$$

The $\gamma_{K^*i}{}^2$ may also be interpreted as the reduced partial widths of the resonance.

The above definition of the coupling constants requires a little clarification. The amplitudes $T_{\pi\pi}$ and $T_{\pi\eta}$ are purely imaginary at the resonance energy; for these amplitudes Eq. (16) is an obvious definition. However, $T_{\eta\eta}$ is complex at $s=s_0$ because of the complex function α_π in $R_{\eta\eta}$. Our definition of $\gamma_{\eta\eta}$ specifies that $\mathrm{Im}R_{\eta\eta}$ be neglected, so that only the imaginary part of $T_{\eta\eta}$ is considered. The validity of this procedure may be seen from the fact that the condition $(s_0-s)^2T_{\pi\eta,r}{}^2=(s_0-s)^2T_{\pi\pi,r}T_{\eta\eta,r}$, implied by Eq. (16), is automatically satisfied for the imaginary parts of the three amplitudes below $\eta+K$ threshold because of the form of the unitarity condition, $\mathrm{Im}T_{ij}=(q_\pi{}^3/s^{1/2})T_{i\pi}{}^*T_{j\pi}$.

In order for a resonance to occur at $s=M^2$, we must have

$$|D_r(M^2)|=0. \quad (17)$$

Applying the definition γ_{K^*i} [Eq. (16)], to the amplitudes, we obtain three relations, which may be written in the form,

$$\frac{\gamma_{K^*i}\gamma_{K^*j}}{4\pi}=-\frac{3}{8}\left[\frac{\mathfrak{N}_{\pi\eta}(s)R_{ij,r}(s)}{\partial|D_r|/\partial s}\right]_{s=M^2}, \quad (18)$$

where the notation is that of Eq. (14). One of the four conditions given in Eqs. (17) and (18) may be derived from the other three; hence, these equations represent three self-consistency requirements on the four parameters M^2, $\gamma_{K^*\pi}{}^2$, $\gamma_{K^*\eta}{}^2$, and $\gamma_{\rho\pi}\gamma_{\rho K}$.

The simplest of the equations represented by Eq. (18) is the one corresponding to the inelastic amplitude $T_{\pi\eta}$. If we divide this equation by $\gamma_{K^*\pi}\gamma_{K^*\eta}$ (which we assume is not zero) the result is

$$1=\tfrac{3}{2}\pi(M^2-s_t)\lambda^{-1}\mathfrak{N}_{\pi\eta}(M^2), \quad (19)$$

where λ is defined by the relation $\lambda=-(M^2-s_t)\times(\partial|D_r|/\partial s)_{s=M^2}$. The convenience of this parameter will become clear shortly. Equations (17) and (18) [with the parameters R_{ij} given by Eq. (15)] may be combined to give the relations

$$1=\alpha_{\pi,r}(M^2)\gamma_{K^*\pi}{}^2+\alpha_\eta(M^2)\kappa_\eta\gamma_{K^*\eta}{}^2, \quad (20a)$$

$$1=\alpha_\eta(M^2)\gamma_{K^*\eta}{}^2+\alpha_{\pi,r}(M^2)[\sqrt{2}\kappa_\rho\gamma_{\rho\pi}\gamma_{\rho K}-\tfrac{1}{3}\kappa_\pi\gamma_{K^*\pi}{}^2]. \quad (20b)$$

These two equations represent the self-consistency equations for the γ^2 in a convenient form.

III. RESULTS

Since we have one more input than output parameter, our solutions depend on one adjustable parameter, which we take to be $\gamma_{K^*\pi}{}^2$. However, Eq. (19) is very insensitive to the choice of $\gamma_{K^*\pi}{}^2$ and may be thought of as the equation for the K^* mass. The only quantity in Eq. (19) that depends on $\gamma_{K^*\pi}{}^2$ is the parameter λ, but λ is very nearly one for all values of $\gamma_{K^*\pi}{}^2$. (If $|D_r|$ were a linear function of s, λ would be equal to one.) The actual dependence of λ on $\gamma_{K^*\pi}{}^2$ is shown in Table I.

TABLE I. Calculated values of $\gamma_{K^*\eta}{}^2$, $\gamma_{\rho\pi}\gamma_{\rho K}$, and λ corresponding to chosen values of $\gamma_{K^*\pi}{}^2$.

$\gamma_{K^*\pi}{}^2/4\pi$	$\gamma_{K^*\eta}{}^2/4\pi$	$\sqrt{2}\gamma_{\rho\pi}\gamma_{\rho K}/4\pi$	λ
0.5	3.45	0.88	0.98
1.0	2.78	1.32	0.98
1.5	2.10	1.77	0.97
1.75	1.76	1.99	0.96
2.0	1.42	2.21	0.96
2.5	0.74	2.66	0.95

We now show that the experimental value of the K^* mass very nearly satisfies Eq. (19). We take for the masses of the various mesons $M=885$ MeV, $m_\rho=750$ MeV, and $m_\eta=550$ MeV. The constants κ may be determined from Fig. 3; they are $\kappa_\rho=1.27$, $\kappa_\pi=0.93$, and $\kappa_\eta=1.22$. We must next decide on an appropriate value for the subtraction energy s_t. In a one-channel problem an appropriate energy is that of the end of the left-hand cut, but we have several left-hand cuts. The ends of the cuts for the processes corresponding to Figs. 1(b) and 1(c) are, in units of $m_K{}^2$, $s=-1.04$ and $s=1.26$, respectively. The corresponding cuts for processes 1(a) and 1(d) include complex regions as well as regions on the real axis; the "ends" of these cuts [points where the argument of the logarithm in Eq.

(7d) vanishes] are $s=-0.1\pm0.9i$ and $s=0.4\pm6.3i$, respectively. Rather arbitrarily we take s_t to be the average of the cut ends for processes 1(b) and 1(c), i.e., $s_t=0.11m_K^2$. If we substitute this value of s_t into Eq. (19) and evaluate $\mathfrak{N}_{\pi\eta}(M^2)$, the right side of Eq. (19) is equal to $0.95\lambda^{-1}$. Since $\lambda\sim1$ $(0.95<\lambda<1$ for reasonable choices of $\gamma_{K^*\pi}^2)$, we see that this equation is very nearly satisfied at the physical value of M^2. The error is less than the variation that would result from different reasonable choices of s_t. Hence, we shall continue to set M^2 equal to the physical value.

The computed values and derivatives of $\alpha_{\pi,r}$ and α_η at $s=M^2$ are found to be $4\pi\alpha_{\pi,r}=0.328$, $4\pi\alpha_\eta=0.198$, $(M^2-s_t)\alpha_{\pi,r}'/\alpha_{\pi,r}=0.94$, and $(M^2-s_t)\alpha_\eta'/\alpha_\eta=1.21$. The values of $\gamma_{K^*\eta}^2$ and $\gamma_{\rho\pi}\gamma_{\rho K}$ corresponding to different values of $\gamma_{K^*\pi}^2$ are determined from Eqs. (20) and are shown in Table I. The $\gamma_{K^*\pi}^2$ is related to the experimental K^* full-width Γ by the formula $\gamma_{K^*\pi}^2/(4\pi)=\frac{3}{8}M^2\Gamma/q_\pi^3$. The experimental value[19] of $\Gamma\sim50$ MeV corresponds to $\gamma_{K^*\pi}^2/(4\pi)\sim0.65$.

As a further test of the self-consistency of our model, we must examine the forces in the $I=\frac{3}{2}$, $\pi+K$ state in order to verify that no resonance is expected in this state. Even though there is no $\eta+K$ channel of isotopic spin $\frac{3}{2}$, we write the equation for $T_{\pi\pi}(I=\frac{3}{2})$ in the form of Eq. (14), in order to facilitate the comparison between isotopic spins, i.e.,

$$T_{\pi\pi}(I=\tfrac{3}{2})=\mathfrak{N}_{\pi\eta}R_{\pi\pi}(I=\tfrac{3}{2})/D(I=\tfrac{3}{2}).$$

The expression for $R_{\pi\pi}(I=\frac{3}{2})$ is similar to Eq. (15b) except that the coefficients are changed, i.e.,

$$R_{\pi\pi}(I=\tfrac{3}{2})=\tfrac{2}{3}\kappa_\pi\gamma_{K^*\pi}^2-\tfrac{1}{2}\sqrt{2}\kappa_\rho\gamma_{\rho\pi}\gamma_{\rho K}.$$

This function is negative for the entire range of $\gamma_{K^*\pi}^2$ shown in Table I, indicating a repulsive force. Hence, our assumption of no input forces from the exchange of an isotopic spin-$\frac{3}{2}$ resonance is self-consistent, as well as consistent with experiment. [We note that if κ_ρ and κ_π are set equal to one, and the ratios of the γ_{li} are taken from unitary symmetry, the quantity $R_{\pi\pi}(I=\frac{3}{2})$ vanishes, as remarked in Ref. 11.]

IV. RELATIONSHIP TO UNITARY SYMMETRY AND CONCLUDING REMARKS

One of the purposes of this paper was to investigate the effects of the *PS* and *V*-meson mass differences on the argument of Ref. 11, in which bootstrap relations were used to predict the *PS-PS-V* meson interaction symmetry of the octet model of unitary symmetry. We have succeeded in isolating the main effects of the mass differences in the equations for the K^*; only the parameters κ and α of Eqs. (19) and (20) depend on these differences. The ratios κ_π/κ_η and $\alpha_{\pi,r}/\alpha_\eta$ measure the main effects of the $\pi-\eta$ mass difference on the dynamics of the K^* problem. Since the experimental masses are much more nearly degenerate for the *V* mesons than for the *PS* mesons, it is very encouraging that the κ_π/κ_η and $\alpha_{\pi,r}/\alpha_\eta$ are much closer to one than the mass ratio m_K/m_π. Furthermore, it seems likely that if the present technique were extended to the ρ and ω mesons, the unitary symmetry would not be broken by the *PS*-meson mass differences to such an extent as to be unrecognizable. For example, if we set $\gamma_{K^*\pi}^2$ equal to $\gamma_{K^*\eta}^2$ (in accordance with unitary symmetry), it is seen from Table I that $\sqrt{2}\gamma_{\rho\pi}\gamma_{\rho K}/(\gamma_{K^*\pi}^2)\sim1.13$, whereas the ratio $\frac{4}{3}$ is predicted by unitary symmetry.

It would be interesting to extend the calculations to the ρ and ω systems, in the hope of eliminating the arbitrary parameter that is present here. However, it seems highly unlikely that such a program would be as fortunate in predicting the ρ and ω masses as we were in predicting the K^* mass. The work of Zachariasen and Zemach shows that the ρ mass may be quite sensitive to other states beside states of two *PS* mesons[4]; it is likely that many states and many contributions to the force play a significant role in actually determining the *V*-meson masses and widths.

As discussed in Sec. III, the K^* width is not predicted by our model. However, if one sets $\gamma_{K^*\pi}^2=\gamma_{K^*\eta}^2$, in accordance with unitary symmetry, then $\gamma_{K^*\pi}^2/(4\pi)=1.75$, which is high compared to the experimental value of ~0.65. Thus, if unitary symmetry is approximately valid, the reduction of the K^* width caused by the coupling to the $\eta+K$ state is insufficient to bring about agreement with experiment. (See the discussion of Sec. I.) The situation is similar for the ρ meson. In the $\rho-\pi\pi-K\bar{K}$ calculation of Ref. 11 (with the $\pi-K$ mass difference neglected) the presence of the $K+\bar{K}$ state reduces the ρ width to $\frac{2}{3}$ the value occurring in the one-channel $\rho-\pi\pi$ model, but the result is still about three times too large. It may be that the *V*-meson widths are further reduced by coupling to states other than those of the *PS* mesons. The prospect of including many states in the calculations seems discouraging at first, since multiple-channel dispersion relations would have to be used. However, if the various strong interactions are related by a symmetry principle, and if this symmetry principle is itself derivable from dispersion relations, one can hope that a method of simplification based on this symmetry will be found, and that it will be possible to reduce the equations to tractable form.

In conclusion, we remark that the agreement between the K^* mass predicted by this model and experiment is encouraging, but many factors not included here may play a significant role in determining the K^* mass and width.

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