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Thermal Conduction in Liquid Helium II. II. Effects of Channel Geometry

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The critical heat-current density W_c in liquid helium II has been measured as a function of temperature in channels of various geometries. For channels of both circular and rectangular cross section, W_c varies almost inversely with channel diameter, but for the same hydraulic diameter W_c is about 25% smaller in rectangular channels. In channels with more irregular cross sections, W_c is much smaller than in cylindrical channels of the same hydraulic diameter. The hydraulic diameter is thus not a useful parameter for comparing data obtained in channels of different shape. The temperature dependence of W_c is in all cases similar to that reported in I for a cylindrical channel, and suggests that W_c may be associated with ordinary turbulence below about 1.7°K but at higher temperatures is caused by some other mechanism. The effects of varying the channel length and shape of orifice were also studied. Shortening one of the channels to 60% of its original length had no observable effect on W_c. W_c was also unaffected by moderate changes in orifice shape, but a severe constriction apparently disrupted the subcritical flow region so completely that W_c could not be measured at all. Various theories of the origin of the critical velocity are discussed in the light of these measurements.

I. INTRODUCTION

IN a previous paper[1] (hereafter referred to as I), measurements of the thermal conductivity and the critical heat-current density W_c of liquid helium contained in a channel of circular cross section were reported. The present paper deals with the dependence of W_c upon a number of geometrical properties of the containing channel: shape and size of cross section, length and shape of orifice. The characteristics of the nineteen different channels which were used in these experiments are described in Sec. II. With the exception of some of the measurements made in the smallest channels (*C*6 and *C*6*a*), all the results were obtained by means of the Vinen technique,[2] which involves the observation of the time τ required for the establishment of the equilibrium temperature gradient when a moderately large heat current is switched on under various initial conditions. The effects of channel geometry on τ have also been qualitatively observed. A full description of the method and experimental apparatus was given in I.

II. DESCRIPTION OF THE CHANNELS

The cross sections of the channels are shown schematically in Fig. 1, and they are described in Table I. The letter designations are intended to indicate the shape of the cross section. The channel used in the experiments described in I is here called *C*6. All the channels with the exception of *C*6*a* were 5.16 cm long; *C*6*a* was made by cutting *C*6 off to a length of 3.17 cm in order to investigate possible length effects. R_1 and R_2 were made of rectangular brass tubing; all the others were made of stainless steel. The inside surface of the tubes was not polished, and may have been moderately rough; no direct measurements of surface roughness were made. All the channels were vacuum jacketed, and were connected at one end to a small chamber containing a heater and resistance thermometer. The other end opened into the helium bath, where a second resistance thermometer was located. The channels were usually mounted horizontally, but in some cases they were mounted vertically such that the heat flow was upwards. The results were independent of channel orientation.

Seven basic channels were constructed, five of circular cross section and two rectangular. These channels are indicated in the table by boldface type. All the other channels were made by modifying these basic channels in some way; these modified channels are listed in the table below the basic channel from which they were made. One channel (*R*1) did not give any useful results because the heater and thermometer leads were introduced by means of glass-to-metal seals in the vacuum jacket that were inclined to leak. The leads in the other channels were either run through the channel itself (*C*1 and *C*2) or through closely fitting capillary tubes parallel to the channel, into which they were sealed with glyptal (*C*3, *C*6, *C*7, and *R*2). Possible errors due

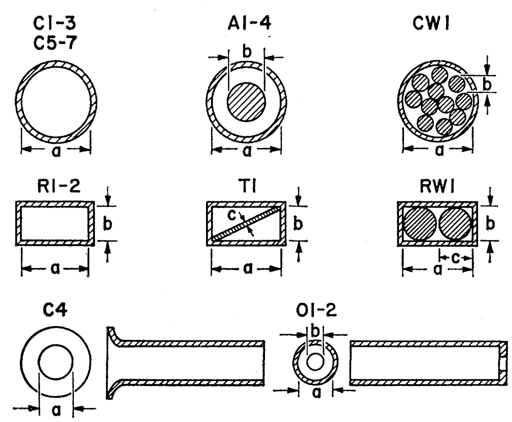


FIG. 1. Cross sections of channels used in the experiments. See Table I.

* Operated with support from the U. S. Army, Navy, and Air Force.
[1] C. E. Chase, Phys. Rev. **127**, 361 (1962).
[2] W. F. Vinen, Proc. Roy. Soc. (London) **A240**, 114, 128 (1957); **A242**, 493 (1957); **A243**, 400 (1957).

TABLE I. Characteristics of channels used in the experiments.[a]

Channel[b]	a(mm)[c]	b(mm)[c]	c(mm)[c]	$4A/p$(mm)	Method of construction
C1	2.62			2.62	[d]
*CW*1	2.62	0.25		0.10	70 stainless steel wires in *C*1
C2	4.04			4.04	[d]
*A*1	4.04	3.17		0.87	Steel rod in *C*2
*A*2	4.04	1.59		2.45	Steel rod in *C*2
*A*3	4.04	3.2		0.8	180-grit silicon carbide cemented to rod in *A*1
C3	4.04			4.04	[e]
*C*4	3.41			3.41	Liner in *C*3; end flared
*C*5	3.41			3.41	Liner in *C*3; end square
*O*1	3.41	1.59		3.41	Liner in *C*3; orifice at "hot" end
*O*2	3.41	1.59		3.41	Liner in *C*3; orifice at "cold" end
*A*4	4.04	3.17		0.87	Steel rod in *C*3
C6	0.80			0.80	[e]Nine parallel tubes
C6a	0.80			0.80	*C*6 shortened to 3.17 cm
C7	1.59			1.59	[e]
R1	1.09	0.41		0.60	Brass; leads through glass seals
R2	1.03	0.51		0.68	[e]Brass; nine parallel tubes
*RW*1	1.03	0.51	0.45	0.14	Two steel wires in each tube of *R*2
*T*1	1.03	0.51	0.07	0.34	Brass strips along diagonals in *R*2

[a] The basic channels used are indicated in boldface type.
[b] All channels except *C6a* are 5.16 cm long. All channels except *R*1, *R*2, *RW*1, and *T*1 are made of stainless steel tubing.
[c] See Fig. 1 for dimensions.
[d] Two 0.028-cm diam leads run through channel. These leads have been ignored in computing $4A/p$.
[e] Leads sealed into 0.033-cm diam capillaries.

to the latter method of introducing leads have been discussed in I. Channels *C*2 and *C*3 were identical except for the arrangement of the leads, and were designed to investigate this question experimentally. The smaller channels (*C*6 and *R*2) consisted of nine identical tubes in parallel; the purpose of this arrangement, as described in I, was to minimize the effects of stray heat currents resulting from bath-temperature variations by increasing the total cross-sectional area of the channel.

The annular channels (*A*1–*A*4) were formed by inserting stainless-steel rods, supported by three small points at each end, into *C*2 or *C*3. These rods were a little longer than the channel and protruded slightly at each end. Channel *A*3 was the same as *A*1 except for a layer of 180-grit silicon carbide cemented to the central rod.[3] *A*1 and *A*4 were identical except for the arrangement of the leads.

Channels *C*4, *C*5, *O*1, and *O*2 were intended to study the effects of changing the shape of the channel orifice, and consisted essentially of stainless-steel tubes, of inside diameter 0.341 cm, lapped to fit smoothly inside *C*3 for its entire length. In order to minimize the flow of superfluid through the resulting annular space, the insert was coated with a layer of vacuum grease. Comparison of the experimental results in *C*4 and *C*5 with those obtained in other cylindrical channels suggests that the errors introduced by such superfluid flow are rather small; in any case, results in these four channels, which were of similar construction, should be comparable.

Channels *T*1, *CW*1, and *RW*1 were designed for the investigation of more irregular cross sections. Channel *T*1 was made by inserting strips of 0.007-cm thick brass shim stock along the diagonals of the nine rectangular channels comprising *R*2 to give a triangular cross section. *CW*1 and *RW*1 were made by inserting 70 0.025-cm-diam stainless-steel wires in *C*1 and two 0.045-cm-diam stainless-steel wires in each of the channels of *R*2.

III. EXPERIMENTAL RESULTS

A. The Delay Time

Nearly all the measurements reported here were made by means of the Vinen technique, which involves the observation of the time τ required for the establishment of the equilibrium temperature gradient when a heat current is switched on. According to Vinen,[2] for initially undisturbed helium τ is given by the expression

$$\tau = aW^{-3/2}, \tag{1}$$

where a is a temperature-dependent parameter and W is the heat-current density. In the course of these experiments, a was measured in a few of the channels, and its temperature dependence is shown in Fig. 2. The curve labelled *C*1 is reproduced from an earlier publication.[4] For clarity, experimental points have been omitted from the curves for channels *C*1 and *A*3.

It was previously reported[4] that the value of a found in *C*1 was appreciably different from that found by Vinen in rectangular channels. Figure 2 confirms that a varies considerably from channel to channel, and in addition, shows that it does not depend in any simple way either on channel size or shape. About all that can be said is that, in general, a is smallest for channels of irregular cross section and is largest for the cylindrical

[3] This channel was used to investigate the importance of surface roughness. No appreciable effect was observed, but the experiment is considered inconclusive because, as is shown below, W_c is determined primarily by the *outer* channel walls and is relatively unaffected by obstructions along the channel axis. The results of these measurements will, therefore, not be discussed in detail.

[4] C. E. Chase, Phys. Rev. **120**, 688 (1960).

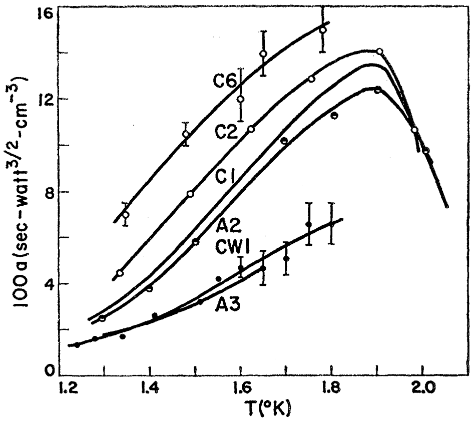


FIG. 2. Temperature dependence of the quantity a in the relation $\tau = aW^{-3/2}$, for various channels. Points have been omitted for $C1$ and $A3$. Results for $C1$ are quoted from Ref. 4.

channels. Until more is known about the factors determining the magnitude of a, therefore, little significance can be attached to the values of this quantity observed in a particular experiment.

B. Dependence on Channel Length

Possible effects of an "inlet length" $v_s\tau$ or $v_n\tau$ on measurements of W_c have been discussed by Vinen[2] and in I. Since the channels used in the present experiments are rather short, it is important to determine whether the observed thermal conductivity and critical heat current are dependent on channel length. In order to investigate this point, channel $C6a$ was made by shortening $C6$ from 5.16 to 3.17 cm.

In I it was shown that, for $W > W_c$, the temperature gradient obeys the relation

$$(\text{grad } T)^* = DW^n, \tag{2}$$

where (grad T)* is the observed temperature gradient minus the contribution due to normal-fluid viscosity and n is somewhat less than 3.5. A convenient way of illustrating deviations from this behavior, as well as emphasizing differences between $C6$ and $C6a$, is to plot the quantity (grad T)*/W^n against W, choosing some average value for n close to the value actually observed. This quantity is almost independent of W in the region where Eq. (2) is obeyed, deviations of n from the chosen average value showing up as small departures of the slope from the horizontal. Such a plot is shown in Fig. 3 for five different temperatures, where n has been set equal to 3.4. The small "bumps" at heat currents just above W_c have already been discussed in I. Apart from a barely discernible tendency for these bumps to be larger and to persist to higher temperatures in the shorter channel, it is clear that the behavior in $C6$ (shown by open circles) and $C6a$ (closed circles) is identical (the small difference in absolute value on the curve at 1.511°K is probably due to uncertainty in the thermometer calibrations, and lies within the anticipated limits of error). These measurements, therefore, show that both the critical heat-current density and the behavior at supercritical heat currents are independent of channel length over the range studied.

C. Effect of Orifice Shape

Various experiments[5–7] have shown that the high thermal resistance observed in the presence of supercritical heat currents often appears first at one end of the channel and then spreads along it with a well-defined velocity. This observation suggests that the shape of the channel orifice might have an important effect upon W_c. The following measurements were undertaken with the object of investigating this question. The channels used for this purpose ($C4$, $C5$, $O1$, and $O2$) were all made by inserting a closely fitting stainless-steel tube, of the same length as the channel, into $C3$. A brass plate soldered to one end of this tube contained an opening that formed the channel orifice. In $C4$ this orifice, which was located at the "cold" end of the channel, was smoothly rounded off to a trumpet-like shape in order to reduce the velocity discontinuity experienced by the superfluid entering the channel. In its companion channel, $C5$, the insert was cut off squarely so that the orifice had sharp corners like all the other channels described in this paper. Channels $O1$ and $O2$ were made with a single insert having a 0.16-cm-thick brass plug with a 0.159-cm-diam hole in its center soldered into one end. In channel $O1$ this constriction was placed at the "hot" end of the channel, whereas in $O2$ it was placed at the "cold" end.

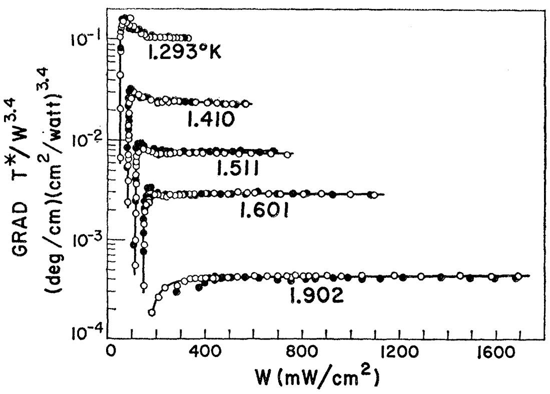


FIG. 3. The quantity (grad T)*/$W^{3.4}$ as a function of W. ○ Channel $C6$; length = 5.16 cm (results taken from I). ● Channel $C6a$; length = 3.17 cm.

[5] K. Mendelssohn and W. A. Steele, Proc. Phys. Soc. (London) A73, 144 (1959).

[6] G. Careri, F. Scaramuzzi, and W. D. McCormick, in *Proceedings of the Seventh International Conference on Low-Temperature Physics, Toronto, 1960* (University of Toronto Press, Toronto, 1961), p. 502.

[7] V. P. Peshkov and V. K. Tkachenko, Zh. Eksperim. i Teor. Fiz. **41**, 1427 (1961) [translation: Soviet Phys.—JETP **14**, 1019 (1962)]; in Proceedings of the Eighth International Conference on Low-Temperature Physics, London, 1962 (to be published), Paper 2.1.

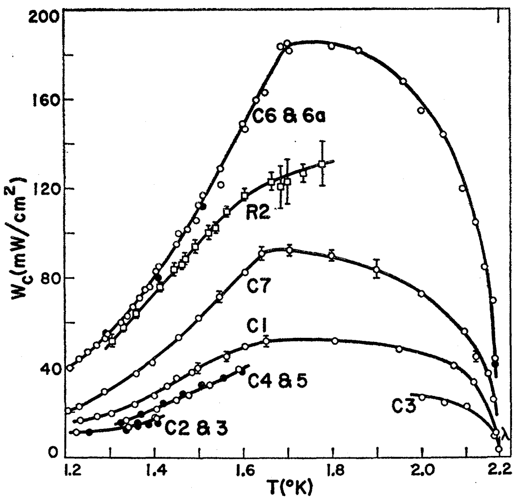


FIG. 4. Critical heat-current density W_c as a function of temperature in channels $C1$–7 and $R2$. Solid points correspond to $C2$, $C4$, and $C6a$.

The results obtained with $C4$ and $C5$, which are included with the other cylindrical channels in Fig. 4 of Sec. III D, are virtually identical. It therefore appears that such moderate changes in orifice shape have no effect on the critical heat current. The behavior of channels $O1$ and $O2$ was, however, very different. The equilibrium time τ was so short in these channels that it was impossible to observe W_c at all; there was no observable difference between $O1$ and $O2$ in this respect. This suggests that such a constriction either reduces W_c to a very small value, or acts in some way to trap turbulence in the channel so that once W_c has been exceeded the recovery time is very long. Further experiments are clearly required before the behavior of such channels can be understood.

D. The Critical Heat-Current Density

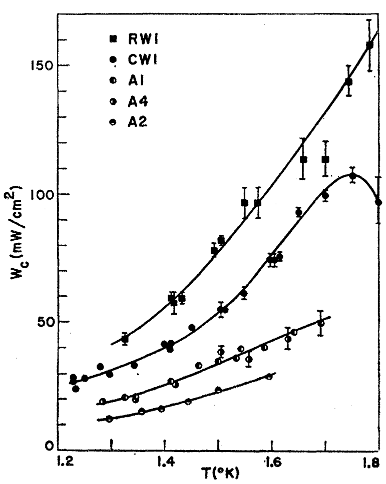
In channels $C6$ and $C6a$, W_c was large enough so that it could be observed directly at nearly all temperatures as an abrupt change in thermal resistance. In the wider channels, however, this was not the case, and W_c could only be measured indirectly by means of the Vinen technique.[2] The problem of assigning a value to W_c by this method when the curve of $\tau(W_1)$ does not exhibit a sharp break was discussed in I. There it was shown, by comparison with direct observations of W_c, that the abrupt change in thermal resistance corresponds not to the point at which τ starts to fall with increasing heat current, but rather to the *bottom* of this decline, where τ levels off at a value of about 0.2 sec. The same criterion has been adopted in the present case. However, in the wider channels the variation of τ is sometimes so gradual that it is impossible to assign any value to W_c at all. This occurs only at intermediate temperatures; even in the widest channels, W_c could always be resolved at the lowest temperatures and near the λ point. In $C2$ and $C3$, however, W_c could not be detected at all between about 1.4 and 2°K.

Figure 4 shows the temperature dependence of W_c in the cylindrical channels $C1$–$C7$ and the rectangular channel $R2$. Measurements near the λ point were made only in a few of the cylindrical channels; the rest were studied only in the low-temperature region. It is to be noted that the temperature dependence of W_c is essentially the same in all the cylindrical channels, although there are some small differences in detail. The "knee" occurring at 1.7°K in $C6$, which was reported in I, is also evident in the curves for $C1$ and $C7$, but it appears to shift to slightly lower temperatures in the larger channels. There is no significant difference between the results obtained in $C2$ and $C3$, which differed only in the method of introducing the heater and thermometer leads. Barring the unlikely possibility that these two different arrangements introduce identical errors in the results, it therefore appears that the effect of the leads can be neglected. The shape of the curve obtained with the rectangular channel $R2$ is rather different from those obtained with the cylindrical channels, but it also appears to change slope abruptly between 1.6 and 1.7°K.

Figure 5 shows the results obtained in channels $A1$, $A2$, $A4$, $CW1$, and $RW1$, all of which have multiply connected cross sections. No measurements were made in any of these channels above 1.8°K. The temperature dependence of W_c is qualitatively the same as in the cylindrical and rectangular channels, but for a given hydraulic diameter (four times the ratio of the cross-sectional area A to the perimeter p) W_c is relatively much smaller. This will be discussed in more detail below. The good agreement between $A1$ and $A4$ provides further confirmation that the effect of the leads can be neglected. Only a few qualitative results were obtained in channel $T1$, because for some reason W_c was hard to resolve in that channel. These results, which have been omitted from the figure, lie midway between those obtained in $R2$ and $RW1$.

Figure 6 shows W_c plotted logarithmically as a function of the hydraulic diameter at a temperature of 1.4°K. The results in channels of circular cross section

FIG. 5. Critical heat-current density W_c as a function of temperature in channels $A1$, $A2$, $A4$, $CW1$, and $RW1$.



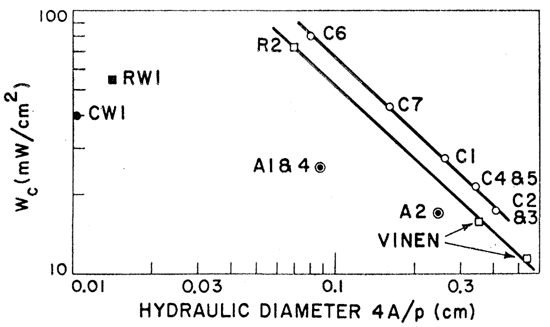


FIG. 6. Critical heat-current density W_c as a function of the hydraulic diameter, $4A/p$, at 1.4°K. Points labeled "Vinen" are quoted from Ref. 2. The area and perimeter of leads through the channels have been neglected in computing the hydraulic diameter for $C1$, $C2$, and Vinen's channels. The slope of the solid lines is approximately -0.93.

all lie on a straight line of slope very nearly -1, in agreement with the well-known fact that the critical velocity varies inversely with channel diameter in wide channels.[8] Vinen's[2] results in rectangular channels have been included in the figure and fall together with $R2$ on a parallel but somewhat lower line. It thus appears that the hydraulic diameter is not a very useful parameter for comparing results obtained in channels of different shape. This situation is even more pronounced in the case of channels with more complicated cross sections; for the most extreme case, $CW1$, W_c is about a factor of 15 smaller than it would be in a cylindrical channel with the same hydraulic diameter.

A possible interpretation of this result is that W_c is determined mostly by the *outer* boundaries of the channel. The insertion of rods into $C2$ and $C3$, giving the annular channels $A1$–4, caused only a slight increase in W_c, and roughening the inner boundary (in channel $A3$) had no effect at all. Even filling $C1$ with 70 wires to construct $CW1$ raised W_c by less than a factor of two, and the conversion of $R2$ into $RW1$ actually *lowered* W_c. This circumstance provides some retrospective justification for the rather inelegant method of bringing leads directly through the channel, which was done in some of the wider channels, and explains the fact that these leads have no observable influence on W_c. Its significance with respect to various theories of the origin of W_c will be discussed below.

IV. DISCUSSION

A. The Critical Reynolds Number

In I, it was suggested that below about 1.7°K W_c could be explained in terms of a critical Reynolds number $\mathcal{R}_1=\rho|\mathbf{v}_n-\mathbf{v}_s|d/\eta_n$ or $\mathcal{R}_2=\rho|\mathbf{v}_n|d/\eta_n$, indicating that W_c might be connected with the onset of a more-or-less classical kind of turbulence. Figure 7 shows the temperature dependence of $\mathcal{R}_2$, computed in terms of the hydraulic diameter, for $R2$, $A1$, $RW1$, $CW1$, and the cylindrical channels. The other channels have been omitted for clarity, since the above illustrate the essential features of the data. The behavior of the cylindrical channels is essentially the same as that reported in I, although there appears to be some slight systematic dependence of $\mathcal{R}_2$ on diameter. It is possible that this is evidence for a length effect too small to have been observed in the measurements of Sec. III B. In the rectangular channel, $\mathcal{R}_2$ is about 25% smaller. Although in ordinary fluids the critical Reynolds number is relatively insensitive to changes in channel shape, this difference is not inconsistent with observations of the flow of water in rectangular channels.[9] In the multiply connected channels $\mathcal{R}_2$ is very much smaller. Relatively few data exist on the flow of ordinary liquids in such channels, and there is no real evidence that the critical Reynolds number for the onset of ordinary turbulence should be the same as in the case of channels whose cross sections are simply connected. On the contrary, Lonsdale[10] reported that, for the flow of water in an annular pipe, the quantity flowing per unit time at the critical velocity was proportional to the outer perimeter, and relatively independent of the inner dimension. For such a case, the critical Reynolds number expressed in terms of the hydraulic diameter would be reduced below that for a circular pipe by the ratio of the total wetted perimeter to the outer perimeter. This is qualitatively what is observed for helium II in the annular channels, although in this case the reduction in $\mathcal{R}_2$ is somewhat greater. If Lonsdale's relation also holds for channels with more complicated multiply connected cross sections, $\mathcal{R}_2$ should be a factor of 8 lower in $CW1$ than in the cylindrical channels, as compared with the experimentally observed factor 15. Considering that Lonsdale's relation is a purely empirical one, this is perhaps as good agreement as one might expect. It thus appears that these measurements are in no way inconsistent with an explanation of W_c in terms of the onset of turbulence at low temperatures.

In a flow experiment designed so that $\mathbf{v}_n\approx\mathbf{v}_s$, Staas,

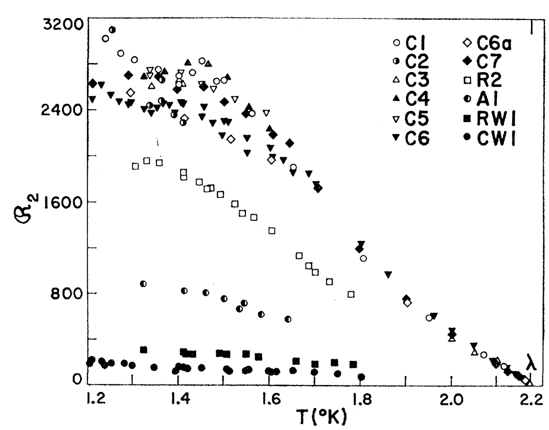


FIG. 7. Critical Reynolds number $\mathcal{R}_2=\rho|\mathbf{v}_n|d/\eta_n$ as a function of temperature.

[8] K. R. Atkins, *Liquid Helium* (Cambridge University Press, Cambridge, England, 1959), p. 199.

[9] H. Schlichting, *Boundary Layer Theory* (McGraw-Hill Book Company, Inc., New York, 1960), 4th ed., p. 517.

[10] T. Lonsdale, Phil. Mag. 46, 163 (1923).

Taconis, and van Alphen[11] often observed the onset of turbulence at $\mathcal{R}_2 \approx 1200$, about half the more usual values reported here at low temperatures. These results are not necessarily contradictory, because the boundary and flow conditions in the two cases are quite different. In particular, at low temperatures $\mathbf{v}_s$ is much larger in Staas' experiment than in the present one, and it is possible to imagine mechanisms whereby turbulence could then be supported at lower Reynolds numbers than is ordinarily the case.

B. Theories of Superfluid Turbulence

The measurements reported in this paper have been shown to be consistent with the idea that W_c below 1.7°K is the result of a transition to turbulence described by a Reynolds number involving either the normal fluid velocity or the relative velocity together with the total density. Perhaps the most telling point in favor of such an explanation is that it predicts both the absolute magnitude and the temperature dependence of W_c within about 20% without the aid of any adjustable parameters. Nevertheless, it is necessary to consider the possibility that this agreement is fortuitous. In this section we, therefore, discuss briefly other theories of the critical heat current.

Vinen[2] developed a detailed theory to account for his results, based on the idea that when $W > W_c$, the superfluid contains a tangled mass of vortex line which interacts with the normal fluid to produce mutual friction. In this theory, W_c itself is determined by a balance between the creation of vortex line, supposed to proceed throughout the body of the fluid, and its annihilation at the walls. Among the attractive features of this idea are its ability to account qualitatively for the long persistence of the supercritical state and the hysteresis which some authors[5,12,13] have observed. The predicted dependence on temperature is also qualitatively, but not quantitatively, correct. However, in this theory W_c should vary inversely with the hydraulic diameter, and should not depend on channel shape. In principle, it might be possible to make the annihilation process depend appropriately on the shape of the channel boundaries as well as their extent. Until some such refinement of the model is carried out, however, it is incapable of explaining the shape dependence of W_c.

Feynman[14] proposed that, qualitatively, W_c might result from the formation of vortex rings at the channel orifice. This model was further developed by Peshkov,[15] who assumed that vortex rings can be formed inside the channel when the superfluid velocity exceeds the value $v_c = E/p$, where E is the energy and p the impulse of the ring. Since for unbounded rings E varies almost linearly with diameter and p quadratically, the critical velocity should correspond to the formation of the largest rings possible—almost as large as the channel. Peshkov's theory contains three adjustable parameters; with reasonable values of these quantities, it explains satisfactorily the dependence of W_c on channel diameter over a very wide range. Moreover, it provides a straightforward reason for the behavior of channels with multiply connected cross sections: Since the vortex rings are supposed to be formed close to the outer channel wall, their energy and impulse would not be affected very much by the presence of additional boundaries close to the channel axis, and the effect of such interior boundaries would be small. There are, however, some difficulties with this theory. First, it provides no explanation of the temperature dependence of the critical velocity. Second, it is difficult to see how such large vortex rings could be formed in the *interior* of a smooth channel, although they might be formed at the orifice. Finally, if the effect of the channel walls on the vortex rings is taken into account,[16] it turns out that E falls toward zero as the ring diameter approaches that of the channel, and the resulting value of v_c is zero (or, at least, much smaller than the observed value, even if nonclassical behavior near the core of the vortex be considered). In order to overcome the latter difficulty, it would presumably be necessary to introduce additional assumptions to prevent the formation of rings too near the walls, or to explain why such rings, if formed, do not contribute to the friction.

All the above models have in common the feature that the supercritical state is characterized by some sort of turbulence, but differ in the details of its inception and in whether it is a property of the whole fluid or of the superfluid only. At present, the description in terms of the critical Reynolds number $\mathcal{R}_2$ seems to the author to come closest to providing a satisfactory picture of the phenomena at low temperatures. However, such a description must be incomplete, because it is totally inadequate above 1.7°K. It seems clear that critical velocities, quite generally, are associated with a transition to some sort of turbulent state (which may or may not be describable in terms of ordinary hydrodynamics). The details of the origin and structure of this state are, however, still uncertain.

ACKNOWLEDGMENTS

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[11] F. A. Staas, K. W. Taconis, and W. M. van Alphen, Physica **27**, 893 (1961).
[12] D. F. Brewer, D. O. Edwards, and K. Mendelssohn, Phil. Mag. **1**, 1130 (1956).
[13] D. F. Brewer and D. O. Edwards, Phil. Mag. **6**, 775 (1961).
[14] R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (Interscience Publishers, Inc., New York, 1955), Vol. **1**, p. 19.
[15] V. P. Peshkov, in *Proceedings of the Seventh International Conference on Low-Temperature Physics, Toronto, 1960* (University of Toronto Press, Toronto, 1961), p. 555; Zh. Eksperim. i Teor. Fiz. **40**, 379 (1961) [translation: Soviet Phys.—JETP **13**, 259 (1961)].
[16] J. C. Fineman and C. E. Chase, in Proceedings of the Eighth International Conference on Low-Temperature Physics, London, 1962 (to be published), Paper 2.6; Phys. Rev. **129**, 1 (1963).