PHYSICAL REVIEW VOLUME 131, NUMBER 5 1 SEPTEMBER 1963

Magnetoresistance of Copper Single Crystals*

A. J. FUNES† AND R. V. COLEMAN
Physics Department, University of Virginia, Charlottesville, Virginia
(Received 29 April 1963)

The anisotropy in the magnetoresistance of copper single crystals has been used to determine the Fermi-surface topology in copper. In particular, the {111} neck thickness in the [1$\bar{1}$0] and [11$\bar{2}$] directions has been determined and found to be 0.29 $(a/2)$ and $0.26(a/2)$, respectively, where a is the reciprocal lattice parameter. Sixteen selected crystals have been measured and the data used to obtain information on the open-orbit regions.

ANISOTROPY in the magnetoresistance of single crystals can be used to obtain information on the Fermi-surface topology of metals, particularly those with open Fermi surfaces such as copper, silver, and gold. Lifshitz *et al.*[1–3] have given the theoretical basis for analysis of magnetoresistance data in terms of open and closed orbits on the Fermi surface. Alekseevskii and Gaĭdukov[4,5] have used single crystals of gold and silver to obtain detailed stereographic projections of the special directions of the magnetic field for which open orbits exist in these metals.

In this investigation we have obtained transverse magnetoresistance data on a large number of single crystals of copper of selected orientations in order to obtain a detailed stereographic projection of the special magnetic field directions in copper. Specimens were cut from large single crystals of copper $\frac{7}{8}$ in. in diameter grown by the Bridgman technique. Specimens with 1 mm^2 cross sections and lengths of 20 to 30 mm were obtained by cutting the crystal into oriented wafers 1 mm thick and then slicing the wafers into specimens 1 mm wide. Orientations were selected with back-reflection Laue techniques to an accuracy of $\frac{1}{2}°$. The cutting of the crystals was done with an acid saw using 10-mil stainless steel wires. Negligible damage was done to the crystals by the sawing process.

Sixteen specimens with axial orientations listed in Table I were measured in a transverse field of 13 500 G. The potential drop across the specimen was measured with a microvolt potentiometer, the unbalance of which was fed into a photoelectric galvanometer and dc amplifier system driving a chart recorder. The sensitivity of the system was 5×10^{-9} V. All meausrements were made at 4.2°K in a bath of liquid helium. Ratios of the room-temperature resistance to the resistance at 4.2°K ranged from 1500 to 1800. Typical rotation diagrams obtained are shown in Fig. 1. The high double peaks occurring symmetrically about the low-index poles [001], [110], and [111] indicate that the Fermi surface is open along the [001], [110], and [111] directions as is also found for gold and silver.[4,5] The resistivity as a function of field has been measured for the peak positions and found to be proportional to H^n where $n=1.8$–2.0, in substantial agreement with the theory of Lifshitz and Peschanskiĭ.[2] The angular distance between the peaks has been used to plot the two-dimensiinal regions of special magnetic field directions located around the low-index poles as shown in Fig. 2. The special field directions for which high maxima in resistance are observed are shown by the solid dots and the directions of deep minima by the open circles. The axes of the crystals used are indicated by ×'s. The major dimensions of the two-dimen-

TABLE I. Orientations of crystals used and measured angles between peaks symmetrically located about low-index poles.

Sample no.	Sample axis	Angle between symmetrical peaks at poles ⟨001⟩ (deg)	⟨110⟩ (deg)	⟨111⟩ (deg)
19	⟨100⟩	22.7	22.7	
13	⟨110⟩	17.7	30	6.6
14	⟨111⟩		24.4	
15	⟨211⟩		18.9	8.5
17	⟨311⟩		18.5	
18	⟨511⟩		20	
16	⟨221⟩		24.4	
26	⟨331⟩		26.3	
27	⟨551⟩		26.7	
12	⟨210⟩	18.1		
20	⟨310⟩	18.1		
22	$\langle 111\rangle \xrightarrow{5°} \langle 221\rangle$		21.2	
25	$\langle 111\rangle \xrightarrow{10°} \langle 221\rangle$		21	
23	$\langle 111\rangle \xrightarrow{6.5°} \langle 211\rangle$		20	
24	$\langle 111\rangle \xrightarrow{13°} \langle 211\rangle$		19.1	
28	$\langle 110\rangle \xrightarrow{4°} \langle 551\rangle$		28.9	

* Research supported by U. S. Office of Naval Research and U. S. Office of Ordinance Research.
† A. J. F. wishes to thank the Argentine Navy for generous support.
[1] I. M. Lifshitz, M. Ia. Azbel, and M. I. Kaganov, Zh. Eksperim. i Teor. Fiz. **31**, 63 (1956) [translation: Soviet Phys.—JETP **4**, 41 (1957)].
[2] I. M. Lifshitz and V. G. Peschanskiĭ, Zh. Eksperim. i Teor. Fiz. **35**, 125 (1958) [translation: Soviet Phys.—JETP **8**, 875 (1959)].
[3] I. M. Lifshitz and V. G. Peschanskiĭ, Zh. Eksperim. i Teor. Fiz. **38**, 188 (1960) [translation: Soviet Phys.—JETP **11**, 137 (1960)].
[4] Yu. P. Gaĭdukov, Zh. Eksperim. i Teor. Fiz. **37**, 1281 (1959) [translation: Soviet Phys.—JETP **10**, 913 (1960)].
[5] N. E. Alekseevskii and Yu. P. Gaĭdukov, Zh. Eksperim. i Teor. Fiz. **42**, 69 (1962) [translation: Soviet Phys.—JETP **15**, 49 (1962)].

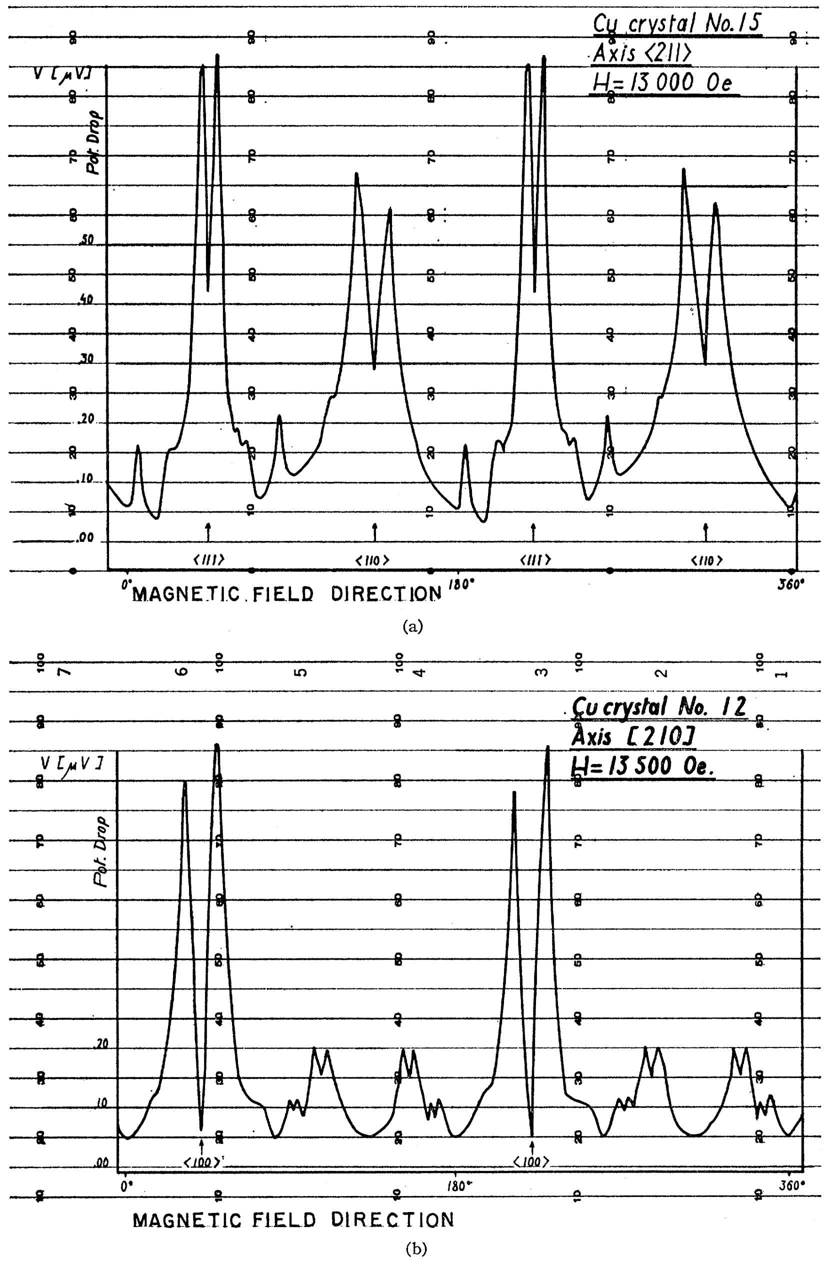

FIG. 1(a) Recorder trace obtained for ⟨211⟩ current axis. Symmetrical peaks occur as magnetic field sweeps through ⟨110⟩ and ⟨111⟩ poles. (b) Recorder trace for ⟨210⟩ current axis. Peaks occur symmetrically about ⟨100⟩ poles.

sional regions are [001]—23 and 18°, [110]—23 and 30°, and [111]—8.5 and 6.5°.

If the angle from the [$1\bar{1}0$] pole at which the resistance is a maximum is called θ, and if this occurs when a "hole" (dog-bone) orbit is just possible, then a polar plot of $\tan\theta$ as a function of current direction in the ($1\bar{1}0$) plane should be a rhomb as shown in Fig. 3. The dog-bone orbit is formed when the plane of the orbits just cuts the four necks at the corners of a rectangle in the ($1\bar{1}0$) plane. The formula for the neck thickness is $\tan\theta \sin\phi = (t/\sqrt{3})/(a/2)$, where ϕ is the angle between the current direction and the [$11\bar{1}$] diagonal of the rectangle in the ($1\bar{1}0$) plane and t is the thickness of the neck. The sides of the rhomb are parallel to the ⟨111⟩ diagonals. The experimental maxima for 14 crystals are shown in the figure with the rhomb corresponding to a neck thickness in the [$1\bar{1}0$] direction of the (111) neck of 0.29 $(a/2)$, where a is the lattice parameter of the bcc reciprocal lattice.

The same diagram for the [001] pole should be a square and this has been compared with experimental points for 4 crystals with axes in the (001) plane. The corresponding thickness of the neck in the [001] direction is 0.21 $(a/2)$. If the neck only confined orbits

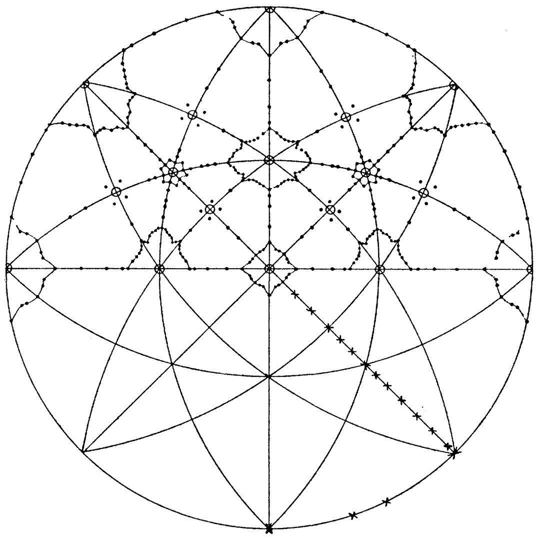

FIG. 2. Stereographic plot of the field directions in which high maxima of resistance are observed. ●—High maxima; ○—deep minima; ×—orientation of crystal axis used.

in the hexagonal face of the zone and if it were circular, this thickness should be 0.237 $(a/2)$ according to the data from the $[1\bar{1}0]$ pole.

In Table II these results are compared to data obtained on the neck thickness of copper by other investigators using a variety of methods.

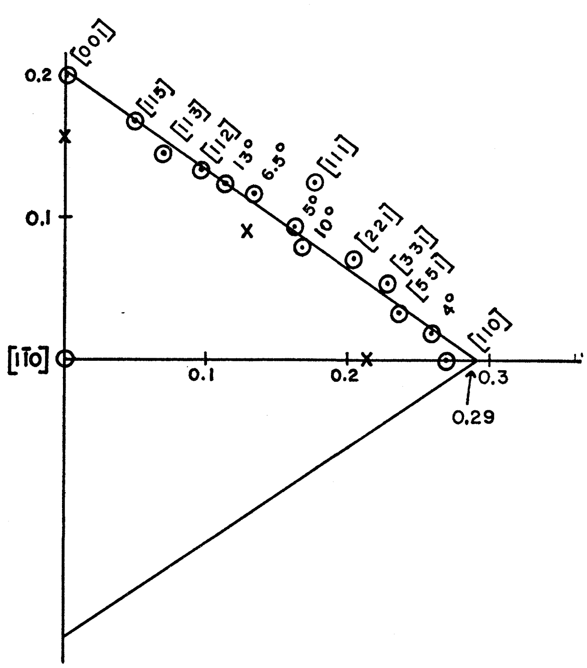

FIG. 3. Polar plot of $\tan\theta$ versus current direction in the $[1\bar{1}0]$ plane for the experimental points from fourteen crystals. θ=angle from $[1\bar{1}0]$ pole at which the resistance is a maximum. Straight lines are drawn for a neck thickness in the $[1\bar{1}0]$ direction of 0.29 $(a/2)$. One-half of the rhomb is shown. X's—data of Alekseevskii and Gaĭdukov.

TABLE II. Calculated neck thicknesses and comparison with other data.

Source of data	Thickness of neck (111) a=reciprocal lattice parameter		Method of measurement
	$[1\bar{1}0]$ direction	$[11\bar{2}]$ direction	
Coleman and Funes	0.29 $(a/2)$	0.26 $(a/2)$[a]	Magnetoresistance
Morse[b]		0.26 $(a/2)$	Acoustic attenuation
	Mean diameter		
Shoenberg[c]	0.31 $(a/2)$		de Haas–van Alphen effect
Alekseevskii and Gaĭdukov[d]	0.26 $(a/2)$		Magnetoresistance

[a] The neck thickness in the $[11\bar{2}]$ direction has been calculated from the data for the two-dimensional region around the [001] pole. (Calculations by J. S. Plaskett.)
[b] R. W. Morse, in *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, Inc., New York, 1960), p. 214.
[c] D. Shoenberg, Phil. Trans. Roy. Soc. London **255**, 85 (1962).
[d] See Ref. 5.

In addition, single peaks occur for the one-dimensional regions corresponding to the magnetic-field directions in the (001), (110), and (111) planes. These are planes for which there are always open orbits for any magnetic field direction lying in the plane. The height of the peaks is proportional to $\cos^2\alpha$, where α is the angle between the current direction and the open-orbit direction.

Data have been taken for current axes such that the magnetic field rotates in the planes corresponding to the one-dimensional regions. For field directions in the (111) plane, deep minima are observed in the resistance at the $\langle 112\rangle$ poles in addition to the $\langle 110\rangle$ singular directions. These deep minima are also found in gold by Gaĭdukov.[4] For the sample with a $\langle 201\rangle$ axis, a double peak was observed as the field crossed the $\langle 112\rangle$ which are not located on the one-dimensional lines. Detailed analysis of the open-orbit topology is being carried out to account for this. The rotation diagram for the $\langle 100\rangle$ axis crystals agrees in detail with that obtained by Klauder and Kunzler.[6] For the (100) plane, the resistance remains uniformly high in the one-dimensional regions falling to deep minima at the $\langle 110\rangle$ and $\langle 100\rangle$ singular directions.

ACKNOWLEDGMENTS

The authors wish to thank Professor John S. Plaskett for many calculations and helpful discussions. Also we wish to thank Dr. John Watkins for his continued interest in the program.

[6] J. R. Klauder and J. E. Kunzler, in *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, Inc., New York, 1960), p. 125.