Shifts and Widths of Some Stark-Broadened Oxygen Lines in an Arc Plasma

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A thermal oxygen plasma was generated in a wall-stabilized arc and its temperature and electron density were determined spectroscopically from the intensity measurement of an oxygen line of known transition probability. Observations were made over the range of electron densities from (1 to 6)$\times 10^{16}$ cm^{-3} and for temperatures around 10 000°K. The profiles of several oxygen lines were scanned end on and side on, and the shifts and half-widths were compared with the results of a recently developed Stark-broadening theory for isolated lines of heavy elements. The agreement in the half-widths is very good, with deviations usually smaller than 10%. The ratio of measured to calculated width averaged over the six investigated multiplets is 1.00 with a standard deviation of ± 0.06. However, the agreement in the shifts is good only in those cases where the shifts are large. For the two strongly broadened multiplets at 6046 and 6455 Å, detailed side-on observations were made at various radial distances. They show the theoretically expected linear dependence of half-widths and shifts on the electron density over the measured range. For the strong multiplet at 6455 Å the intensity distribution in the distant line wings was measured, too, and could be represented by power laws with exponents of -1.96 for the blue wing and -1.91 for the red wing, in close agreement with theoretical asymptotic formulas.

INTRODUCTION

THE diagnostics of dense plasmas from the study of line profiles received renewed attention after some significant advances in the theory of Stark broadening were reported in the last few years. Numerical computations were initially confined to hydrogen[1] and helium,[2] and very good agreement with measured line profiles was achieved.[3,4] Recently, the Stark-broadening calculations have been extended by Griem to isolated lines of some heavier elements.[5] The accuracy is expected to decrease here somewhat, since larger uncertainties arise in the computation of the perturbations of the levels. The broadening is almost entirely due to interactions between the upper excited level and its neighbors. All these levels are usually very far removed from the core of the atom, so that a hydrogen-like approximation for the computation of the relevant wave functions should give good results. Thus, Griem used for his line-broadening calculations the Coulomb approximation of Bates and Damgaard.[6] Experimental data on line profiles were already available then for cesium[7] and argon.[8] For cesium, where the Coulomb approximation should give good results since only the excited electron is outside a closed shell, very good agreement is obtained. In the case of argon, however, the validity of the Coulomb approximation is questionable and the agreement with the experiment becomes good only if considerable modifications of the experimental data are introduced.[5] Transition probabilities, measured in the argon experiment, are then modified, too, and agree less well with those determined by other authors.[9,10] It seems, therefore, that the question of the accuracy of the Stark-broadening calculations for complex atoms still remains somewhat unsettled.

The present experiment serves as a further check of the theory using an element with an outer electron shell which is intermediate in complexity between cesium and argon. The results of an experimental study of the shifts and widths of six oxygen lines are presented and a comparison with the theoretical results is made. Furthermore, the intensity distribution in the far wings of a strong oxygen line is measured and compared with theoretical asymptotic formulas.

EXPERIMENTAL

The oxygen plasma was generated in a wall-stabilized arc at atmospheric pressure which has been described in detail in an earlier report.[4] For this experiment the arc channel was widened to 5 mm since oxygen exhibits a relatively broad arc column. The electrode regions were, as before, blanketed in argon to prevent erosion of the carbon electrodes. The flow rates of the two gases, which were of the order of 500 ml/min, were adjusted such that the diffusion of argon into the observation part was kept below one percent, which was checked spectroscopically using recently measured f values.[10] The arc was always operated at the same flow rate, voltage, and current (50 A) settings. It burned extremely stably and quiescently for periods as long as 2 h. During the detailed observations of line profiles, the total intensity of a strong oxygen line was monitored and could

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[1] H. R. Griem, A. C. Kolb, and K. Y. Shen, Phys. Rev. **116**, 4 (1959), and Astrophys. J. **135**, 272 (1962).

[2] H. R. Griem, M. Baranger, A. C. Kolb, and G. Oertel, Phys. Rev. **125**, 177 (1962).

[3] H. F. Berg, A. W. Ali, R. Lincke, and H. R. Griem, Phys. Rev. **125**, 199 (1962).

[4] W. L. Wiese, D. R. Paquette, and J. E. Solarski, Phys. Rev. **129**, 1225 (1963).

[5] H. R. Griem, Phys. Rev. **128**, 515 (1962) and to be published. We are indebted to Professor Griem for furnishing us with a preprint of his results.

[6] D. R. Bates and A. Damgaard, Phil. Trans. Roy. Soc. London **A242**, 101 (1949).

[7] P. M. Stone and L. Agnew, Phys. Rev. **127**, 1157 (1962).

[8] W. E. Gericke, Z. Astrophysik **53**, 68 (1961).

[9] H. W. Drawin, Z. Physik **146**, 295 (1956).

[10] H. N. Olsen J. Quant. Spectry., Radiative Transfer **3**, 59 (1963).

be kept constant within 2% by maintaining the current and the gas flow rates within narrow limits. The deviations of the intensities from the mean value were used as correction factors. The reproducibility from run to run was found to be very good and is reflected in the small standard deviations for the shift and width measurements.

The spectroscopic observations of the arc column were made side on as well as end on. The lines were recorded with an Ebert-type spectrometer with a linear dispersion of 10 Å/mm while a second spectrometer was used for the monitoring. The entrance and exit slit widths were varied from 10 to 30 μ depending on the width of the various oxygen lines. Since the arc column has cylindrical symmetry, the temperatures and particle densities are only defined for concentric layers around the axis, and spatial resolution is required. In the end-on observations this is achieved by using a very narrow aperture in the imaging lens (1:120) thereby admitting only the radiation very close to the axis ($r<0.12$ mm) into the spectrometer. The intensities measured side on, however, always include contributions from a number of different radial layers and have to be inverted into radial intensities, which is accomplished utilizing the Abel integral equation. A recently constructed electronic data-processing system[11] has been applied to the execution of this inversion process. This instrumentation in connection with an electronic computer permits a rapid, fully automatic, and very precise Abel transformation.

The intensity of the 6157-Å line was calibrated on an absolute scale with a tungsten strip lamp placed at the position of the arc. Reflection and absorption losses from the quartz window enclosing the arc were measured and taken into account.

For the shift measurements precise wavelength determinations of the oxygen lines were required. For this purpose, the lines were compared with helium, neon, or thorium lines, depending on the availability of the latter in the various spectral regions. The comparison lines were produced in Geissler tubes or electrodeless discharges. Extended wavelength ranges around the oxygen lines, which included several comparison lines, were scanned. Beam splitters or removable mirrors imaged both the arc and the comparison source on the spectrograph slit and permitted continuous, uninterrupted scanning. The wavelengths of the comparison lines were taken from recent literature.[12,13]

PLASMA DIAGNOSTICS

For the relatively high electron densities and low temperatures encountered in this experiment, deviations from local thermodynamic equilibrium (LTE) should be extremely small. This is not only indicated from the numerical application of the equilibrium criteria for arcs,[14] which give deviations of the order of 1%, but also by the results of other arc experiments done under similar conditions. Furthermore, the existence of LTE is suggested by the good agreement between measured and calculated line-broadening parameters over the whole range of electron densities.

At the conditions of this experiment, namely for temperatures around 10 000°K and electron densities between 1 and 6×10^{16} cm^{-3}, the dissociation of molecular oxygen is practically complete, the first ionization of oxygen atoms is appreciable (between 1 and 15%), but the second ionization is still negligible as found from the numerical application of the relevant mass-action laws.[15] Thus, only neutral and singly ionized oxygen atoms and electrons are present in significant quantities, whereas the concentrations of all other oxygen species like O_2, O_2^+, etc., are extremely small. The particle densities of the remaining simple system (O,O^+,e^-) were calculated as functions of temperature from the condition of local electrical neutrality, Dalton's law of partial pressures, and the Saha equation governing the degree of ionization. High-density corrections[16,17] were taken into account.

Thus, the experimental determination of the temperature remained. Utilizing the strong dependence of line intensities on temperature, this was accomplished by measuring the absolute intensity of the oxygen I multiplet $3p\ ^5P-4d\ ^4D^\circ$ at 6157 Å, using a transition probability measured recently by one of us.[15] The numerical value was slightly changed to take into consideration two new results: First, the relation taking into account the lowering of the ionization potential (the only important high-density correction here) was changed after it became apparent from recent theoretical studies that the formula used in the earlier report is not applicable in this density region.[16,17] Second, the values of the electron densities, on which the transition probability was based, were slightly altered: These electron densities were obtained from line-profile measurements of the Balmer line H_β using results of an improved Stark-broadening theory.[1] In the meantime, the Stark broadening of this line was studied experimentally under conditions very similar to those in this experiment, and showed a small difference between the electron densities obtained from line-broadening theory and from precise line-intensity measurements.[4] The deviation of the line-broadening result from the line-intensity measurements was applied as a correction factor. As a result of these two corrections, the transition probability changed by 4% to $A=7.33\times10^{16}$ sec^{-1}. It is of interest to note that this agrees within 2% with the value obtained from the Coulomb approximation by Bates and Damgaard[6]

[11] D. R. Paquette and W. L. Wiese (to be published).
[12] Massachusetts Institute of Technology, *Wavelength Tables* (John Wiley & Sons, Inc., New York, 1939).
[13] R. Zalubas, Nat. Bur. Std. U. S. Monograph **17** (1960).
[14] W. Finkelnburg and H. Maecker, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 22.
[15] W. L. Wiese and J. B. Shumaker, J. Opt. Soc. Am. **51**, 937 (1961).
[16] H. R. Griem, Phys. Rev. **128**, 997 (1962).
[17] G. Ecker and W. Kröll, Phys. Fluids **6**, (1963).

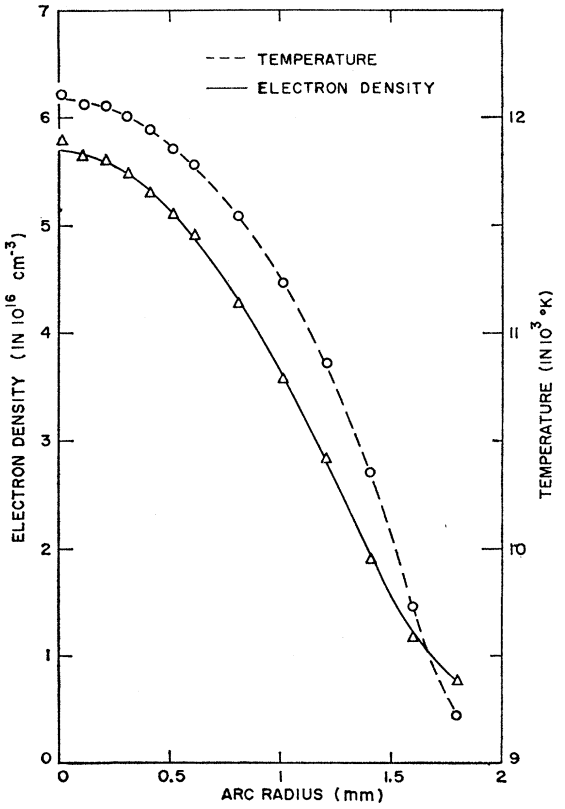

FIG. 1. Radial distribution of temperatures and electron densities.

and within 8% with the result of an experiment by Jürgens.[18]

The method of temperature determination and the line-profile measurements require implicitly that the lines be emitted from an optically thin layer. This condition was checked by comparing the intensity at the line centers with the calculated intensity of a blackbody at the same temperature as that prevailing in the arc, and was always found to be fulfilled.

The temperature at the arc axis was determined several times independently and a mean value of 12 080°K with a standard deviation of 80°K was obtained. Furthermore, for the experimental run having an axis temperature closest to this mean value, the temperatures and electron densities at many radial positions were measured from the Abel inverted intensities. These are presented graphically in Fig. 1.

It was attempted to measure the shifts and widths for all lines in the visible region of the spectrum for which line-broadening parameters were available, but it soon became apparent that some lines were so weak above the continuous background that they had to be omitted. In the end-on observations, interference with argon lines originating from the argon layers around the electrode regions reduced the number of available oxygen lines even further.

[18] G. Jürgens, Z. Physik **138**, 613 (1954).

Besides Stark broadening, the thermal Doppler broadening and the finite resolution of the spectrometer might cause additional broadening under the experimental conditions and had to be considered. In order to correct for Doppler broadening, the profile resulting from Stark broadening, which is approximately of the dispersion type, was "folded" with the Gaussian profile resulting from Doppler broadening.[19] However, even for the narrowest lines the Doppler width was only about one tenth of the total width and the resultant Voigt profiles had within 0.1% or less the same half-widths as the Stark profiles themselves, i.e., the effect of Doppler broadening on the half-widths was always negligible. The situation was somewhat different for the apparatus profile, which had to be experimentally determined first. For this purpose, the profiles of thorium and neon lines emitted from low-pressure electrodeless discharges were scanned. The intrinsic widths of these lines were small compared with the apparatus widths, which were found to be in the range from 0.15 to 0.30 Å (full half-widths) depending mainly on the slit openings used, and were from 5 to 30 times smaller than the oxygen linewidths. The apparatus profiles turned out to be very nearly of triangular shape. Thus, in order to correct for their influence, a folding of a dispersion profile with a triangular shape was undertaken. Since only the half-widths and not the whole resulting profiles are of interest, simpler trial and error calculations to determine these were sufficient here.[20] The results showed that around 30% of the apparatus widths must be subtracted. This correction amounted to 7% of the total linewidths in the most unfavorable case.

Some linewidth measurements require an additional correction before the comparison with the line-broadening theory can be undertaken. This is due to slightly different situations in experiment and theory: The line-broadening calculations refer to the components of the multiplets, which have, within the accuracy of the theory, all the same width[5]; but experimentally most multiplets are observed as a single broadened line, because the components lie so close together that the individual profiles completely overlap. A small wavelength spread between the components causes, therefore, an additional broadening in the observed over-all profile, and the measured half-width must be reduced to the component width, to which the line-broadening data apply.

For four of the six investigated multiplets the wavelength spread between the components was extremely small compared to the half-widths, and did not need to be considered. But this was not so for the multiplets $3s\,^5S^\circ - 4p\,^5P$ and $3p\,^5P - 5s\,^5S^\circ$ at 3947 and 6455 Å, where the over-all wavelength spread was 0.29 and 2.37 Å against measured half-widths of 0.9 and 6.90 Å, respectively (at the axis position). Thus, a correction

[19] A. Unsöld, *Physik der Sternatmosphären* (Springer-Verlag, Berlin, 1955).

[20] H. F. Berg, Ph.D. thesis, University of Maryland, College Park, Maryland, 1961 (unpublished).

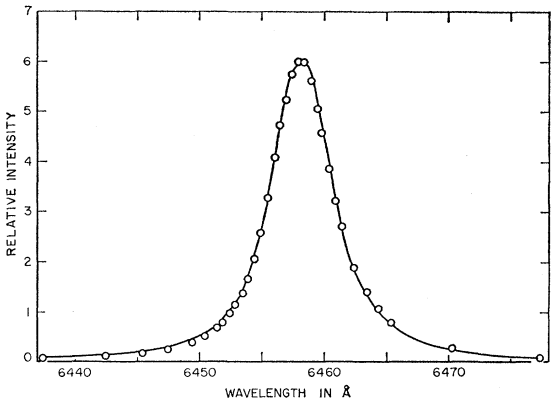

FIG. 2. Comparison of the observed over-all profile for the 6455 Å multiplet with the one constructed from the three overlaping lines at 6453.64, 6454.48, and 6456.01 Å for the case of the best fit in the half-widths.

was applied in the following manner: Assuming that *LS* coupling holds, the intensity ratios between the components could be obtained.[21] With the further assumption that all components have dispersion profiles[22] of the same half-widths and with the known wavelength spread between the components, the resulting profiles could be constructed. These were then compared with the experimental shapes until by adjusting the component half-widths the best fit was obtained. The very good agreement between the constructed and observed profiles for the case of the best fit is illustrated in Fig. 2 for the 6455 Å multiplet. The widths thus determined were found to be 20% smaller than the over-all measured widths for the 3947 Å line and 20 to 30% smaller for the 6455 Å line.

The component spread was also taken into account in the line-shift measurements by using the center of gravity of the multiplets as the point of reference. The center of gravity was determined by weighting the components, again according to *LS* coupling rules.

The assumption of *LS* coupling, aside from being justified by the good agreement, is also supported by observations at low electron densities, i.e., very large radial distances, where the three components of the 6455 Å multiplet appear almost separated and an intensity ratio of 3:4:8.6 is observed, which agrees fairly well with the theoretical ratio of 3:5:7 considering that the reconstruction of the wings is somewhat left to personal choice.

A. Comparison of Shifts and Half-Widths

End-on measurements were undertaken for the two broad lines at 7254 and 6455 Å and served mainly as a check against the extensive side-on measurements. For the end-on measurements, the aperture was limited to such an extent (1:120) that only radiation from the regions closer than 0.12 mm from the arc axis was admitted to the spectrometer. Thus only those parts of the plasma were observed where the change of electron density relative to the axis value is smaller than 1.5%, as follows from Fig. 1. For the comparison, the theoretical shifts and widths were calculated from the approximate formulas given by Griem [Eqs. (12) and (13) of Ref. 5], with the parameters scaled and interpolated according to the measured electron densities and temperatures. The validity criteria are always fulfilled, i.e., the lines are isolated as found from inspection of the energy level tables,[23] and the electron density is under the critical value where Debye shielding becomes important. Furthermore, the approximate formulas used should be very accurate in this case, since the ion broadening parameter α is always very small ($\alpha \ll 0.5$), and the ratio R of mean ion distance and Debye radius is below the required limit of 0.8 ($R=0.52$). The results and the comparison with theory are presented in Table I. It is seen that the agreement for these broad lines is about as good for the shifts as for the widths. The measured values are somewhat smaller than the calculated ones, but this tendency is not preserved throughout the side-on measurements. The indicated errors are derived from the calculated or estimated experimental uncertainties. For the strong multiplet at 6455 Å, standard deviations of 3.5% in the mean values for the width and shift are obtained from four independent measurements. To the error in the linewidth measurement an estimated uncertainty of 2% in the correction for the component spread is added. Larger error limits are assigned to the weak and very broad line at 7254 Å, since it proves to be difficult to determine precisely the point where the line merges into the continuous background. The errors listed under the column "Calculated' correspond to uncertainties in the electron-density measurement, which in turn are caused mainly by uncertainties in the intensity measurements and in the transition probability for the 6157 Å line. To these figures an estimated uncertainty of 20% from the theory should be added.[5]

Detailed side-on measurements were undertaken for the two multiplets at 6046 and 6455 Å. These lines were so strongly broadened that, with narrow spectrograph slits, point-by-point intensity determinations at a large number of wavelength settings across their profiles could

TABLE I. Comparison of calculated linewidths and shifts with the results of the end-on observations ($N_e=5.70\times10^{16}$ cm^{-3}, $T=12\,080$°K).

Multiplet	λ (Å)	Full half-width (Å) Measured	Calculated	Shift (Å) Measured	Calculated
$3p\ ^5P-5s\ ^5S°$	6455	5.6±0.3	7.4±1.2	3.6±0.1	4.0±0.7
$3p\ ^3P-5s\ ^3S°$	7254	7.4±2.0	9.5±1.5	4.5±1.0	5.1±0.8

[21] C. W. Allen, *Astrophysical Quantities* (The Athlone Press, London, 1955).

[22] This condition should be closely fulfilled, since these lines have very small ion-broadening parameters.

[23] C. E. Moore, in *Atomic Energy Levels*, National Bureau of Standards Circular No. 467, Vol. I (U. S. Government Printing Office, Washington 25, D. C., 1949).

be made. After performing the Abel transformation to radial intensities, the line shapes were obtained graphically for a number of radial positions. A typical profile indicative of the precision achieved is shown in Fig. 2. After applying the relevant corrections, the measured half-widths and shifts were compared with the theoretical data, which correspond to the electron densities and temperatures determined from the intensity measurement of the 6157 Å line.

The comparison is presented in Figs. 3, 4, and 5. The solid lines represent the calculated values and the experimental points are indicated by dots. The theory gives practically straight lines in all cases. Deviations from linearity due to the ion contribution and the temperature variation across the arc are very small and partly compensate for each other. The experimental points represent straight lines, too. The deviations are almost always within the error limits. The slight bends in the experimental points seem to be accidental; they are in opposite directions for the two lines. The Abel transformation is probably in part responsible for this, since it provides a certain amount of coupling between neighboring radial points. The results for the 6046 Å multiplet are estimated to be slightly less accurate than for the 6455 Å multiplet, since the former line is rather weak and some overlap of the red wing with a stronger oxygen line at 6106 Å occurs. Error bars, given for a few selected points, indicate the estimated uncertainty in the measurements of shifts or widths. The experimental precision gradually decreases towards smaller electron densities, i.e., larger radial distances. The horizontal error bars on the theoretical curves represent the estimated uncertainties in the electron-density measurements.

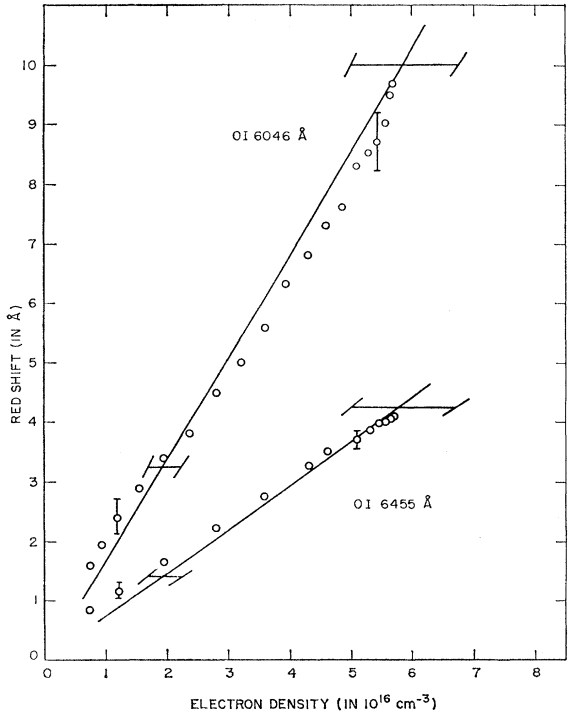

FIG. 3. Measured and calculated shifts for the multiplet $3p\ ^3P-6s\ ^3S°$ at 6046 Å and the multiplet $3p\ ^5P-5s\ ^5S°$ at 6455 Å as a function of electron density. The solid lines represent the calculated values and the experimental points are indicated by circles. The vertical error bars represent the estimated uncertainties in the shift measurements; the horizontal error bars indicate the uncertainties in the electron density measurements.

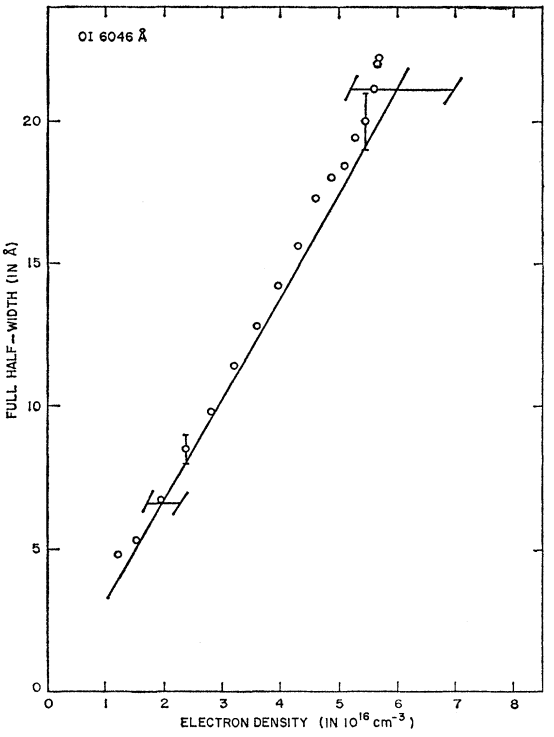

FIG. 4. Measured and calculated half-widths for the multiplet $3p\ ^3P-6s\ ^3S°$ at 6046 Å as a function of electron density. The notation is analogous to Fig. 3.

It is seen that the axial values for the shifts and widths of the 6455 Å multiplet agree very closely with the results of the end-on measurements given in Table I.

The remaining oxygen lines exhibit much narrower profiles so that an inversion to radial intensities with the method discussed above was not feasible. Instead, viewing again from the side, the line profiles were scanned versus wavelength for the position of maximum intensity (arc axis). This was also done for the strong, broad multiplet at 6455 Å for which already the detailed, spatially resolved measurements were undertaken. Hence, the width and shift of this line were measured for the central position both Abel-inverted as well as not inverted, and the ratios between the two results were applied as correction factors to the shift and width measurements of the other lines. This procedure should lead to accurate results since all investigated lines have, within 3%, the same upper excitation potentials as the 6455 Å line, i.e., the intensity distributions of the lines over a cross section of the arc are practically alike. Furthermore, the radiation in the spatially unresolved scans comes predominantly from the central region of

TABLE II. Comparison of calculated linewidths and shifts with the results of the side-on measurements ($N_e = 5.70 \times 10^{16}$ cm^{-3}, $T = 12\,080$°K).

Multiplet	λ (Å)	Full half-width (Å) Measured	Full half-width (Å) Calculated	Shift (Å) Measured	Shift (Å) Calculated
$3s\,^3P^\circ - 5p\,^3P$	3692	2.72±0.3	$2.84 \pm 0.4_5$	0.18±0.05	0.07 ±0.01
$3s\,^5S^\circ - 4p\,^5P$	3947	0.83±0.08	0.70±0.09	−0.13±0.05	$+0.043 \pm 0.01_5$
$3s\,^3S^\circ - 4p\,^3P$	4368	1.08±0.06	1.05±0.17	0.20±0.07	0.30 ±0.05
$3p\,^3P - 5s\,^3S^\circ$	7254	8.9 ±0.5	9.5 ±1.5	5.30±0.6	5.1 ±0.8

the arc in which the electron density changes by only a few percent. Values of 0.82 and 0.94 for the ratios of spatially unresolved to spatially resolved shifts and widths were obtained.

The results for the four multiplets are listed in Table II. In all cases several independent experiments were made. Very good reproducibility from run to run was obtained which is reflected in the small statistical error. The experimental error contains the standard deviations of the mean values (which are mostly of the order of 3%) and the estimated uncertainty in the ratio of radially inverted to not inverted parameters. As before, under the column "Calculated" the error in the electron density measurement is listed. The corresponding error from the temperature determination is negligible. Again, the estimated theoretical uncertainty of[5] 20% is not included in this figure.

The agreement with the calculated widths is always well within the experimental and theoretical error estimates. For the shifts the agreement is good only if the shifts are of about the same size as the widths, while for smaller shifts the deviations become large. This is theoretically expected from the application of a quantum-mechanical dispersion relation to the shifts and widths of isolated lines.[24] A result of this study is that uncertainties in the calculated shifts should be of the order of 20% of the (half) half-widths. Thus, for the multiplets at 3692 and 3947 Å, where the shifts amount only to one tenth or less of the (half) half-width, uncertainties of the order of several hundred percent must be expected; for the 4368 Å line, with a shift of about one-half of the width, the uncertainty should be about 40%; and for the broad lines at 6046, 6455, and 7254 Å, which have shifts about as large as the (half) half-widths, the uncertainties in the calculated shifts should decrease to 20%. This general trend is clearly confirmed from the experimental data.

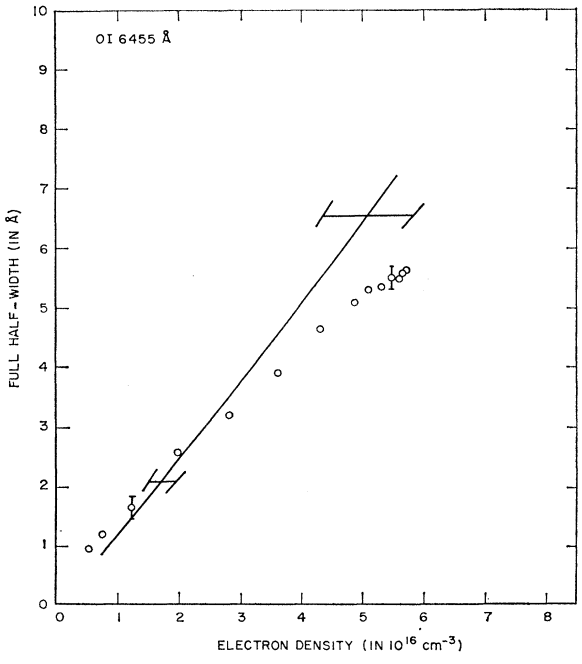

FIG. 5. Measured and calculated half-widths for the multiplet $3p\,^5P - 5s\,^5S^\circ$ at 6455 Å as a function of electron density. The notation is analogous to Fig. 3.

In the case of the 3947 Å multiplet, a small blue shift is observed, whereas the theory predicts an equally small red shift. However, according to theory the shifts for this line are relatively temperature sensitive and reverse sign at slightly lower temperatures, so that the discrepancy is not too serious.

It is of interest to note that of the six investigated multiplets three have a single upper term (those at 6046, 6455, and 7254 Å) and the other three have an upper term consisting of three levels (at 3692, 3947, and 4368 Å). For the multiplets of the first group the line-broadening parameters for the lines within the multiplet should be identical regardless of the complexity of the lower terms, since practically all the broadening originates from the upper term. For the multiplets of the second group the line-broadening parameters are calculated by using only one upper level and by assuming that the parameters for all lines in the multiplet are the same within the accuracy of the calculations. This assumption is strongly supported by the experimental results, i.e., no difference in accuracy between the two groups of multiplets is observed.

B. Comparison of Line Wings

For the strong 6455 Å line the detailed point-by-point intensity measurements across the profile were extended to the far wings of the line for a study of the intensity distribution in this region. This is the range from 5 to 12 times the half-width in which good measurements still can be made and tabulated values of the reduced

[24] H. R. Griem and C. S. Shen, Phys. Rev. **125**, 196 (1962).

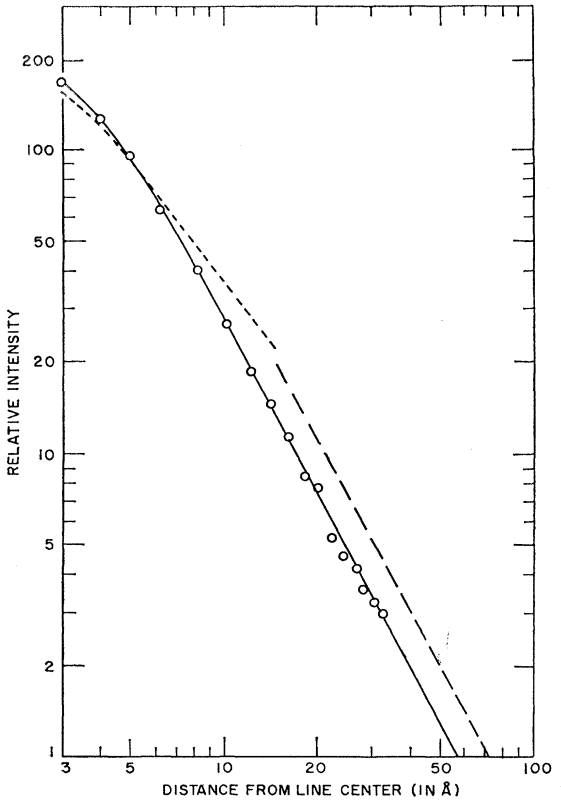

FIG. 6. Comparison of the measured and calculated intensity distributions for the red wing of the 6455 Å multiplet in logarithmic representation. The solid line is the best fit for the experimental points, which are indicated by open circles, and the two broken lines represent the theoretical results. Tabulated theoretical values were available from Ref. 5 for small wavelength distances, and for larger distances the asymptotic formulas of Ref. 2 were used.

profile are no longer available. The theoretical intensity distribution at very large distances $\Delta\lambda$ from the line center follows a power law of the form $I(\Delta\lambda)=C\Delta\lambda^{-n}$ with the exponent n assuming the value 2 for the blue wing and gradually approaching 7/4 for the red wing.[2] Hence, graphic presentations of $I(\Delta\lambda)$ versus $\Delta\lambda$ on a logarithmic scale were undertaken which should result in straight lines with the slope given by the exponent n. The critical experimental problem was the determination of the continuous background. As a first approximation, the latter was assumed to be the value of the measured point farthest away from the line center and to be constant over the region of the line. But the plots of $\ln I(\Delta\lambda)$ versus $\ln\Delta\lambda$ showed that the tail ends of the curves for large wavelength distances ($\Delta\lambda>25$ Å) deviated increasingly from straight lines. Assuming that the general form of the intensity distribution, namely the power law, is correct, the outermost parts were forced approximately on a straight line by lowering the constant background further. The slopes of the experimental distributions which gave the best fit to a straight line were graphically determined as -1.96 for the blue wing, and -1.91 for the red wing. The theoretical values corresponding to the experimental conditions in this region are -2.00 and -1.89, respectively. The theoretical and experimental results are compared in Figs. 6 and 7 separately for the red and blue wings. The solid line is the best fit for the experimental points, which are indicated by open circles, and the broken lines represent the theoretical results. For small wavelength distances tabulated values were available,[5] and for the larger wavelength distances the asymptotic formulas[2] had to be used. The curves were put on the same scale by normalizing the areas and by converting the reduced theoretical wavelengths to experimental wavelengths using the experimentally determined electron density. The differences of 30 and 50% for the red and blue wings between the experimental and theoretical intensities are probably to a large extent due to uncertainties in the construction of the experimental background.

SUMMARY

The results of this experiment confirm the estimated accuracy of 20% for the recent Stark-broadening calculations of isolated lines of heavy elements with respect to half-widths and line shapes. They also confirm the theoretical expectation that uncertainties in the shifts become large as the shifts—expressed in terms of the

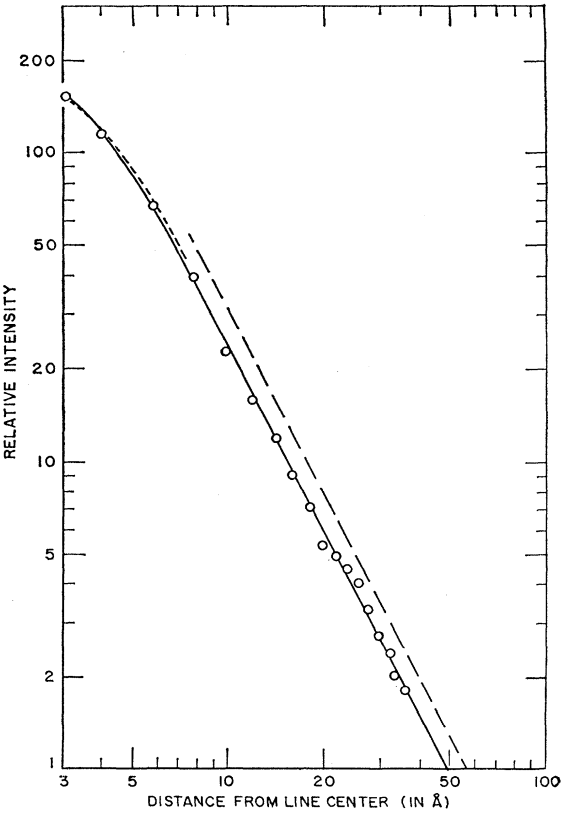

FIG. 7. Comparison of the measured and calculated intensity distribution for the blue wing of the 6455-Å multiplet. For further explanations, see Fig. 6.

half-widths—decrease. Stark-broadening calculations have thus made possible a different and very convenient approach for determining the electron densities in dense plasmas from linewidth measurements with an accuracy comparable to other spectroscopic methods. The advantages of this method are that the temperature needs to be only roughly determined since the broadening is mainly a density effect, and that the width (and shift) measurements are quickly and rather precisely done, since only relative intensity measurements are required. It is advisable to determine the widths of several lines to obtain a smaller statistical error. For example, the ratio of measured to calculated widths averaged over all six multiplets is in this experiment 1.00 with a standard deviation of ± 0.06.

In comparing the cesium and oxygen experiments no significant deterioration in the results is observed, which means that the use of the Coulomb approximation does not seem to introduce additional significant errors in the case of a more complex atom.

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Elastic and Inelastic Collision Cross Sections in Hydrogen and Deuterium from Transport Coefficients*

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By means of a numerical solution of the Boltzmann equation, elastic and inelastic collision cross sections have been derived for electrons in H_2 and D_2 subjected to a dc electric field. The cross sections for momentum transfer, rotational excitation, vibrational excitation, electronic excitation, and ionization are investigated by comparing experimental and theoretical values of transport coefficients. The same momentum transfer cross section previously obtained for H_2 by Frost and Phelps has been found to be valid for D_2. Good agreement is secured between experiment and theory by multiplying the theoretical rotational cross sections of Gerjuoy and Stein by approximately 1.5, provided the polarization factor of Dalgarno and Moffett is used. The final cross section for vibrational excitation of H_2 has a threshold at 0.52 eV and a peak of 7.7×10^{-17} cm^2 at 4.5 eV, whereas that of D_2 has a threshold at 0.36 eV and a peak of 6.6×10^{-17} cm^2 at 4.7 eV. The derived electronic excitation cross sections are the same for both H_2 and D_2. The ionization cross section was taken from the experimental results of Tate and Smith. Calculated transport coefficients for electrons subjected to crossed electric and magnetic fields, and high-frequency ac electric fields are in agreement with recent experimental and theoretical results.

I. INTRODUCTION

ELASTIC and inelastic collisions of low-energy electrons with molecular gases have been the subject of considerable theoretical and experimental investigation[1–3] in recent years. In this paper we extend the cross-section determinations of Frost and Phelps[4] to higher energies in H_2, and to D_2. We include the processes of elastic scattering, and rotational, vibrational, and electronic excitation as well as ionization. Stated somewhat differently, we shall take into account both elastic and inelastic collisions involving electrons with energies up to 100 eV.

Our method of calculation is essentially the same as that of Frost and Phelps (hereafter called I). We numerically solve the Boltzmann transport equation for the distribution function, f, of electron energies taking into consideration both elastic and inelastic collisions. In the case of only a dc electric field present the three transport coefficients[5] of principal interest are the diffusion-coefficient, D, the mobility, μ, and the Townsend primay ionization coefficient α_i. These coefficients are found by taking the appropriate average over f. Cross sections are determined by successive adjustments to initial estimates until theoretical and experimental values of the transport coefficients are brought into good agreement. The results are by no means unique, but they certainly do represent a consistent and realistic set of elastic and inelastic collision cross sections.

It is possible to consider separately three distinct regions of electron energy. In our calculation the electron energy is characterized by an experimentally measurable quantity, the characteristic energy ϵ_K, where

$$\epsilon_K = eD/\mu, \tag{1}$$

and e is the electronic charge.

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