Nuclear Structure Studies in Isotopes of Nickel and Iron*

R. H. FULMER AND A. L. MCCARTHY
University of Pittsburgh, Pittsburgh, Pennsylvania
(Received 29 April 1963)

The levels of Fe^{55}, Fe^{58}, Ni^{62}, Ni^{63}, and Ni^{65} are studied with (d,p) reactions; angular distributions and absolute cross sections are analyzed with the aid of distorted-wave Born-approximation calculations. Essentially all levels in the odd isotopes up to about 1-MeV binding energy are assigned to a shell-model state and their reduced widths are measured. From this, the "center of gravity" of each shell-model state is determined; the results for Fe^{55}, which has one neutron above a closed shell, are $p_{3/2} \gtrsim 0.1$ MeV, $f_{5/2} - 1.4$ MeV, $p_{1/2} \sim 3.2$ MeV, $g_{9/2} - 3.8$ MeV, $d_{5/2} \sim 6.7$ MeV, and $s_{1/2} \sim 7.3$ MeV. The shell-model states in the Ni isotopes follow the same energy ordering except that the $f_{5/2}$ state decreases in energy to become the ground state of Ni^{65}, but the energy range of the spectra is compressed for greater neutron excess. The $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ states increase in "fullness" with increasing neutron excess, while the other states remain completely empty. The widths of the energy distributions of levels belonging to single shell-model states agree well with the predictions of the giant resonance theory of Lane, Thomas, and Wigner.

I. INTRODUCTION

THE (d,p) stripping reaction is a useful tool for investigating nuclear single-particle states because it excites nuclear levels only insofar as these levels have a single-particle nature.[1] When the stripping reaction is used with an energy resolution sufficient to distinguish individual nuclear levels, the location and excitation strength of the nuclear levels directly indicate the location and strength of the single-particle states.

A recent paper from this laboratory has applied such a (d,p) stripping process to a study of single-particle levels in medium mass nuclei.[2] That study was concerned primarily with levels in Ni^{59}, Ni^{61}, and Fe^{57}. The present paper is an extension of that study to other isotopes of nickel and iron.

II. EXPERIMENTAL PROCEDURE

The basic experimental method has been described previously. Briefly, it consists of the following: Protons which are produced in (d,p) reactions induced by 15-MeV deuterons are passed through a wedge magnet spectrograph and detected by the tracks they produce in a photographic emulsion located in the focal plane of the spectrograph. In the present series of experiments, spectra for each target were taken at two settings of the magnetic spectrograph to cover a sufficiently large energy range. The resolution of the system was about 45 keV. The targets bombarded were Fe^{54}, Fe^{57}, Ni^{61}, and Ni^{64}. All the targets were isotopically enriched metal foils about 2-mg/cm^2 thick. For each target, data were taken at several scattering angles in the range 9° to 50°. A typical spectrum is shown in Fig. 1.

The spectra for a given target were used to construct angular distributions of cross sections for the proton groups observed. From the angular distribution data, a value of the angular momentum transfer, l, was assigned to each nuclear level by reference to distorted-wave Born-approximation (DWBA) predictions.[3] Representative angular distribution curves are shown below in connection with the discussion of results for individual levels.

Relative cross sections used in the above angular distributions are estimated to be in error by less than 15%. In determining absolute cross sections, uncertainties in geometrical factors and target thicknesses introduce an error of about 20%.

III. EVEN-ODD FINAL NUCLEI

A. Results

The experimental results for Fe^{55}, Ni^{63}, and Ni^{65} are summarized in Tables I to III, respectively. The tables list, in successive columns, the energies of the observed nuclear levels and corresponding values of the angular momentum l, the (d,p) cross section, and the "reduced" cross section $(2j+1)S$. This last factor is found by the relation

$$d\sigma/d\omega = [(2j+1)/(2I+1)]\sigma(l,\theta,Q)S, \qquad (1)$$

in which $d\sigma/d\omega$ is the experimental absolute cross section, j the spin of the state observed in the stripping process, I the spin of the target nucleus, and S the spectroscopic factor for the observed level. The quantity $\sigma(l,\theta,Q)$ is a reaction function calculated in the present case by DWBA methods. It is a function of the angular momentum transfer, the scattering angle θ, and the reaction Q value.

Tables I and III also present, for the levels observed in this study, values of excitation energies, l values, and

* This work was done in the Sarah Mellon Scaife Radiation Laboratory and supported by the National Science Foundation and the U. S. Office of Naval Research.

[1] B. L. Cohen, R. H. Fulmer, A. L. McCarthy, and P. Mukherjhee, Rev. Mod. Phys. 35, 332 (1963).

[2] B. L. Cohen, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 126, 698 (1962).

[3] G. R. Satchler, R. Bassel, R. Drisko, and E. Rost (private communications). The authors are greatly indebted to Dr. Satchler and his group for performing the DWBA calculations for the cases of interest here. They are based on the theory of Tobocman [Phys. Rev. 94, 1655 (1954); 115, 99 (1959)]. The optical potentials used are of the Saxon form; the values of these and other parameters are given in Ref. 2. See also, R. M. Drisko, R. H. Bassel, and G. R. Satchler, Oak Ridge National Laboratory Report ORNL-3085, 1962 (unpublished).

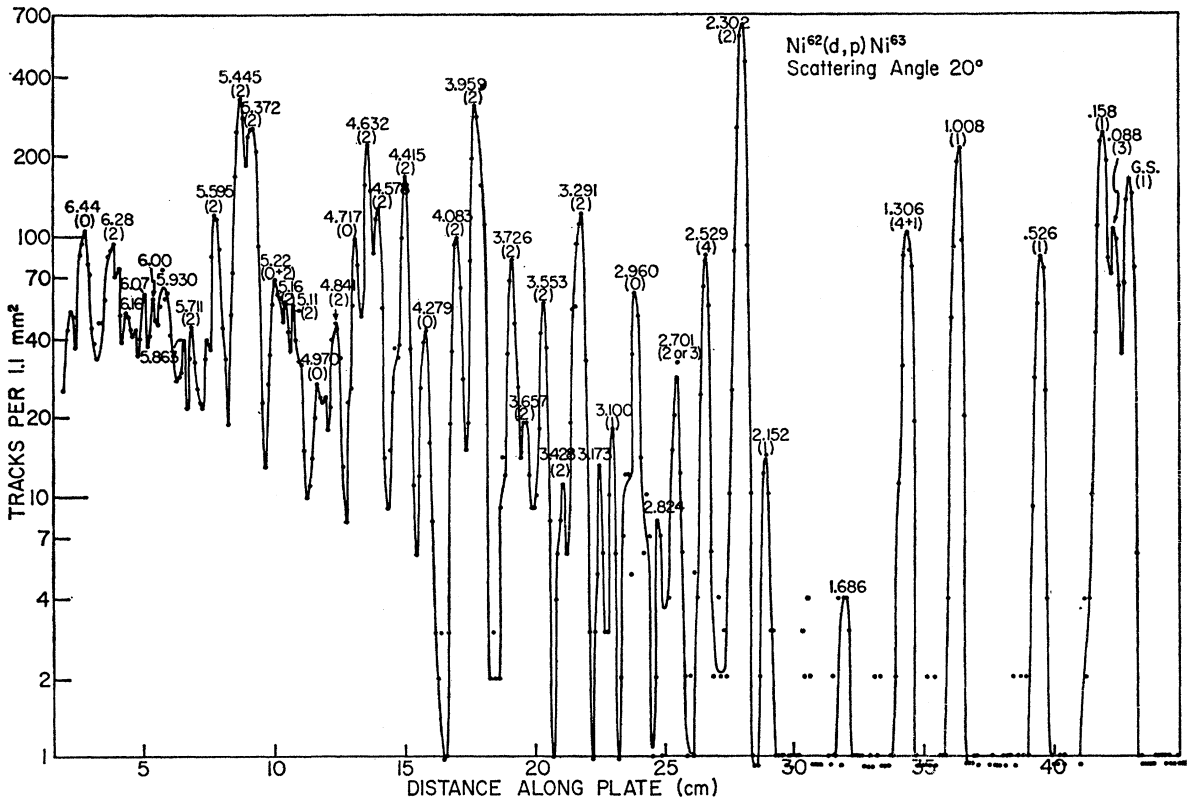

FIG. 1. Measured proton energy spectrum from $Ni^{62}(d,p)Ni^{63}$. Numbers above peaks are excitation energies in Ni^{63} in MeV, and l values assigned to these peaks are in parentheses. Energy resolution here is typical of the experiment.

reduced widths obtained at MIT in Refs. 4 and 5. In general, the angular momentum assignments of the present work agree well with those of Refs. 4 and 5 for low-lying levels; the agreement is less complete for states of higher excitation energies. In particular, levels which are assigned in the present work as $l=2$ have been frequently assigned in the MIT reports as $l=1$. These levels agree well with our DWBA curves for $l=2$ (see, for example, Fig. 2); they fail to match angular distributions for known $l=1$ levels.

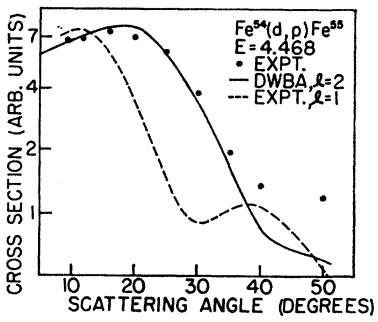

FIG. 2. Comparison of experimental points with angular distributions expected from $l=2$ and $l=1$. The $l=2$ curve is a DWBA calculation; the $l=1$ curve is empirically determined.

For some proton groups, the experimental angular distribution of cross section cannot be identified with any single angular momentum transfer. These are cases in which two or more nuclear levels cannot be resolved, and it is usually possible to consider the angular distribution as a superposition of angular distributions for two different l values.

Figure 3 illustrates the decomposition of an observed angular distribution into that for two l values. The possibilities $l=1$ plus $l=4$ and $l=1$ plus $l=3$ are compared. Note that assigning the secondary level as $l=4$ gives essentially perfect agreement with all the data points, while an $l=3$ assignment gives poorer agreement, especially at 25°, 40°, 50°. On the basis of the comparison of Fig. 3, the secondary level is assigned as $l=4$.

This assignment is greatly strengthened by the fact that the level in question has just the intensity and energy expected for the $g_{9/2}$ state from the systematics of this mass region (see Figs. 5 and 6, respectively), whereas the assignment $l=3$ would lead to an anomalously large intensity and excitation energy for the $f_{5/2}$ state.

[4] $Fe^{54}(d,p)Fe^{55}$ reactions were reported by A. Sperduto and J. Rapaport, Massachusetts Institute of Technology Laboratory for Nuclear Science Progress Report, (NYO-2668) 1961 (unpublished), p. 137.

[5] $Ni^{64}(d,p)Ni^{65}$ reactions were reported by J. Ross and H. A. Enge, Massachusetts Institute of Technology Laboratory for Nuclear Science Progress Report, (NYO-2669) 1962 (unpublished), p. 61.

From Table I, the $l=4$ assignment is further strengthened by the MIT data of Ref. 4, which include an $l=1$ level and an isotropic level in the energy range of the proton group of Fig. 3. An $l=4$ angular distribution is fairly isotropic, much more so than is an $l=3$ angular distribution, so that the isotropic level observed in Ref. 4 is presumably the $l=4$ level found in the present study by the superposition technique.

Another example of superposing angular distributions for two l values to reproduce the experimental points is shown in Fig. 4. This example is the most uncertain case in which l values were assigned by the superposition technique. The proton group under consideration is fit by a relatively strong $l=2$ level and a weaker $l=0$ level. However, since an $l=0$ component is needed to fit only one experimental point, while a pure $l=2$ curve can account for the other points, the presence of an $l=0$ level

TABLE I. Results of the $Fe^{54}(d,p)Fe^{55}$ reactions.

	This paper			MIT reports		
(1) Excitation energy (MeV)	(2) l	(3) $(d\sigma/d\omega)_{max}$ (mb/sr)	(4) $(2j+1)S$	(5) Excitation energy (MeV)	(6) l	(7) $(2j+1)\gamma$ (keV)
0	1	17.2	3.25	0	1	78.3
0.417	1	6.63	1.20	0.413	1	28.0
0.935	3	1.69	3.60	0.933	3	57.1
1.327	3	0.166	0.329	1.322	(3)	8.9
1.413	(3)	~0.07	~0.14	1.413	~isotropic	
1.926	1	1.63	0.255	1.925	1	3.6
2.061	1	2.45	0.377	2.058	1	9.6
2.159	3	0.514	0.885	2.151	3	17.9
2.218	...	~0.07		2.218	~isotropic	
2.490	1	5.13	0.766	2.478	1	19.5
2.597	3 or 4	0.114	0.184 or 0.414	2.585	4	0.015
3.040	1	0.594	0.085	3.035	1	2.7
3.556	1	3.86	0.525	3.559	1	14.0
3.802	1 and 4	~10.8	~1.4	3.800	1	36.0
		~2.44	~7.4	3.860	~isotropic	
3.923	1	0.533	0.070	3.916	1	2.4
				4.028	~isotropic	
4.039	3	0.264	0.336	4.057	isotropic	
4.468	2	4.28	1.03	4.463	1	13.0
				4.507	1	2.2
4.54	(1)	~0.7	~0.09	4.538	1	4.3
4.716	2	1.33	0.306	4.707	1 or 2	...
5.001	...	0.432	...	4.999	1 or 2	...
5.123	1	0.382	0.046	5.124	1	1.4
5.406	2	0.516	0.106	5.394	1	3.0
5.564	2	0.188	0.037	5.564	2	0.55
5.792	0	2.81	0.187	5.687	0	0.005
5.796	(1)	0.911	0.104	5.775	1	4.2
5.91	2	0.751	0.140	5.900	1	3.6
5.96	(0)	0.414	0.028			
6.069	2	1.06	0.193			
6.167	2	2.58	0.460			
6.275	0	4.06	0.270	6.282	1	0.7
6.487	2	3.34	0.565			
6.639	2	2.05	0.342			
6.778	2	3.08	0.496			
6.852	...	~1.5	...			
6.908	2	~1.8	~0.29			
6.96	0	~0.71	~0.05	6.962	0	...
7.254	2	4.03	0.603			
7.367	2	1.43	0.212	7.310	1	...
7.413	2 and (0)	~0.75	~0.12	7.419	0	...
		~0.64	~0.04			
7.614	2	1.90	0.276			
7.808	2 and (0)	~1.2	~0.16			
		~0.91	~0.06			
7.853	2	~0.91	~0.13			
7.938	...	0.935	...			
8.028	2	0.594	0.082			
8.13	(2)	~1.35	~0.18			
8.18	(2)	~1.35	~0.18			
8.264	2	1.45	0.195			
8.00	...	~2.5	...			
8.40	...	~1.7	...			
8.514	2	0.795	0.106			
8.56	(0)	~0.98	~0.065			
8.796	2	1.12	0.141			
8.843	0	~2.2	~0.15			
8.910	0	~2.1	~0.14			
9.007	2	3.40	0.420			

TABLE II. Results of the $Ni^{62}(d,p)Ni^{63}$ reactions.

(1) Excitation energy (MeV)	(2) l	(3) $(d\sigma/d\omega)_{max}$ (mb/sr)	(4) $(2j+1)S$
0	1	5.00	0.747
0.088	3	1.47	2.39
0.158	1	7.22	1.065
0.526	1	2.14	0.306
1.008	1	4.83	0.663
1.306	4 and	~2.0 ~0.65	~6.1 ~0.09
1.686	~isotropic	~0.07	...
2.152	1	0.297	0.037
2.302	2	7.27	1.66
2.529	4	0.955	2.70
2.701	2 or 3	~0.8	~0.16 ~0.7
2.824	~isotropic	~0.09	...
2.960	0	5.63	0.375
3.100	1	0.529	0.061
3.173	isotropic	0.1	...
3.291	2	2.06	3.96
3.428	2	0.148	0.027
3.553	2	0.545	0.100
3.657	(2)	0.28	0.05
3.726	2	1.23	0.212
3.80	2	0.100	0.018
3.959	2	4.22	0.739
4.083	2	1.52	0.258
4.279	0	2.70	0.180
4.415	2	1.93	0.309
4.473	2	0.564	0.087
4.578	2	1.07	0.166
4.636	2	2.58	0.396
4.717	0	4.60	0.306
4.841	2	0.708	0.106
4.907	(0)	0.36	0.028
4.933	(2)	0.14	0.020
4.972	0	1.09	0.073
5.11	2	~0.48	~0.07
5.16	2	~0.46	~0.07
5.20	0	~1.5	~0.1
5.24	2	~1.0	~0.14
5.372	2	4.12	0.585
5.445	2	5.36	0.743
5.595	2	2.34	0.320
5.711	2	0.599	0.081
5.863	...	0.520	...
5.930	2	0.885	0.118
6.00	...	~0.46	...
6.07	2	~0.76	~0.10
6.16	...	1.42	...
6.28	2	1.38	0.178
6.32	...	~0.47	...
6.44	0	2.2	0.15
6.50	2	~2	0.2

TABLE III. Results of the $Ni^{64}(d,p)Ni^{65}$ reactions.

This paper				MIT reports		
(1) Excitation energy (MeV)	(2) l	(3) $(d\sigma/d\omega)_{max}$ (mb/sr)	(4) $(2j+1)S$	(5) Excitation energy (MeV)	(6) l	(7) $(2j+1)\theta^2 \times 10^{-3}$
0	3	1.01	1.49	0	3	13
0.065	1	8.74	1.23	0.063	1	15
0.315	1	1.27	0.173	0.311	1	1.9
0.699	1	4.62	0.615	0.691	2	13
1.021	4	2.88	8.31	1.017	3	23
1.33	1	0.130	0.017			
1.417	1	1.11	0.140	1.417	1	1.8
1.603	2	0.390	0.089			
1.779	1	0.122	0.015			
1.919	(2)	~6.3	~1.3	1.923	1	10
2.153	(1)	0.837	0.099	2.148	1	1.3
2.338	...	0.765	...	2.340	3	10
2.794	2	4.01	0.742	2.795	1	4.9
3.032	2	0.874	0.156	3.021	1	0.74
				3.046	0	0.67
3.13	(2)	0.234	0.041	3.113	0	0.40
				3.187	0	0.43
3.276	2	0.141	0.024	3.273	0	0.29
3.350	2	1.57	0.266	3.359	2	5.1
3.401	0	4.02	0.267	3.416	0	9.2
3.558	2	3.21	0.523	3.512	0	0.17
				3.570	2	15
3.740	2	1.67	0.263	3.750	2	7.2
3.892	2	2.74	0.422	3.915	1	3.0
4.006	2	0.669	0.101	3.968	3	4.1
				4.018	1	0.58
4.113	2	0.360	0.054	4.115	...	...
4.196	0	4.11	0.273	4.203	0	11
4.23	(2)	0.362	0.054	4.258	0	1.4
4.392	2	2.54	0.369	4.392	0	...
4.43	0	1.6	0.1	4.446	...	...
4.48	(2)	0.093	0.013	4.488	...	...
4.567	2	1.57	0.224	4.572	...	...
				4.597	...	...
4.64	2	3.46	0.489	4.635	...	...
				4.650	0	2.0
4.828	2	1.37	0.190	4.814	0	6.0
4.878	0	1.33	0.088	4.887	0	4.3
4.975	2	2.65	0.358	4.980	...	...
5.061	2	1.60	0.216	5.037	1	0.52
5.118	2	0.99	0.131			
5.191	2	2.15	0.285			
5.346	2	4.11	0.541			
5.50	...	0.58	...			
5.58	...	1.03	...			
5.62	...	0.86	...			
5.70	...	0.71	...			
5.81	...	1.23	...			
5.86	...	0.89	...			
5.94	...	0.88	...			
6.00	...	2.89	...			
6.09	...	1.58	...			
6.13	...	1.41	...			
6.21	...	2.24	...			
6.34	...	1.27	...			
6.44	...	3.29	...			
6.59	...	0.94	...			
6.69	...	1.08	...			
6.75	...	1.81	...			

in the observed proton group must be regarded as uncertain, and is marked as much in Table I. Although the $l=0$ level is assumed present in the computation of the center of gravity and spectroscopic factor for the $s_{1/2}$ state of Fe^{55} (see below), there will be practically no change in these quantities if the level were neglected.

It has been suggested on the basis of (d,t) and (He^3,α) reactions[6,7] that Fe^{55} has an $l=1$ level at about 1.4-MeV excitation energy. As Table I indicates, no $l=1$ level at that energy is observed in (d,p) reactions.

To obtain information on single-particle states from the data of Tables I–III, it is useful to assign values of the spin j, as well as of the angular momentum, to the nuclear levels. The (d,p) stripping reactions, taken alone, can determine only values of l, not of j, for these levels. In the present case, however, simple shell-model considerations eliminate the ambiguity in spin assignments for $l=0$ ($s_{1/2}$), $l=3$ ($f_{5/2}$), and $l=4$ ($g_{9/2}$). Ambiguities remain for $l=1$ ($p_{3/2}$ or $p_{1/2}$) and $l=2$ ($d_{5/2}$ or $d_{3/2}$).

Some indication of the spins of p levels may be obtained by comparing cross sections of (d,p) and (d,t) reactions proceeding to the levels in question. Specifically, the ratio of (d,t) to (d,p) cross sections should be approximately the same for all levels of a given spin, when corrections are made for Q dependence of cross sections.[8] Moreover, the ratio for spin-$\frac{3}{2}$ levels should be greater than the corresponding ratio for spin-$\frac{1}{2}$ levels by a large factor, the specific factor depending on the fullness of the $p_{3/2}$ and $p_{1/2}$ levels.

For purposes of comparison, supplementary (d,t) reactions were run to obtain cross sections for the low-lying states of[9] Fe^{55} and Ni^{63}. In Fe^{55}, the 0.417-MeV

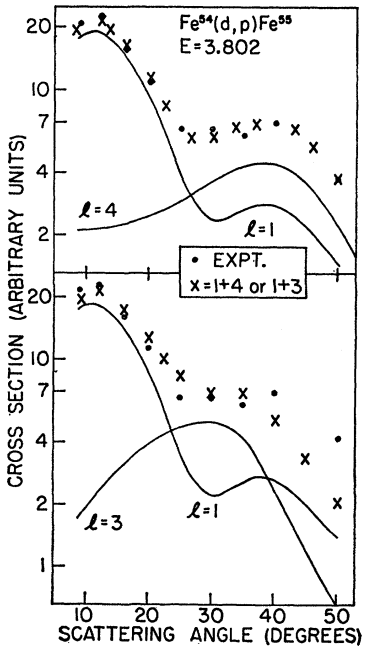

FIG. 3. Fitting of 3.802-MeV state of Fe^{55} by combination of empirically determined curves for $l=1$ plus $l=4$ and $l=1$ plus $l=3$. The circles are data points; the crosses are the summed contributions of the two l values.

[6] B. Zeidman, J. L. Yntema, and B. J. Raz, Phys. Rev. **120**, 1723 (1960).
[7] A. G. Blair and H. E. Wegner, Phys. Rev. **127**, 1233 (1962).
[8] Recent experimental evidence [B. L. Cohen and O. Chubinsky, Phys. Rev. **131**, 2182 (1963)] suggests that the validity of this assumption is limited when the nuclear levels are weakly excited.
[9] The authors are indebted to J. E. Hay for making this measurement.

level has the same $(d,t)/(d,p)$ cross-section ratio as the ground state, within 11%. Since the ground state is known to be $p_{3/2}$,[10] the comparison method indicates that the 0.417-MeV level is also $p_{3/2}$. The cross-section ratios for the three other p levels with less than 2.6-MeV excitation energy are smaller than the ratio required for $p_{3/2}$ levels by a factor of about 10; hence, these states probably have spin $\frac{1}{2}$. No (d,t) cross sections were available for levels beyond 2.6 MeV; it is conditionally assumed below that all p levels above this energy have spin $\frac{1}{2}$. From the requirement of equal spectroscopic factors for the $p_{3/2}$ and $p_{1/2}$ states in Fe^{54}, however (see Sec. IIIB), it seems unlikely that this assumption is correct; at least one p level above 2.6-MeV excitation energy in Fe^{55} probably has spin $\frac{3}{2}$.

In Ni^{63}, the p states below 1.1-MeV excitation energy were observed in (d,t) reactions. The $(d,t)/(d,p)$ cross-section ratios, in arbitrary units, are 5.0 (0), 16.6 (0.158), 15.9 (0.526), and 4.1 (1.008), where the numbers in parentheses are the excitation energies of the levels in MeV. Note that the ratios fall into two categories which differ by a factor of about 3.6. We tentatively conclude then, according to the above discussion, that the two levels with the larger ratio have spin $\frac{3}{2}$, while the two with the smaller ratio have spin $\frac{1}{2}$. If these conclusions are correct, the ground-state spin and parity of Ni^{63}, which has not been assigned a definite value previously, is $\frac{1}{2}^-$. As in Fe^{55}, p levels not investigated by (d,t) reactions are assumed to have spin $\frac{1}{2}$ because of their relatively high excitation energy.

The levels of the third even-odd isotope in this study, Ni^{65}, cannot be reached by (d,t) reactions, and so the comparison method cannot be used. A crude assignment of spins could be made by assigning the first two p levels as $p_{3/2}$ and the others as $p_{1/2}$. This arrangement of spins is roughly the same as that in the isotonic nuclide Zn^{67}, in which the spins of many low-lying levels are known.[10]

The second value of the angular momentum which permits two possible spin assignments is $l=2$. In this study, each observed level having $l=2$ is assigned as $d_{5/2}$.

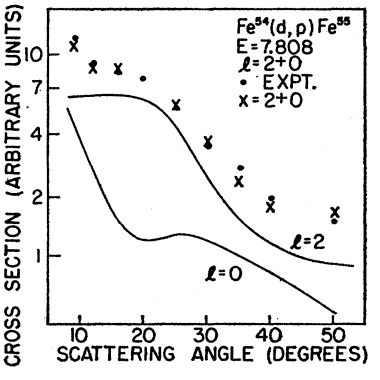

FIG. 4. Fitting of 7.808-MeV state of Fe^{55} by combination of empirically determined curves for $l=0$ plus $l=2$. The symbols are the same as in Fig. 3.

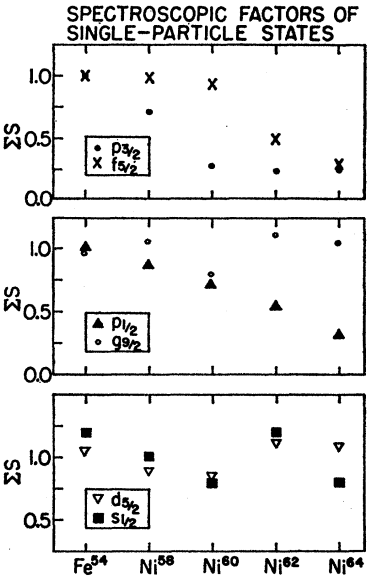

FIG. 5. "Normalized" spectroscopic factors of the single-particle states of the nuclei investigated.

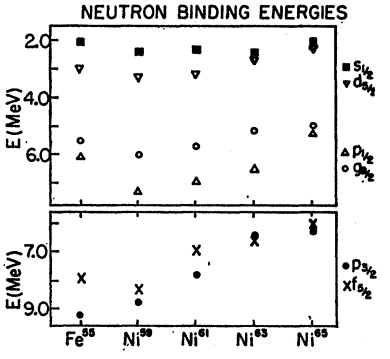

FIG. 6. Location of single-particle levels in the nuclei investigated. Locations of the $s_{1/2}$ level in Fe^{55}, Ni^{63}, and Ni^{65} are somewhat uncertain. See discussion in text.

This procedure is very probably valid for the low-lying d levels, since the shell-model $d_{5/2}$ state is expected to have an excitation energy about 3 MeV less than the $d_{3/2}$ state,[11] but the method becomes increasing unreliable as the excitation energy increases. Approximate limits on the number of $d_{5/2}$ levels are given by (1) the spectroscopic factor for the $d_{5/2}$ single-particle state, which ought to be unity (see below), and (2), the expected range of splitting for the $d_{5/2}$ state, about 3 to 4 MeV (see below). Each of these conditions is met by the assignment of all d levels as $d_{5/2}$. However, it is still quite likely that at high excitation energies some $d_{3/2}$ levels have been observed, since the $d_{3/2}$ state is also expected to be split over a large energy range.

B. Analysis of the Results

The results of Tables I–III can readily be used to find the total spectroscopic factors $\sum S$ of the single-particle states of the target nuclei; the value of $\sum S$ is just the sum of the individual spectroscopic factors of all nuclear

[10] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D. C.),

[11] B. L. Cohen, P. Mukherjee, R. H. Fulmer, and A. L. McCarthy, Phys. Rev. 127, 1678 (1962).

TABLE IV. Spectroscopic factors.

(1) Single-particle state	(2a) Fe^{54} obs	(2b) Fe^{54} norm	(3a) Ni^{58} obs	(3b) Ni^{58} norm	(4a) Ni^{60} obs	(4b) Ni^{60} norm	(5a) Ni^{62} obs	(5b) Ni^{62} norm	(6a) Ni^{64} obs	(6b) Ni^{64} norm
$p_{3/2}$	1.11 (1.36)	1.0	0.98	0.72	0.40	0.29	0.34	0.25	0.35	0.26
$f_{5/2}$	0.90	1.0	0.89	0.99	0.84	0.93	~0.46	~0.51	0.25	0.28
$p_{1/2}$	1.86 (1.36)	1.0	1.18	0.87	0.98	0.72	0.75	0.55	0.44	0.32
$g_{9/2}$	0.76	0.97	0.83	1.06	0.62	0.79	0.88	1.12	0.83	1.06
$d_{5/2}$	1.13	1.06	0.95	0.89	0.90	0.85	1.19	1.12	1.15	1.08
$s_{1/2}$	~0.6	1.2	~0.5	1.0	~0.4	0.8	~0.6	1.2	~0.4	0.8

levels belonging to a given single-particle state. Values of the total spectroscopic factors obtained in this way are listed in columns (2a), (5a), and (6a) of Table IV. For completeness, Table IV also lists, in columns (3a) and (4a), similar values for Ni^{58} and Ni^{60} from Ref. 12. The single-particle states of Table IV are listed approximately in the order of increasing energy.

It has been shown[13,14] that for even-even target nuclei, $\sum S(j)$ is identical with the quantity U_j^2 of pairing theory, where U_j^2 is the normalized probability that the single-particle state j is completely unpopulated. Since the twenty-eight neutrons of Fe^{54} form a closed shell, all the single-particle states investigated in this nucleus should be entirely empty and, hence, should give rise to $U_j^2=\sum S=1$.

The deviations from unity of the total spectroscopic factors of Fe^{54} can be assumed due to combined errors in the magnitudes of the DWBA predictions of $\sigma(l,\theta,Q)$, in the experimental absolute cross sections, and, for the p and d states, in erroneous spin assignments. For example, that more p levels should be assigned spin $\frac{3}{2}$ is indicated, as was said above, by the inequality of the p-state spectroscopic factors, $\sum S(p)$. The values of the spectroscopic factors of the p states may be made independent of spin assignments by requiring that, since the two states are equally empty, the spectroscopic factor should be the same for each. This requirement leads to the value 1.36 for $\sum S(p)$.

The spectroscopic factor with the largest deviation from unity is that for the $s_{1/2}$ state. The spectroscopic factor for this state is more difficult to determine than that for other single-particle states. Nuclear $s_{1/2}$ levels occur in an energy region of relatively high background and level density. The cross section can be measured best at only low scattering angles, where the intensity is high, but the theoretical value $\sigma(l,\theta,Q)$ is especially unreliable for low angles because the slope of the angular distribution there is very large (see Fig. 4). At higher angles, where the angular distribution is more smoothly varying, the low intensity of the levels against a generally high background adds an unusually large uncertainty to the determination of cross sections. The uncertainty is increased by the fact that $s_{1/2}$ levels occur up to the end of the investigated energy range. The total spectroscopic factor should be increased by an amount corresponding to the strength of any $s_{1/2}$ levels which might occur beyond this range. For the nuclei presently studied, the spectroscopic factors of the $s_{1/2}$ states, as listed in Table IV, are averages over three angles. The resulting values of about 0.6, in view of the uncertainties discussed above, are in fair agreement with the expected value of unity.

TABLE V. Single-particle level excitation energies in MeV.

Single-particle level	Fe^{55}	Ni^{63}	Ni^{65}
$p_{3/2}$	0.1	0.3	0.1
$f_{5/2}$	1.4	0.1	0
$p_{1/2}$	3.3	0.6	1.0
$g_{9/2}$	3.8	1.7	1.0
$d_{5/2}$	~6.7	4.1	3.7
$s_{1/2}$	~7.3	4.4	4.1

Many of the difficulties due to the uncertainties in DWBA calculations and other factors are avoided by "normalizing" the observed spectroscopic factors so that the empty states have a $\sum S$ equal to one. By this procedure the number unity is assigned to $\sum S$ for the $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ states of Fe^{54} and to the average value of $\sum S$ for the $g_{9/2}$, $d_{5/2}$, and $s_{1/2}$ states. The results of this normalization are listed in columns (2b)–(6b) of Table IV and are illustrated in Fig. 5.

The spectroscopic factors of Table IV and Fig. 5 show the expected trends. As the neutron number increases, the spectroscopic factors decrease for the states of lowest energy, $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$, indicating that the added neutrons populate these states. The spectroscopic factors for the other states seem to remain constant with increase in neutron number, indicating that the states of higher excitation energy do not fill appreciably in the mass region investigated.

In addition to spectroscopic factors, the energies, E_j, of the single-particle levels may be obtained from Tables I–III. We take these energies to be the "centers of gravity" of the reduced widths, or

$$E_j=\sum_i E_i S_i/\sum_i S_i, \quad (2)$$

[12] R. H. Fulmer, A. L. McCarthy, B. L. Cohen, and R. Middleton (to be published).
[13] S. Yoshida, Phys. Rev. **123**, 2122 (1961).
[14] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

where the summation is over all nuclear levels belonging to a particular shell-model state j. The values of E_j for the spin assignments discussed above are given in Table V and are illustrated, in the form of neutron binding energies, in Fig. 6. Figure 6 also includes similar values for the nuclei Ni^{59} and Ni^{61} from Ref. 12.

The binding energies of Fig. 6 show the energy gap expected between the major shells, that is, between the $g_{9/2}$ and $d_{5/2}$ levels. The energies also show a tendency to decrease with increase in neutron number. This effect indicates that the shell-model potential well may become shallower as the neutron excess increases.[15]

The data of Tables I–III may also be used to determine the energy distribution of levels belonging to a single shell-model state. The half-width of this distribution is expected to be about W, the depth of the imaginary potential in the optical model. Values of W are well determined at high excitation energies from elastic scattering studies; a crude extrapolation of these values to the low excitation energy region gives[2]

$$W \approx 0.33 E_j. \quad (3)$$

The experimental energy distributions are illustrated in Fig. 7, in which the various nuclear levels belonging to a particular shell-model state are shown by lines of height

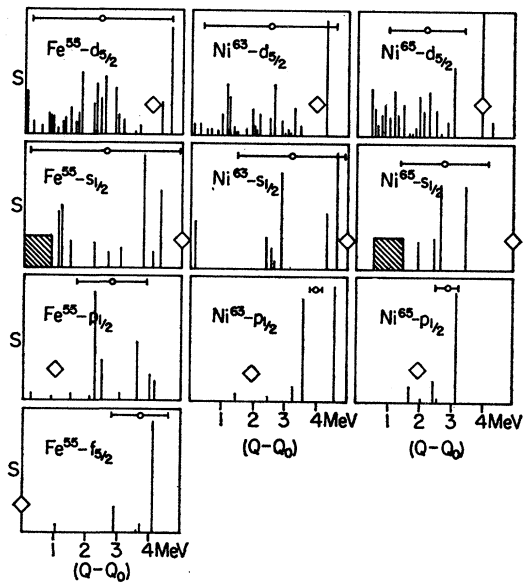

FIG. 7. Nuclear levels found in this work belong to the shell-model states designated. Vertical lines represent positions of levels, and their heights are proportional to the S values of Tables I–III; the latter are roughly proportional to the cross sections. The cross-hatched areas represent regions not investigated in these experiments. The open circles designate the center of gravity of these levels reported in Table V; the horizontal bars centered on the open circles designate the width of the single-particle levels expected from giant resonance theory. The origin of the abscissa scales, Q_0, is defined independently for each diagram. Its value may be found from the condition that for each diagram the open diamond is centered at $Q=2$ MeV.

[15] B. L. Cohen (to be published).

TABLE VI. Results of the $Fe^{57}(d,p)Fe^{58}$ reactions.

This paper (1) Excitation energy (MeV)	(2) l	(3) $(d\sigma/d\omega)_{max}$ (mb/sr)	(4) S'	(5) J^π	MIT (6) Excitation energy (MeV)
0	1	0.20	0.04	0^+	0
0.806	1	0.59	0.11	2^+	0.800
	and				
	3	0.12	0.29		
1.682	1	1.59	0.44	2^+	1.664
2.143	3	0.20	0.39		2.124
2.54	···	0.05	···		2.593
2.787	1	2.90	0.45		2.775
2.892	···	0.26	···		2.870
3.098	1	2.52	0.38		3.077
3.255	···	0.14	···		3.224
3.522	1	1.0	0.15		3.532
3.652	1	0.86	0.12		3.623
3.908	···	0.60	···		3.896
4.034	···	0.96	···		4.009
4.175	1	3.24	0.44		4.156
4.342	···	1.68	···		4.316
					4.348
4.454	···	1.05	···		4.437
					4.470
4.552	···	3.92	···		4.543
4.853	···	0.68	···		4.810
					4.829
4.912	(2)	0.91	0.24		4.928
5.008	1	2.59	0.33		4.998
5.113	1	0.45	0.06		5.144
5.332	···	0.68	···		5.383
5.428	···	1.61	···		5.414

proportioned to their reduced widths. The values of W predicted by (3) are shown in Fig. 7 as horizontal bars centered on the open circles, which denote E_j. In general, the agreement is quite good; this serves to support our assumptions that nearly all the $l=2$ states are $d_{5/2}$.

IV. EVEN-EVEN FINAL NUCLEI

A. Results

The results of the investigation of Fe^{58} and Ni^{62} by (d,p) reactions are presented in Tables VI and VII, respectively. Column (1) of these tables lists the energies, column (2) the angular momentum transfers, and column (3) the cross sections of the levels observed. The fourth column contains values of S', defined as the observed cross section divided by the DWBA parameter σ of Eq. (1); the fifth column lists the known spins; and the last column gives MIT energy values from Refs. 16 and 17.

Note from Table VII that the known 3^- level[18] at 3.77 MeV in Ni^{62} is not observed in these (d,p) reactions.

[16] Energy levels of Fe^{58} are reported by A. Sperduto, Massachusetts Institute of Technology Laboratory for Nuclear Science Progress Report, 1957 (unpublished), p. 53.

[17] Energy levels of Ni^{62} are reported by C. H. Paris, Massachusetts Institute of Technology Laboratory for Nuclear Science Annual Progress Report, 1958 (unpublished), p. 74.

[18] R. K. Jolly, E. K. Lin, and B. L. Cohen, Phys. Rev. **128**, 2292 (1962).

TABLE VII. Results of the $Ni^{61}(d,p)Ni^{62}$ reactions.

This paper					MIT
(1) Excitation energy (MeV)	(2) l	(3) $(d\sigma/d\omega)_{max}$ (mb/sr)	(4) S'	(5) J^π	(6) Excitation energy (MeV)
0	1	1.97	0.45	0^+	0
1.176	1	1.62	0.31	2^+	1.172
2.067	1	0.49	0.085	0^+	2.048
2.339	1	0.22	0.037	2^+	2.302
	and				
	3	0.26	0.55	4^+	2.336
2.896	1	0.77	0.12		2.888
3.170	1	0.72	0.11		3.155
	and				
	3	0.12	0.22		3.175
3.286	1	0.49	0.076		3.254
	and				
	3	0.46	0.82		3.267
3.393	1	1.67	0.26		3.367
3.536	1	2.09	0.32		3.516
3.867	1	2.58	0.38		
3.91	1	0.81	0.12		
3.963	1	0.51	0.075		
4.198	···	1.04	···		
4.449	···	0.34	···		
4.712	···	0.37	···		
4.894	···	1.51	···		
5.066	···	1.70	···		
5.324	2	1.38	0.38		
5.50	···	0.78	···		
5.62	2	1.27	0.32		
5.86	···	0.81	···		
6.11	2	2.78	0.63		
6.33	···	1.39	···		
6.39	···	0.92	···		
6.58	···	1.80	···		
6.74	0	1.52	···		

The (d,p) cross section for forming this level must, therefore, be less than ~0.1 mb/sr.

B. Discussion of Results

Estimates of spectroscopic factors for Fe^{58} and Ni^{62} can be obtained from the reactions $Fe^{57}(d,p)Fe^{58}$ and $Ni^{61}(d,p)Ni^{62}$. With the above definition of S', we have

$$S' = (2j+1)S,$$

where j is the spin transfer in the (d,p) reaction and S is the same as in Eq. (1) for reactions on even targets. Summing S' over all levels formed by a spin transfer j gives

$$\sum S'(j) = \sum (2j+1)S(j) = (2j+1)U_j^2. \quad (4)$$

Hence, $\sum S'$ indicates the "emptiness" of the shell-model state j. Values of $\sum S'$ for Fe^{58} and Ni^{62} predicted by Eq. (4) are presented in Table VII columns (2) and (4), respectively.

The spectroscopic factors of Fe^{58} obtained from Eq. (4) should be comparable with those of the isotonic nucleus Ni^{60}, as determined from the reaction $Ni^{60}(d,p)Ni^{61}$; and the spectroscopic factors of Ni^{62} obtained from (4) should be the same as the ones found from the $Ni^{62}(d,p)Ni^{63}$ reaction. Values of $\sum S'$ from the $Ni^{60}(d,p)$ and $Ni^{62}(d,p)$ reactions, obtained from Table IV, are listed in columns (3) and (5), respectively, of Table VIII.

TABLE VIII. Values of $\Sigma S'$ for Fe^{58} and Ni^{62}.

	A. Fe^{58}		B. Ni^{62}	
(1) Single-particle state	(2) $Fe^{57}(d,p)Fe^{58}$	(3) $Ni^{60}(d,p)Ni^{61}$	(4) $Ni^{61}(d,p)Ni^{62}$	(5) $Ni^{62}(d,p)Ni^{63}$
$p_{3/2}+p_{1/2}$	1.9	2.6	1.7	2.1
$f_{5/2}$	0.8	5.6	1.8	3.1
$d_{5/2}$	0.2	6.0	1.3	6.0

As Table VIII indicates, the $\sum S'$ obtained from the (odd neutron → even neutron) reactions are less than the corresponding values found from the (even neutron → odd neutron) reactions. We interpret this inequality as indicating that not all the proton groups from (odd neutron → even neutron) reactions were identified. Identification of the reactions is complicated by the fact that in many cases reactions to a nuclear state may proceed by more than one value of the angular momentum transfer. In such cases, the resulting angular distributions are superpositions of those due to single l transitions, and in the superposition it is often difficult to detect the presence of l transfers with a relatively small cross section. This fact would account for the large discrepancy in $\sum S'$ for the $f_{5/2}$ states and the smaller discrepancy for the p states, since in angular distributions which are admixtures of $l=1$ and $l=3$, the $l=1$ component is almost always stronger and, therefore, easier to detect, and small admixtures of $l=3$ might not be detected. The discrepancy in the values of $\sum S'$ for the $d_{5/2}$ state can be ascribed to the fact that the (odd neutron → even neutron) reactions were not studied over the entire energy range in which $d_{5/2}$ angular-momentum transfers should occur.

If the fullness of the shell-model state j is indicated by V_j^2, then from the definition of U_j^2 above, $U_j^2+V_j^2=1$. An upper limit on the fullness V^2 of the $p_{3/2}$ single-particle state in Ni^{62} can be obtained by applying to the $Ni^{61}(d,p)Ni^{62}$ ground-state reaction the relation[14]

$$d\sigma/d\omega \leq \sigma(l,\theta,Q)V^2, \quad (5)$$

where the inequality sign holds if the ground state is not the only $p_{3/2}$ level in Ni^{61}. Equation (5) predicts that for Ni^{62}, $U^2(p_{3/2}) \leq 0.67$. The equation may also be applied to the $Fe^{57}(d,p)Fe^{58}$ ground-state reaction, giving the result that $U^2(p_{1/2}) \leq 0.97$ for Fe^{58}. Both of the above limits are consistent with the summed spectroscopic factors of Table IV.

ACKNOWLEDGMENTS

The authors are greatly indebted to Bernard L. Cohen for many helpful discussions during the course of this work. Thanks are due also to R. Sass and R. Patell for assistance in the analysis of data and to F. Karasek and B. Zeidman of Argonne National Laboratory for providing the Ni^{61} target on loan.