Properties of Capture Gamma-Ray Spectra of *s*-Wave and *p*-Wave Resonances in Zr, Nb, and Mo†

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The capture gamma-ray spectra of neutron resonances in Zr^{91}, Nb^{93}, and Mo^{95} have been studied in order to make unambiguous identification of individual resonances resulting from *p*-wave capture. Features of the capture spectra which are expected to be correlated with the parity of the capture state are discussed. For the nuclides studied, *p*-wave capture was detected by observing anomalously high intensities of transitions from the resonant state to a group of low-lying states of positive parity. For Mo^{95}, previous measurements have been extended to 1 keV and resonances at 108, 215, 330, and 1010 eV have been assigned negative parity. For Zr^{91}, the total intensity of transitions to the ten states with lowest energy in Zr^{92} has been measured for resonances in Zr^{91} below 2050 eV. The total intensities for the resonances at 247 and 448 eV, and at least one resonance at 2050 eV, are approximately 10 times that of the other resonances and have been assigned a negative parity. For Nb, the interpretation of the spectra is complicated by the presence of a 3^- state at 40 keV in Nb^{94}. However, for transitions to states within 700 keV of the ground state of Nb^{94}, resonances at 34, 42, 94, 244, and 524 eV have total intensities 5 times that of the remaining resonances and have been interpreted as *p*-wave resonances. From the assignments for Nb^{93}, a value of $(4.6_{-1.6}^{+7.3})\times10^{-4}$ was obtained for the neutron *p*-wave strength function.

I. INTRODUCTION

FOR most nuclei, an interaction with neutrons having an orbital angular momentum greater than zero is expected to be inhibited by the centrifugal barrier in the nuclear potential to such an extent that the interactions will be insignificant at energies below 1 keV. Thus, the parities of low-energy resonances in such targets can be assigned with a high degree of confidence by assuming all resonances to be excited by *s*-wave neutrons. Exceptions to this rule are expected to occur in the mass region around $A=90$, where the optical model predicts a giant resonance in the neutron *p*-wave strength function. In agreement with this expectation, transmission measurements[1–3] on Zr, Nb, and Ag have revealed the presence of an anomalously large number of resonances with very small neutron widths; and these resonances have been interpreted as evidence for appreciable *p*-wave resonance capture. However, because of the nature of the Porter-Thomas law governing the distribution of neutron widths, a parity assignment based on a small neutron width cannot be certain; at best, it is only highly probable. In view of the current interest in *p*-wave capture and in the systematics of the *p*-wave strength function, a more positive means of separating resonances according to their parity is desirable. Such a means of identification would make possible a direct measurement of the resonance parameters of *p*-wave resonances, parameters which heretofore have had to be inferred from the average cross sections in the kV region. Of particular value to both nuclear structure theory and reactor physics would be a direct measurement of the total radiation width for *p*-wave capture and a direct determination of the *p*-wave strength function from the parameters of individual resonances.

In exploring possible means for determining the parity of resonances, one must note the considerable effort recently devoted to determining characteristics of capture gamma-ray spectra, which can be of use in the isotopic identification[4] of and spin assignment[5] to neutron resonances. In each of these cases, methods based on the study of capture spectra have been successfully applied to a broad range of nuclides. This success suggests the possibility of parity assignments on the basis of distinctive features of the capture-gamma-ray spectra. Two experiments were considered: (1) measurement of the angular distribution of primary capture gamma rays with respect to the neutron beam and (2) determination of the intensity of primary transitions to a group of levels of known parity. The first method was rejected because of the amount of time required to obtain data of sufficient accuracy to establish the existence of an anisotropic angular distribution typical of *p*-wave capture. An exploratory measurement of resonances in Mo^{95} based on the second method, which has already been reported,[6] indicates that the primary intensities of transitions to low-lying states exhibit an observable correlation with the parity of the radiating state. This paper reports subsequent measurements, the purpose of which was to develop a systematic technique for assigning parity and also to obtain a substantial group of *p*-wave resonances which can be useful for studying the dependence of the resonance parameters on the parity of the resonant state.

† Work performed under the auspices of the U. S. Atomic Energy Commission.
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[2] J. S. Desjardin, J. L. Rosen, W. W. Havens, Jr., and J. Rainwater, Phys. Rev. **120**, 2214 (1960).
[3] J. Morgenstern, C. Corge, V. D. Huynh, J. Julien, and F. Netter, Bull. Am. Phys. Soc. **7**, 288 (1962).
[4] H. E. Jackson and L. M. Bollinger, Phys. Rev. **124**, 1142 (1961).
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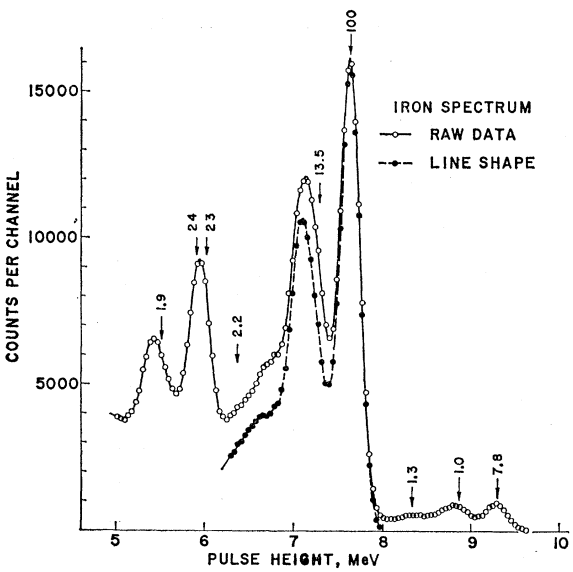

FIG. 1. Response function (solid circles) to 7.64-MeV gamma rays for a NaI(Tl) crystal 8 in. in diameter by 6-in. deep. It was obtained by correcting the observed spectrum (open circles) resulting from capture of thermal neutrons by iron. The numbers and arrows on the curve give the relative intensities and positions of known transitions.

In order to obtain a significant number of resonances that are excited by p-wave capture, the three nuclides chosen for study were Zr^{91}, Nb^{93}, and Mo^{95} which, according to average total cross-section measurements,[7] are in the immediate region of a peak in the p-wave strength function. In each of these cases the target nuclide is in a ground state of positive parity. At the same time, in the product nucleus the levels within 2 MeV of the ground states of the even-even nuclei Zr^{92} and Mo^{96} result from collective nuclear excitations and will, therefore, be of positive parity. The level scheme of Nb^{94}, an odd-odd nucleus, is more complex; but a qualitative argument will be presented to infer the dominance of levels of positive parity near the ground state. Thus, for these nuclides, p-wave neutron capture will result in negative-parity states which can decay directly by $E1$ radiation to low-lying states of the proper spin and positive parity, while s-wave capture will excite positive-parity states which must emit $M1$ radiation in transitions to the same states. Since a preliminary experiment[6] established the absence of $E2$ radiation, higher-order radiation can be neglected within the sensitivity of the present experiment. Therefore, to the extent that $E1$ radiation is stronger than $M1$ radiation, the intensity of transitions to low-lying states for which dipole radiation is permitted will depend on the parity of the neutron captured. For a nucleus with $A=90$, the Weisskopf estimate[8] of the ratio of the average partial width of an $E1$ transition to that of an $M1$ transition of the same energy from a state of the same spin is 105. Thus, it would appear that the parity of a resonance could be established by comparing the intensities of individual transitions to low-lying positive-parity states.

Unfortunately, individual transitions of a given multipolarity show wide fluctuations from resonance to resonance. These fluctuations are so large that, in spite of the large difference in average values, there is a considerable overlap in the distributions of the two types of widths[8] for nuclei of the same mass. Thus, in cases in which individual transitions must be considered, a conclusive parity assignment can be made only when the observed strength of a transition is much too large to be consistent with $M1$ radiation. If, however, one can measure the total intensity of transitions to a band of levels of positive parity, fluctuations will be suppressed by the averaging over many levels and the resultant intensities will be strongly correlated with the parities of the resonances. For these reasons, two variations of the method of parity assignment based on the observed intensities of primary transitions will be presented in this paper. One emphasizes individual intensities, the other the average intensity of transitions to a band of levels. Although the primary object of the paper is to describe the assignments made on the basis of these two methods, the results for each nuclide will be discussed as illustrative of the particular method used.

II. EXPERIMENTAL PROCEDURE

The basic experimental arrangement was the same as that used for the preliminary measurements. For details the reader is referred to Ref. 6. Briefly, the Argonne fast chopper was used to select neutrons of a known energy on the basis of their time of flight. One measurement of resonances in Mo^{95} above 100 eV was made with a flight path of 60 m and a time-of-flight resolution of about 40 nsec/m. For all other runs the resolution was approximately 80 nsec/m. Capture photons were observed in a large NaI crystal 8 in. in diameter and 6 in. deep. The Argonne three-variable magnetic-tape recording system[9] was used to record and analyze the data. It should be noted that use of this system permitted all resonance spectra for a given element to be measured in a single run under identical conditions of pulse-height gain and time-of-flight calibration. This feature permitted the simultaneous analysis of all resonance spectra in a given nuclide by use of only a singlet set of energies for the primary transitions and only one gamma-ray response function for the NaI crystal and minimized systematic errors in the measured

[7] H. W. Newson, R. C. Block, P. F. Nichols, A. Taylor, and A. K. Furr, Ann. Phys. (N. Y.) 8, 211 (1959); E. G. Bilpuch, K. K. Seth, C. D. Bowman, R. H. Tabony, R. C. Smith, and H. W. Newson, *ibid.* **14**, 387 (1961).

[8] For a discussion of single-particle estimates of radiation widths, see G. A. Bartholomew, Ann. Rev. Nucl. Sci. **11**, 259 (1961).

[9] C. C. Rockwood and M. G. Strauss, Rev. Sci. Instr. **32**, 1211 (1961).

values of the relative intensities of the transitions from resonance to resonance.

Because the absolute intensities of individual transitions in resonant spectra of Mo^{96} were of interest, the relative intensities were measured for each resonance and compared with the values for resonances at 45 and 160 eV to obtain absolute values. The procedure was as follows: (1) The relative strengths of primary transitions were determined by a simultaneous least-squares fitting of all the experimental resonance spectra by use of a measured response function of the NaI crystal and the relative positions of the transitions as given by the known level scheme for Mo^{96}. (2) The resulting numbers were normalized by using the number of counts between 2 and 3 MeV in each resonance spectrum as a measure of the relative number of captures. (3) The absolute intensities previously reported[6] for the 8.38-MeV transition in resonances at 45 and 160 eV were used as standards to obtain all the intensities in photons per capture.

The same type of analysis was performed for Zr and Nb, for which only the average intensity to a band of levels was of interest. A complete analysis to obtain individual intensities was necessary because the energy range spanned by the response function (Fig. 1) of the NaI crystal for monoenergetic gamma rays was larger than the region of interest. Consequently, the total number of counts in the interval is an underestimate of the total intensity and the relationship of the true intensity to the number of counts observed will depend upon the relative position of the lines. This problem was avoided by analyzing the spectra for the relative intensities of individual transitions in the same manner as for Mo^{96}. The resulting numbers were then added to obtain the average intensity of transitions to the region of interest.

III. RESULTS

In the Introduction, two variations of the method of assigning parity on the basis of the measurement of the intensities of primary transitions were mentioned—one emphasizing individual intensities, the other the average intensity to a band of levels. In this section each method is presented in detail and in application to the resonance capture spectra of a particular nuclide. Moreover, where necessary to complement the discussion, data are presented which was gathered to obtain information on the level scheme of the nuclide under study for use in the analysis of the resonance spectra. The errors and experimental limitations of each case will be pointed out and the results discussed as illustrative of the particular method used.

Mo^{95}

The preliminary work on Mo^{95} already reported[6] was concerned with resonances below 215 eV and only with the intensities of the 9.16- and the 8.54-MeV transitions.

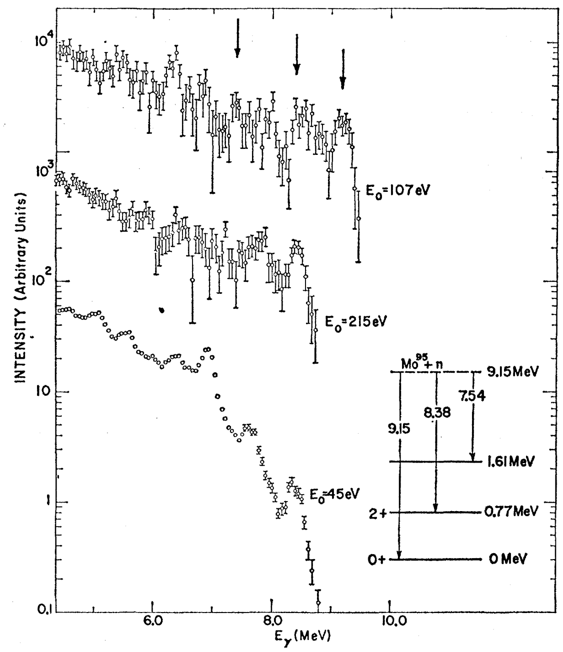

FIG. 2. Gamma-ray spectra from resonance capture in Mo^{95}. The arrows indicate the positions of the transitions which are shown in the level scheme for Mo^{96} (lower right).

In the present experiment, the study was extended to resonances up to 1100 eV and the spectra were analyzed for the intensity of the 7.54-MeV transition as well as the two of higher energy. The resonance at 45 eV can definitely be assigned even parity because of the large value of $g\Gamma_n^0$ and because the total cross section[10] in the off-resonance regions shows interference between the resonance and potential scattering. The measured values for the intensities of the transitions under study are small for this resonance and are consistent with s-wave neutron capture. The first strong transition in the resonance spectrum is observed at 6.92 MeV. It is likely that this transition is $E1$ and the 2.25-MeV level which it feeds is the first negative-parity level. Thus, at most only the five known levels below 2.25 MeV can be assumed to have positive parity. Consideration has been limited to the first three levels in order to obtain intensities free from systematic errors resulting from the high-energy tails of strong transitions to the energy region containing energy levels of both parities.

Three resonance spectra, chosen as characteristic of the possible cases which can occur in Mo^{96}, are shown in Fig. 2. The spectrum for the 45-eV resonance typifies the spectrum shape for s-wave capture. The intensity of radiation to the region above 7 MeV is low relative to that below 7 MeV. In contrast, the spectrum for the resonance at 215 eV does not show the sharp break above 7 MeV, and the intensity of transitions to the

[10] L. M. Bollinger, L. M. Bogan, R. T. Carpenter, R. E. Coté, H. E. Jackson, J. P. Marion, and G. E. Thomas, Argonne National Laboratory Report ANL-6534, 1962 (unpublished), p. 7.

TABLE I. Parameters for resonances in Mo^{95}.

Resonance energy (eV)	Absolute intensity (photons per capture) $E_\gamma=7.54$ MeV	$E_\gamma=8.38$ MeV	$E_\gamma=9.16$ MeV	k_{M1} (MeV^{-3}) $E_\gamma=7.54$ MeV ($\times10^{-1}$)	$E_\gamma=8.38$ MeV ($\times10^{-1}$)	Parity
45	0.014±0.002	0.006±0.001	0.0000±0.0005	0.21	0.09	+
107	0.016±0.008	0.019±0.004	0.041 ±0.008	...	...	−
118	0.012±0.006	0.018±0.005	−0.002 ±0.004	0.22	0.25	...
160	0.011±0.003	0.014±0.003	0.000 ±0.001	0.15	0.19	...
215	0.039±0.013	0.057±0.012	0.000 ±0.002	0.72	0.78	−
330	0.003±0.012	0.072±0.015	0.011 ±0.007	0.05	0.96	−
570	0.003±0.003	0.005±0.002	0.000 ±0.003	0.05	0.07	...
1010	0.008±0.004	0.001±0.004	>0.012 ±0.003	...	...	−

high-energy region. The third spectrum also fails to show a break above 7 MeV and, in addition, contains a high-energy component due to the transition directly from the capture state to the ground state.

The possible spin states of Mo^{96} formed by capture of p-wave or s-wave neutrons are shown in Fig. 3, together with the transitions which were studied. Although de-excitation from the compound state to the 0.77- and the 1.61-MeV states after s-wave neutron capture occurs by $M1$ radiation as noted in Sec. I, these transitions are expected to have measurable intensities in many s-wave resonances because of fluctuations in the corresponding partial radiation widths. This expectation is confirmed by the spectrum of the 45-eV resonance, in which a sizable primary transition to the 1.61-MeV state occurs. Clearly, a negative parity can be assigned to a resonance only when the observed strength of a transition is too large to be consistent with $M1$ radiation; and the multipolarity of a gamma ray of an observed strength can be assigned only if some statistical criterion is established. Because the average widths for $M1$ radiation in Mo^{96} were not known, the quantity determined was the reduced width which corresponded to the observed strength of each transition. The reduced width for $M1$ radiation is defined[8] here as $k_{M1}=\alpha\Gamma_\gamma/DE_\gamma^3$, where Γ_γ is the total radiation width of the resonance, α is the intensity of the transition, D is the spacing of states of the same spin and parity as the initial state, and E_γ is the gamma-ray energy. The reduced width, which is assumed to be independent of E_γ, facilitates comparison of the intensities of transitions in neighboring nuclei. A transition was somewhat arbitrarily considered to be inconsistent with s-wave capture if its reduced width was more than 8 times the observed mean value for nuclei between $A=70$ and $A=120$. The probability that a state which decayed by $M1$ radiation would have this width is less than 0.01 if the partial widths are assumed to be distributed according to the Porter-Thomas distribution. The experimental value of the mean reduced width for $M1$ radiation, taken from the compilation of Bartholomew,[8] is 0.9×10^{-2} MeV^{-3}. It is probably an overestimate since many small widths are missing because of the experimental difficulty in observing such weak transitions. In this respect the criterion used here to establish p-wave capture is conservative. Interpretation of the intensity of the primary transition to the ground state is less ambiguous. Of the possible compound states formed in s-wave and p-wave capture, dipole radiation to the ground state can occur only from a 1^- state. Therefore, the presence of an observable ground-state transition in a capture spectrum indicates p-wave capture in a resonance with spin and parity 1^-.

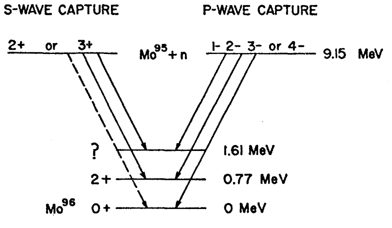

FIG. 3. Possible states formed by resonant capture of s-wave and p-wave neutrons by Mo^{96}. Also shown are the primary transitions to the first three states of Mo^{96}. The lowest order radiation which can be emitted in the dashed transition is $E2$, and is not observable within the sensitivity of this experiment. The spin of the 1.61-MeV level is not known.

Table I lists the intensities of the 7.54-, 8.38-, and 9.16-MeV transitions for resonances in Mo^{95} between 45 and 1010 eV and the reduced widths for the 7.54- and 8.38-MeV transitions. In addition to the experimental error tabulated for each intensity, all the intensities contain a systematic error which is directly proportional to error in the assumed value of the total radiation width. The value $\Gamma_\gamma=210\pm60$ MeV was used in this experiment. This error does not affect the reduced width which to first order is independent of Γ_γ.

The resonance at 1010 eV was only partially resolved in time of flight from neighboring resonances. Nevertheless, as noted in Table I, a clearly measurable ground-state transition indicates a spin and parity of 1^- for it, as well as for the resonance at 107 eV. The reduced widths for both transitions in the 215-eV resonance are more than eight times the assumed mean value for $M1$ radiation and, accordingly, a negative parity was assigned to the resonance. The 330-eV resonance contains a very strong 8.38-MeV transition for which the reduced width is more than ten times the mean value, so that the resonance has also been assigned a negative parity. Only one resonance in Mo^{95} (the one at 345 eV, which could not be resolved from a resonance in a different nuclide) was not included in Table I. A significant feature of our results is the large fraction of the resonances (almost half) which are p wave. In fact, this is a lower limit since the method used isolates only p-wave capture with strong $E1$ intensities for the transitions measured.

Zr^{91}

The low-lying levels of Zr^{92} and the available spin and parity assignments[11] are shown in Fig. 4. The first negative-parity level occurs at 2.35 MeV. Below this are ten levels, six of which already have been assigned positive parities and the indicated spins. For the purposes of this experiment, all ten were assumed to be of positive parity. In considering the statistical behavior of transitions to these ten states, the radiative decay of a compound state to a particular level has been assumed to be a single-channel process, with a partial width distributed according to a χ-squared distribution with 1 degree of freedom. To date, there is no experimental evidence for the correlation of partial widths of neighboring levels. If the individual partial widths are independent, then for n levels fed by a single-channel process the distribution of the sum of the individual widths will be χ squared with n degrees of freedom and the relative variance will be $(2/n)^{1/2}$. Consequently, the total intensity of transitions to a band of levels of definite parity will show less fluctuation from resonance to resonance and will be a statistically more accurate estimator of the mean partial width than the individual widths. Hence, in studying Zr^{91}, fluctuations were suppressed by measurement of the sum of the intensities of transitions from the capture state to the lowest ten states of Zr^{92}. To demonstrate the value of this procedure, the probability distribution function of the total intensity for s-wave and p-wave resonances was calculated approximately. The following five assumptions were made: (1) The sequence of spins and parities for the first 10 levels was based on the assumption that the spins and parities assigned by Marten *et al.*[12] for a sequence of levels built upon the ground state of a Zr^{90} core would also hold for a sequence of levels built upon the 0+ first excited state of Zr^{90}. Six of the spin values have been observed experimentally among the first ten levels of Zr^{92}. (2) The density of resonances with a given J is proportional to $(2J+1)$. As in Mo^{95}, the ground state of Zr^{91} has $J=\frac{5}{2}$ and the values for the resonance

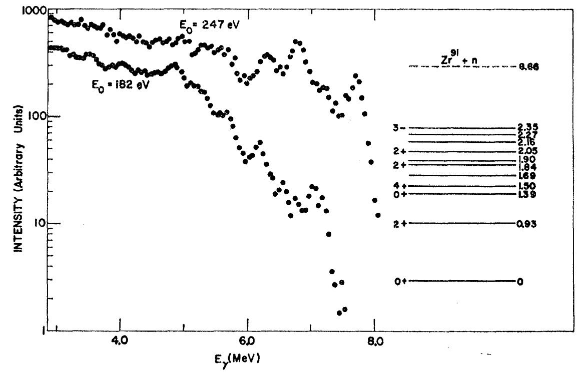

FIG. 4. Gamma-ray spectra from resonance capture in Zr^{91}. The spectra shown for resonances at 182 and 247 eV are typical of the two spectral shapes observed for resonances in Zr^{91} and have been interpreted as s-wave and p-wave spectra, respectively. Shown at the right is the Zr^{92} level scheme used in the analysis of the spectra.

spins which can result from s-wave and p-wave capture are identical to those shown for Mo^{95} in Fig. 1. (3) The ratio of the mean values of the partial radiation widths for $E1$ and $M1$ radiation is equal to the ratio of the Weisskopf estimates. (4) The individual partial widths for a given transition are distributed from resonance to resonance according to a Porter-Thomas distribution. (5) The mean values of the partial widths are proportional to the spacing of the states of the same spin and parity as the initial state.

The resulting distributions are shown in Fig. 5. Essentially all of the possible values of the relative intensity for s-wave resonances are included in the shaded region. The clear separation of the area under the p-wave curve from that under the s-wave curve implies that, in the absence of experimental difficulties, resonances can be separated on the basis of the average intensity of transitions to this band of states into two groups representing s-wave and p-wave capture. The reader should note that the present experimental evidence[13,14] on the distribution of partial radiation widths for a particular transition only limits ν, the number of degrees of freedom corresponding to the correct χ-squared distribution, to a number less than about two. If ν is appreciably larger than one, the separation of the distributions of the total intensity for s-wave and p-wave capture will be more favorable than that of Fig. 5.

Experimental difficulties preclude accurate measurement of intensities of transitions to more than the first few levels. These difficulties result from the broad multiple-peaked response function of the NaI crystal. In the present analysis, the relative positions of the first ten transitions were determined from the level scheme of Fig. 4. The spectra were then fitted by a

[11] D. T. Goldman and C. R. Lubitz, Knolls Atomic Power Laboratory Report KAPL-2163, 1961 (unpublished), p. 7.

[12] H. J. Martin, Jr., M. B. Sampson, and R. L. Preston, Phys. Rev. **125**, 942 (1962).

[13] L. M. Bollinger, R. E. Coté, and T. J. Kennett, Phys. Rev. Letters **3**, 376 (1959).

[14] R. E. Chrien, H. H. Bolotin, and H. Palevsky, Phys. Rev. **127**, 1680 (1962).

TABLE II. Parameters for resonances in Zr^{91}.

Resonance energy (eV)	Intensity (photons per capture) $E_\gamma=7.16$ MeV	$E_\gamma=7.27$ MeV	$E_\gamma=7.70$ MeV	$\sum_{i=1}^{10} I_i$	Parity
182	0.011 ± 0.002	0.002 ± 0.001	0.001 ± 0.001	0.03 ± 0.01	+
247	-0.022 ± 0.022	0.024 ± 0.010	0.101 ± 0.010	0.31 ± 0.03	−
448	0.073 ± 0.044	-0.014 ± 0.027	0.165 ± 0.009	0.25 ± 0.03	−
700	0.001 ± 0.007	0.004 ± 0.005	0.010 ± 0.002	0.02 ± 0.01	+
930	0.025 ± 0.013	0.005 ± 0.010	-0.002 ± 0.002	0.05 ± 0.02	+
1550	0.045 ± 0.020	-0.006 ± 0.013	0.012 ± 0.005	0.04 ± 0.02	+
2050	0.008 ± 0.020	0.000 ± 0.015	0.109 ± 0.005	0.32 ± 0.03	−

least-squares procedure in which the individual intensities and the position of the ground-state transition were free parameters. The errors in the intensities of the highest energy lines have a cumulative effect at lower photon energies, because each lower energy transition rides on the sum of the low-energy portions of the line shapes for the higher energy transitions. Because of this correlation in the errors of successively lower transitions, the values of the individual intensities are not significant. On the other hand, the sum of the intensities is determined with sufficient accuracy by simply adding the individual intensities.

In light of these difficulties, Table II lists only the intensities of the three transitions of highest energy and the total intensity of the first ten transitions for each resonance. Absolute values of intensities were obtained by calibrating the intensities in photons per capture for the 182-eV resonance according to the procedure used in the preliminary work on Mo^{95}. The absolute calibration was an approximate one, the error being about 30%. The sum of the individual intensities was also determined in photons per capture. Spectra typical of the two classes observed are shown in Fig. 4. A much larger component due to transitions above 6 MeV is present in the 247-eV resonance. The intensities and the parity assignments are listed in Table II. One resonance in Zr^{91} at 300 eV was not included in the tabulation because it was not resolved from another in Zr^{90} or Zr^{96}. The two classes qualitatively suggested by the spectra for the seven resonances are verified quantitatively by the observed values of the sum intensity. The average total intensity of the resonances at 247, 448, and 2050 eV is 8.3 times that of the remaining resonances. Thus, the separation of resonances predicted in Sec. II for Zr^{91} is verified, and the resonances with the high total intensities have been assigned negative parities.

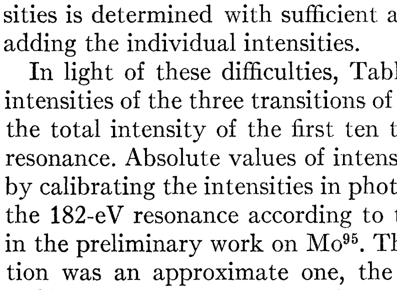

FIG. 5. Probability distributions for the sum of the partial widths for primary transitions to the first ten levels of Zr^{92} following resonance capture by p-wave and s-wave neutrons. The area under the distribution for s-wave capture is contained in the shaded region.

Nb^{93}

Nb^{93}, the third nucleus studied, differs from Zr^{91} in three respects that are relevant to the interpretation of the capture spectra of Nb^{94}. First, the compound state formed by p-wave capture is an odd-odd nucleus; and the levels near the ground state of Nb^{94} are characteristically much more closely spaced than those of Zr^{92} and Mo^{96}. Second, a negative-parity state[15] with $J=3$ and an energy of 42 keV has been observed in Nb^{94}. Third, the resonances at 194 and 384 eV are known with certainty to be s wave because transmission measurements[1] show interference between resonance and potential scattering. The small level spacing allowed restriction of these studies to transitions to states within 700 keV of the ground state while still keeping enough levels to suppress fluctuations. In the absence of detailed information on the level structure, the qualitative prediction of the shell model can be used to estimate the relative strength of positive-parity and negative-parity states. For Nb^{94}, the unpaired proton and neutron can have one of two nucleon configurations of opposite parities, $(g_{9/2})(d_{5/2})$ and $(p_{1/2})(d_{5/2})$, whose energy corresponds approximately to the ground state. The splitting of these levels caused by the interaction of the nucleon spins will result in three times as many states of positive parity as of negative parity. Therefore, in spite of the presence of the 3^- level at 40 keV, positive-parity states are expected to predominate by a factor of 3. To the extent that this is true, the average transition strength will reflect the parity of the capture state as it did in the spectra of Zr^{92}. Niobium possesses the additional advantage that the spectra of the resonances at 194 and 384 eV can be used to confirm the consistency of the procedure. The results for Nb indicate a more favorable discrimination than was anticipated from these arguments—probably because the nucleon

[15] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington, D. C.), NRC 60-5-106.

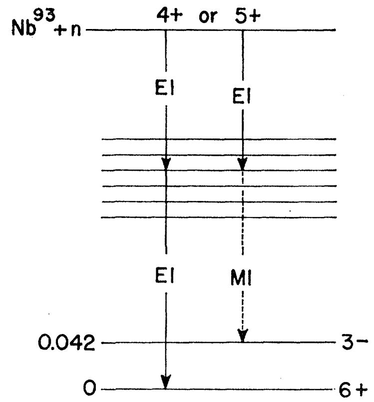

FIG. 6. Two-photon gamma-ray cascades to the ground state and first excited state of Nb^{94} following thermal neutron capture in Nb^{93}.

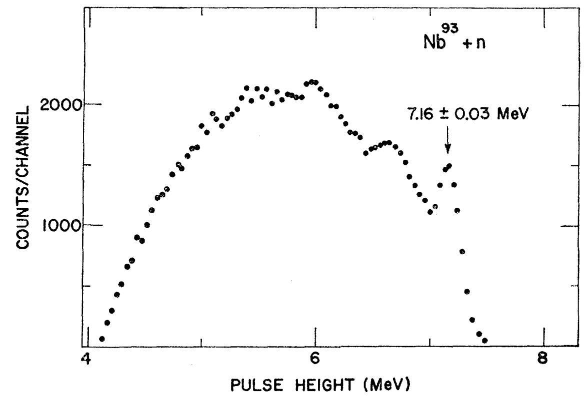

FIG. 7. Sum-pulse distribution corresponding to the total energy of two-photon gamma-ray cascades following thermal neutron capture in Nb^{93}. The peak corresponding to the cascade to the ground state is shown by an arrow.

configuration of next higher energy for the levels studied also had positive parity.

All the available information about the level structure of Nb^{94} comes from two studies. A 3-isomeric level[45] at 42 keV was observed in the β decay of Nb^{94}, and the gamma-ray spectrum[16] from thermal-neutron capture in Nb^{93} indicates primary transitions with energies of 5.90, 6.85, and 7.19 MeV. The accepted value of the binding energy is based mainly on the assumption that the 7.19-MeV transition goes directly to the ground state. However, if the thermal capture was dominated by a 4+ state, an $E1$ transition to the 42-keV state would be expected to dominate. In an effort to obtain a more reliable value of the binding energy, the sum-pulse spectrum[4] of two-photon cascades following thermal capture in Nb^{93} was measured. It was assumed that s-wave neutron capture dominates the thermal spectrum. The absence of p-wave capture is suggested by Bartholomew's measurement of the thermal capture spectrum of Nb^{93}, which shows only weak primary transitions to the region within 1.3 MeV of the ground state; their intensities are less than 0.008 photons per capture. Figure 6 shows the types of two-step cascades that are expected for s-wave capture. The sum-pulse spectrum was obtained by observing cascades through levels between 2 and 5 MeV above the ground state. In the region of the binding energy of Nb^{94}, the sum-pulse spectrum will be dominated by the cascade to the ground state since this cascade can proceed by the emission of two $E1$ photons in contrast to the cascade to the 0.042-MeV state in which an $E1$ and $M1$ photon will be emitted. The pulse heights were observed in two 8-in.×6-in. NaI crystals, and the individual coincidence spectra were added digitally by means of the magnetic-tape-analyzing system to obtain the sum-pulse spectrum which is shown in Fig. 7. The peak of highest energy (corresponding to the ground-state cascade) is 7.16±0.03 MeV. This number is consistent with the binding energy given in the nuclear data sheets, 7.20±0.03 MeV. For the analysis of this experiment we, therefore, used an average of the two, 7.18±0.02 MeV.

In order to obtain a more detailed level scheme, two more measurements were made. The first measured the spectra of coincident gamma rays resulting from capture of thermal neutrons. The range of energies accepted was approximately 60 keV to 2.5 MeV for both crystals. A resulting coincidence spectrum is shown in Fig. 8. Strong lines are present at 110 and 255 keV. The data were extensively studied in order to determine which lines were observed in coincidence; but the results were inconclusive. Furthermore, levels isolated by this procedure were not of primary interest in this experiment since they are not necessarily fed directly by primary transitions from the capture state.

In an effort to obtain data more directly related to analysis of the singles spectra, a second measurement was made. Coincidence spectra from resonance capture were recorded for events in which a primary transition with $E>6.0$ MeV was observed in coincidence with a

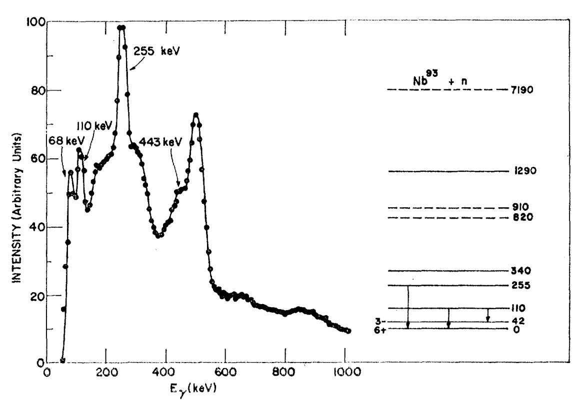

FIG. 8. Low-energy gamma-ray spectrum from thermal-neutron capture in Nb^{93}. Also shown at the right is the level scheme (energies in keV) obtained from subsidiary experiments (see text). The arrows on the level scheme indicate the principal transitions in the accompanying spectrum.

[16] G. A. Bartholomew and B. B. Kinsey, Can. J. Phys. 31, 1025 (1953).

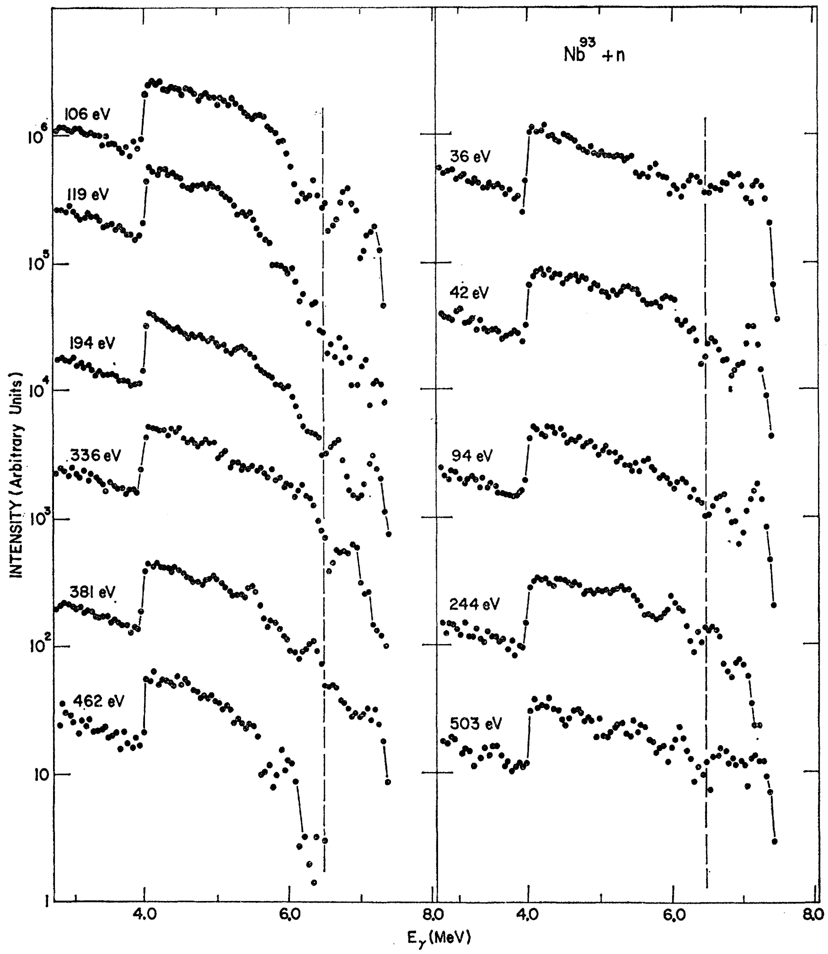

FIG. 9. Gamma-ray spectra from resonance neutron capture in Nb^{93}. The spectra have been arranged in two groups. In the group on the left, the total relative intensity of transitions with energies to the right of the dashed line is in the range 0.0–0.3; in the group on the right it is in the range 0.8–1.2.

low-energy photon. The capture rate in resonances was several orders of magnitude less than that at thermal energy. However, the uncorrelated fluctuations from resonance to resonance in the partial widths of the transitions result in the dominance of different lines in each resonance spectrum and compensates for the poor statistical accuracy of the individual spectra. A strong transition with an energy of 6.94 ± 0.02 MeV in coincidence with the strong 255-keV line of the thermal spectrum was observed in the spectrum of the 34-eV resonance. The value of the binding energy, 7.18 MeV, together with these data implies the existence of a state at 255 keV. A transition at 7.14 ± 0.05 MeV in the spectrum of the 44-eV resonance was observed to be in coincidence with the 110-keV transition observed in the low-energy thermal spectrum. A level was placed at 110 keV. A pair of levels was approximately located at 0.910 ± 0.02 and 0.820 ± 0.02 MeV by observing transitions of 6.27 and 6.36 MeV in each of two resonances at 44 and 384 eV. In addition, Bartholomew's observations of the 6.85- and 5.90-MeV transitions in the thermal spectrum were used to locate levels at 0.340 and 1.29 MeV. The level scheme resulting from these data, shown in Fig. 8, should not be regarded as complete but only as containing those states fed by strong primary transitions whose precise energies are important for the purpose of analysis of the singles spectra for the different resonances. In point of fact, the evidence from the singles spectra indicates at least two more primary transitions to the region of interest, and the results of the coincidence measurements for thermal neutrons indicate the presence of several states not included in Fig. 8.

The single-photon capture spectra with the background subtracted is shown in Fig. 9 for eleven resonances. Because only every fourth event was recorded for gamma rays with energies less than 4 MeV, a break in the curve is observed at that energy. The average intensity of gamma rays below this energy was used to determine the relative capture rate for each resonance in the analysis of the total intensity of transitions above 6.5 MeV. A qualitative measure of the relative intensity of the high-energy component can be obtained directly from Fig. 9 by comparing the average level of the high-energy region with that of the low-energy region. It is immediately apparent that the spectra comprise two families, one of which contains a more intense high-energy component. The spectra have been divided in this manner in Fig. 9. To establish this difference quantitatively, the spectra were analyzed in the same manner as were the data on Mo and Zr to obtain the individual relative intensities of transitions to the region of interest. These values were then added to obtain the total intensity for each resonance. The level scheme of Fig. 8 was used, but additional states at 620 keV and 1.13 MeV were necessary to obtain statistically consistent fits to the experimental data. The results are given in Table III. The classification of the spectra in Fig. 9 is confirmed. The relative intensity of the high-energy component ranges from 0.0 to 0.3 for one group and 0.8 to 1.2 for the second group. The mean intensity of the latter is 5 times that of the first group. Interpreting the high-intensity group as p-wave resonances leads to the assignments of the last column. These confirm two assignments[1] of even parity made by means of transmission measurements. Also tabulated is $2g\Gamma_n^0$, the reduced neutron width multiplied by the statistical factor. Of the seven resonances with the smallest values of $2g\Gamma_n^0$, five are p wave. Thus, the present results are consistent with the expectation that the neutron width is small for p waves because of the penetration factor for the centrifugal potential. Again, almost half the resonances in the interval studies are excited by p-wave capture.

V. DISCUSSION

As already pointed out, the neutron widths of p-wave resonances are expected to be small. However, this fact cannot be used in transmission measurement to isolate p-wave capture, because the Porter-Thomas distribution of neutron widths predicts a large number of s-wave resonances in a given population with small widths. Yet, with the recent improvements in the resolution and sensitivity of experimental techniques in neutron spectroscopy, small resonances—previously undetected—have become accessible to experimental study. To what extent studies of small resonances will be of significant value remains to be seen, but the existence of these resonances and the ambiguity introduced by ignorance of their parities casts doubt on even the existing data on the nuclides studied. The most striking feature of the present results for Zr^{91}, Nb^{93}, and Mo^{95} is that almost half of the resonances in the energy interval studied are excited by p-wave neutrons. The assumption of s-wave excitation for all resonances is probably adequate for determination of the s-wave strength function since the dominant contribution comes from large resonances which are excited by s-wave neutrons; but a parameter such as the resonance spacing has no meaning in the absence of definite parity assignments. The type of assignments demonstrated in this paper is also prerequisite to another study of current interest, namely, the variation of radiation width from resonance to resonance and the relation between this width and the parity of the capture state.

The successful parity assignments of large numbers of neutron resonances in Zr^{91} and Nb^{93}, as well as for neighboring nuclides, will also be of value in studies of the systematics of the interaction of p-wave neutrons. With sufficient resolution and neutron intensity, a direct determination of the p-wave strength function from resonance parameters of individual resonances is feasible. The p-wave strength function, as defined by Saplakoglu *et al.*,[1] was recalculated on the basis of the limited data available. The result was $S_1=(4.6^{+7.3}{}_{-1.6})\times10^{-4}$. One resonance at 320 eV was ignored in the analysis since its parity is unknown; the resulting ambiguity is included in the error quoted. The result is not substantially different from that quoted by Saplakoglu. As more extensive data become available, a precise measurement of S_1 using the parameters of

TABLE III. Parameters for resonances in Nb^{93}.

Resonance energy (eV)	Relative intensity of all transitions with $E_\gamma>6.5$ MeV	$2g\Gamma_n^0\times10^6$ (eV)	Parity assignment: From total cross section	Parity assignment: From γ-ray spectrum
36	1.2±0.1	25		−
42	0.8±0.1	20		−
94	0.8±0.1	36		−
106	0.3±0.1	37		+
119	0.1±0.1	257		+
194	0.2±0.1	2160	+	+
244	1.0±0.2	105		−
336	0.3±0.2	1190		+
381	0.3±0.1	5500	+	+
462	0.0±0.1	362		+
503	1.2±0.1	333		−

resonances whose parities are determined by the methods outlined here will be feasible; and comparison with the values of S_1 inferred from measurements of average cross sections will be of wide interest.

In summary, these experiments have demonstrated the success of two methods that use capture gamma-ray spectra to determine the parity of neutron resonances. Each has certain limitations. In the first method, which is based on the measurement of the intensity of individual transitions, anomalously large values of the reduced width k_{M1} are taken as evidence for p-wave interactions. Two disadvantages of this procedure are that (1) the partial width of a transition fluctuates from resonance to resonance so that not all p-wave resonances of a given spin can be isolated and (2) an individual transition is useful only for resonances of specific spin value. In the second method, the total intensity of transitions to a band of levels near the ground state is measured. This approach is useful only if most of the levels near the ground state have the same parity. This is not a serious disadvantage, however, since most nuclides satisfy this requirement. The second method has the advantage that assignments are unambiguous for all resonances since fluctuations and variations caused by the spin of the capture state are suppressed.

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Nuclear Spectroscopy of Bi^{210} with Stripping Reactions*

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Forty-four single neutron states are located in Bi^{210} with $Bi^{209}(d,p)$ reactions. These are compared with the levels predicted from the j–j coupling shell model. Of the ten levels predicted from a coupling of the $h_{9/2}$ proton and the $g_{9/2}$ neutron, nine are observed including the ground state. One of them is assumed to be composed of two close-lying levels, thus, accounting for all the ten levels. Proton angular distributions and total cross section for exciting these ten levels suggest that they have a very pure $(h_{9/2})_p(g_{9/2})_n$ configuration. A comparison of the relative cross sections for exciting these ten levels with the $2J+1$ statistical factor gives consistent spin values for these levels. Angular distributions of protons from several prominent groups in $Bi^{209}(d,p)$ reactions are compared with those from $Pb^{208}(d,p)$ reactions, identifying the $(d_{5/2})$, $(s_{1/2})$, and $(d_{3/2})$ neutron states in Bi^{210}. Effect of configuration mixing is discussed. The level density above 2.5-MeV excitation is found to be small compared to the shell-model prediction.

INTRODUCTION AND EXPERIMENTAL

FROM shell-model considerations Bi^{210} has a particularly simple configuration, a proton in the $h_{9/2}$ state and a neutron in $g_{9/2}$ state outside a doubly closed shell of protons and neutrons. The low-energy states in Bi^{210} will be determined by the single-particle levels available to the outer neutron and proton. In recent months a great deal of information on the location of neutron states in the $N=126$–184 shell has been obtained[1,2] as a result of (d,p) reactions on Pb^{208}. All the seven single-particle (s.p.) states in this shell are located. Starting from these s.p. states one can very easily build up the possible shell-model states in Bi^{210}, provided the corresponding proton levels are also known. Unfortunately, only a few such proton states are known[3] in Bi^{209}. Apart from the ground state of Bi^{209}, which is $h_{9/2}$, it has been established that the first excited state at 0.90 MeV is $f_{7/2}$. Several well-separated states are known in Bi^{209} from (n,n') and (p,p') reactions.[3] These include the above-mentioned $f_{7/2}$ state. The next level occurs at 1.60 MeV. From shell-model predictions this should be the $i_{13/2}$ proton state. Throughout our discussions in the present work we have assumed the 1.60-MeV state as $i_{13/2}$. As will be evident later, the (d,p) reaction analysis in Bi^{209} suggests that this level is not an odd-parity state (e.g., $p_{3/2}$ or $f_{5/2}$). If it were so, there would have been more states in Bi^{210} excited in (d,p) reactions near the $(d_{5/2})_n$ group.

It is apparent that in $Bi^{209}(d,p)$ reactions only those states in Bi^{210} are excited which have a large $(h_{9/2})_p(l_j)_n$ admixture. l and j stand for the orbital angular momenta and the total angular momenta, respectively, for the seven neutron states $g_{9/2}$, $i_{11/2}$, $j_{15/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$, in the $N=126$–184 shell. So, corresponding to a particular j-neutron state in Pb^{209}, one should observe in $Bi^{209}(d,p)$ reactions states with angular momenta ranging from $|j-\frac{9}{2}|$ to $j+\frac{9}{2}$. Since the neutron binding energy should be of the same order in Pb^{209} as in Bi^{210}, the above-mentioned levels should be observed approximately at the same Q values as in $Pb^{208}(d,p)$. Over and above these neutron states, one should expect to excite several other states in Bi^{210}, where the proton may be in the excited states $f_{7/2}$, $i_{13/2}$, etc. This becomes possible because of strong configuration mixing between the near lying $[(h_{9/2})_p(lj)_n]_{J\Pi}$ and $[(f_{7/2})_p(l'j')_n]_{J\Pi}$ states. A study of such states will lead us to understand the extent of configuration mixing in Bi^{210}.

Another helpful feature of the (d,p) reactions in this region at our available deuteron energy (14.8 MeV) is the marked difference in the angular distribution of the emitted protons at forward angles ($\theta<40°$) for the different l values of the captured neutrons. (See, for example, Fig. 3 in Ref. 1). Thus, a comparison of the

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