TABLE I. Summary of results.

Nuclide	Q_α (MeV)	Half-life	Alpha branch	δ^2 (MeV)
Er^{152}	4.93±0.02	10.7±0.5 sec	$0.90^{+0.05}_{-0.20}$	0.091
Er^{153}	4.80±0.02	36 ±2 sec	$0.95^{+0.05}_{-0.20}$	0.13
Er^{154}	4.26±0.02	4.5±1.0 min	...	...

level, however, these differences would tend to become smaller and as a result the reduced widths would be expected to become larger. Further work in progress on thulium, ytterbium, lutetium, and hafnium alpha emitters near the 82-neutron closed shell may indicate more clearly how δ^2 varies as more protons are added beyond $Z=64$.

The results obtained on the erbium alpha emitters are summarized in Table I.

ACKNOWLEDGMENTS

We would like to thank Dr. J. Alexander and Dr. G. Simonoff for their contributions by way of informative discussions and also C. Corum, J. Johnston, R. Latimer, W. Goldsworthy, A. Wydler, and M. Nakamura who assisted in developing the experimental system used in this work. The cooperation of the Hilac personnel is gratefully acknowledged.

PHYSICAL REVIEW VOLUME 131, NUMBER 5 1 SEPTEMBER 1963

Electromagnetic Properties of Li^7

J. H. VAN DER MERWE
Department of Applied Mathematics, University of the Witwatersrand, Johannesburg, South Africa
(Received 16 April 1963)

The energy level and magnetic data of Li^7 are more or less adequately explained by assuming an $(1p)^3$ configuration. However, there seems to be some discrepancy between the value for the quadrupole moment as predicted from this assumption and the measured value given by Kahalas and Nesbet. This discrepancy is explained in terms of a weak particle-surface coupling which need only affect the electric quadrupole operator.

RECENT investigations by Kahalas and Nesbet[1] have led them to assign a definite value to the quadrupole moment of Li^7. The purpose of the present paper is to consider this result together with the other well-known low-level electromagnetic properties of the Li nucleus, in order to determine whether they can be adequately accounted for in terms of the usual single-configuration assumption, so successfully employed in energy-level calculations of the $1p$ shell.[2,3] It will be shown that the introduction of configuration mixing of the kind manifesting itself as a weak coupling between individual-particle and nuclear surface motion is probably all that is needed to explain the data satisfactorily.

Assuming Li^7 to be adequately described by the single configuration $(1p)^3$, the most general wave function that one can write for the ground state is

$$\psi(J=3/2)=C_1\,{}^{22}P[3]+C_2\,{}^{22}P[21]+C_3\,{}^{24}P[21]+C_4\,{}^{22}D[21]+C_5\,{}^{24}D[21], \quad (1)$$

where the notation is ${}^{2T+1,\,2S+1}L[\lambda]$ and λ designates the spatial symmetry properties of the wave function. The magnetic moment μ and quadrupole moment Q are then given, respectively, by

$$\mu=3.12C_1^2-1.054C_1C_2-0.282C_1C_4-0.01C_2^2+3.98C_2C_3-0.56C_3^2+0.80C_3C_5+0.81C_4^2+5.328C_4C_5+0.39C_5^2 \quad (2)$$

in units of nm, and

$$Q/e\langle r^2\rangle=-0.24C_1^2+0.252C_1C_2-0.112C_1C_4-0.358C_2C_4-0.16C_3^2-0.48C_3C_5. \quad (3)$$

In energy-level calculations[2,3,4] with central and spin-orbit forces the ground state is predominantly ${}^{22}P[3]$, with the result that μ and Q are very insensitive to the variation of the parameters involved. These parameters usually are, in standard notation, $W,M,B,H,L/K,a/K$, with $W+M+B+H=1$. Thus, taking the force mixture to be that used by Inglis[2] and Kurath[3] (i.e., $M=0.8$, $B=0.2$), we obtain, after diagonalization of the 5×5 energy matrix[5] with which the ground state is associated and extraction of the eigenvector corresponding to the ground-state energy value:

[1] S. L. Kahalas and R. K. Nesbet, Phys. Rev. Letters **6**, 549 (1961). The quadrupole moment is given there by $Q/e=(-3.56\times10^{-26}\pm10\%)$ cm^2, but, according to a private communication from Dr. Kahalas, this value has been revised to -4.4×10^{-26} cm^2, with no real error estimate that can be associated with this value. Our conclusions, originally based on the first-mentioned value, were strengthened by this revision.

[2] D. R. Inglis, Rev. Mod. Phys. **25**, 390 (1953).

[3] D. Kurath, Phys. Rev. **101**, 216 (1956).

[4] J. M. Soper, Phil. Mag. **2**, 1219 (1957).

[5] See, for example, J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953).

For

$$4 \leq L/K \leq 8 \quad \text{and} \quad 1 \leq a/K \leq 2,$$
$$3.15 \leq \mu \leq 3.21;$$
$$-0.26 \leq Q/e\langle r^2\rangle \leq -0.24;$$
$$0.95 \leq C_1 \leq 0.997.$$

Using the rather different force mixture of Soper,[4] namely, $W=0.40$, $M=0.33$, $B=0.17$, $H=0.10$, gives results scarcely different from these. Even the inclusion of a weak tensor force should not seriously affect the predominance of the ${}^{22}P[3]$ state in the ground-state wave function.

Single-configuration analysis, therefore, gives a value for the magnetic moment which is quite close to the experimental value 3.256 nm. Taking into account that there are various effects,[6] most notably those arising from the presence of velocity-dependent interactions (e.g., spin-orbit coupling and exchange forces), which have been neglected above in setting up the magnetic moment operator but which contribute to the magnetic moment an amount often estimated to be of the order of 0.1 nm,[7] it can be stated with reasonable certainty that the measured value of the magnetic moment is consistent with the energy-level data. It may be noted also that no corrections have to be applied to the magnetic (or any other) moment on account of the center-of-mass motion of the alpha core, since, as Elliott and Skyrme have shown,[8] the center of mass always moves in an $1s$ state.

As an experimental value for the quadrupole moment we assume[1] $Q/e=-4.4\times10^{-26}$ cm^2. It is difficult to make an estimate of the value of $\langle r^2\rangle$, the mean-square radius of the $1p$ nucleons; however, considering the results given by various authors,[9] it seems reasonably safe to assume $\langle r^2\rangle<10^{-25}$ cm^2, which is sufficient for our purpose. Therefore, $|Q/e\langle r^2\rangle|>0.44$. Allowing for uncertainties and effects which are undeterminable but expected to be comparatively small, we conclude that the observed quadrupole moment is of the order twice (or more) the calculated value. It may be noted that at least the effects due to velocity-dependent interactions are of negligible importance here; because of gauge invariance the quadrupole moment operator is independent of this type of interaction.[10]

For an $M1$ transition the radiation width $\Gamma(M1)$ is given by[11]

$$\Gamma(M1)=2.76\times10^{-3}E^3\Lambda(M1), \tag{4}$$

where $\Gamma(M1)$ is in eV, E in MeV, and where[12]

$$\Lambda(M1)=\frac{1}{(2J_i+1)}\sum_{M_iM_f}|\langle J_fM_f|\mu_q|J_iM_i\rangle|^2 = |\langle J_i\|\mu\|J_f\rangle|^2 \tag{5}$$

is the transition strength, a dimensionless quantity, μ_q being the magnetic dipole operator in units $e\hbar/2Mc$. Taking the experimental value of the mean lifetime $\tau(=\hbar/\Gamma)$ of the 0.478-MeV level to be $(1.20\pm0.1)\times10^{-13}$ sec,[13,14] gives $16.8\leq\Lambda(M1)_{\text{Li}}\leq19.9$. Also available is the lifetime of the 0.431-MeV level of Be^7, as measured by Bunbury *et al.*,[15] who found the lifetime of this level to be $(2.7\pm1.0)\times10^{-13}$ sec. This gives $8.1\leq\Lambda(M1)_{\text{Be}}\leq17.5$.

The $E2$ transition probability of the forementioned level of Li^7 has been measured by Stelson and McGowan,[16] who obtained a value of 1.5×10^{-9} sec for the half-life. Corresponding to Eq. (4) one has[11]

$$\Gamma(E2)=8.08\times10^{44}E^5\Lambda(E2), \tag{6}$$

where

$$\Lambda(E2)=|\langle J_i\|Q\|J_f\rangle|^2, \tag{7}$$

with

$$Q_\mu=(16\pi/5)^{\frac{1}{2}}\textstyle\sum_j\frac{1}{2}[1-\tau_z(j)]r_j^2Y_\mu^2(j). \tag{8}$$

(The static quadrupole moment operator is equal to eQ_0.) The quoted half-life value corresponds to $\Lambda(E2)=1.51\times10^{-50}$ cm^4, remembering that $\tau_{1/2}=0.693(\hbar/\Gamma)$. The possible error given is 20%.

To compare with theory, we write the excited wave function as

$$\psi(J=\tfrac{1}{2})=C_1^*\,{}^{22}P[3]+C_2^*\,{}^{22}P[21]+C_3^*\,{}^{24}P[21]+C_4^*\,{}^{24}D[21]+C_5^*\,{}^{22}S[111]. \tag{9}$$

Then

$$\begin{aligned}\Lambda(M1)_{\text{Li}}=[&4.293C_1^*C_1+0.430C_2^*C_1+0.430C_1^*C_2\\&-0.835C_2^*C_2+1.283C_3^*C_2+0.8164C_5^*C_2\\&-4.054C_2^*C_3-1.536C_3^*C_3-0.289C_4^*C_3\\&-0.577C_1^*C_4+3.845C_4^*C_4+0.645C_3^*C_5\\&-2.061C_4^*C_5]^2\end{aligned} \tag{10}$$

and

$$\Lambda(M1)_{\text{Be}}=[0.620C_1^*C_1+0.620C_2^*C_2+0.981C_3^*C_3+1.316C_4^*C_5-(\Lambda(M1)_{\text{Li}})^{1/2}]^2. \tag{11}$$

Carrying through the energy matrix diagonalization procedure for the ground as well as for the excited

[6] R. J. Blin-Stoyle, Rev. Mod. Phys. **28**, 75 (1956).

[7] See, for example, A. M. Lane, Proc. Phys. Soc. (London) **A68**, 189 (1955).

[8] J. P. Elliott and T. H. R. Skyrme, Proc. Roy. Soc. (London) **A232**, 561 (1955).

[9] R. A. Ferrell and W. M. Visscher, Bull. Am. Phys. Soc. **1**, 17 (1956); R. Hofstadter, Rev. Mod. Phys. **28**, 214 (1956); B. C. Carlson and I. Talmi, Phys. Rev. **96**, 436 (1954); J. H. Fregeau and R. Hofstadter, *ibid.* **99**, 1503 (1955).

[10] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).

[11] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).

[12] The notation in this article is that used, for example, in M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[13] C. P. Swann, V. K. Rasmussen, and F. R. Metzger, Phys. Rev. **114**, 862 (1959).

[14] W. L. Mouton, J. P. F. Sellschop, and R. J. Keddy, Phys. Rev. **128**, 2745 (1962).

[15] D. St P. Bunbury, S. Devons, G. Manning, and J. H. Towle, Proc. Phys. Soc. (London) **A69**, 165 (1956).

[16] P. H. Stelson and F. K. McGowan, Bull. Am. Phys. Soc. **5**, 76 (1960).

states yields the proper wave function for each state, and hence the transition strengths can be determined. For the same variation of L/K and a/K as before, we get, again using the Inglis-Kurath force mixture, $18.2 \leq \Lambda(M1)_{\mathrm{Li}} \leq 18.7$ and $13.6 \leq \Lambda(M1)_{\mathrm{Be}} \leq 13.8$ with $0.95 \leq C_1, C_1^* \leq 0.997$. Hence, in the same way as for the magnetic moment, the measured lifetimes are approximately in agreement with the theoretical estimates. Since the same operator is involved in the transition and magnetic moment calculations, one can expect, for example, interaction effects to contribute to both in a small way.

It makes almost no difference to the transition strengths if the states involved are assumed to be pure $^{22}P[3]$ states. Doing this we get for Li^7 $\Lambda(E2) = (72/125) \times (\langle r^2 \rangle)^2$, which gives $\Lambda(E2) < 0.58 \times 10^{-50}$ cm^4.

Collecting all the evidence, it seems that disagreement between theory and experiment exists only for those quantities involving the electric quadrupole operator. This fact can be explained by introducing a very weak coupling between particle and surface motion, the theory and applications of which have been discussed by various authors.[17–20] Being very near to a closed shell, the collective type surface oscillations of $A=7$ nuclei have very high frequencies compared to the particle frequencies and the two types of motion are approximately independent, being coupled by a small perturbing interaction, which is usually written to first order as

$$H_{\mathrm{int}}(\alpha_{2\mu}, x) = -k \sum_{\mu i} \alpha_{2u} Y_\mu{}^2(\theta_i, \varphi_i). \qquad (12)$$

Here k is the coupling constant, α_{2u} the collective degrees of freedom measuring the surface deformation and $x = (r_i, \theta_i, \varphi_i)$ the coordinates of the loose particles, three in this case. The customary treatment is to expand the wave function in terms of the uncoupled states; in the weak-coupling limit we can write

$$\psi(I=J, M) = \psi(\alpha J; 00; JM) + \sum_{\alpha' J' NR} A_{\alpha' J' NR} \psi(\alpha' J'; NR; JM), \qquad (13)$$

where

$$\psi(\alpha' J'; NR; IM) = \sum_{\mu\mu'} C(J'RI; \mu'\mu M) \psi_p(\alpha' J' \mu') \psi_c(NR\mu). \qquad (14)$$

Here IM denotes the total angular momentum quantum numbers of the nucleus, αJ and $\alpha' J'$ denote the quantum numbers characterizing the individual-particle states, while N = number of phonons and R = total angular momentum characterize the states of the collective oscillation. The coefficients $A_{\alpha' J' NR}$ are small compared to unity on account of the great difference in phonon and individual-particle energies.

The collective part of the magnetic dipole operator is proportional to R_μ, which is diagonal with respect to N and R. Therefore, the contribution of the collective motion to the magnetic moment and $\Lambda(M1)$ is of second order only. The collective quadrupole operator, being a tensor of rank 2, contributes to Q and $\Lambda(E2)$ in first order, so that only these two are affected appreciably if the $A_{\alpha' J' NR}$ are sufficiently small.

The effect of surface coupling may be described in terms of the tendency of each nucleon outside the closed shell to deform the surface of the shell to match its own anisotropic distribution. The quadrupole moment thus induced is proportional to, and of the order of, the mass quadrupole moment of the particle state causing it. Weak surface coupling implies only a small perturbation of the particle motion of the closed shell, hence the effect is additive if there are several particles outside the closed shell. In this case one can deal with the effect mathematically by assigning to every nucleon outside the shell an additional charge, the same for protons and neutrons, proportional to the coupling constant k.[17] Thus, replacing the term $\frac{1}{2}[1-\tau_z(i)]e$ in the quadrupole operator by $\{\frac{1}{2}[1-\tau_z(i)]+C\}e$ and considering pure $^{22}P[3]$ states, we get

$$Q/e\langle r^2 \rangle = -\frac{6}{25}(1+3C), \quad \Lambda(E2) = \frac{72}{125}(1+3C)^2(\langle r^2 \rangle)^2.$$

Since only half the number of core particles are charged, $C \lesssim \frac{1}{2}$,[21] i.e., of the order that is needed.

Note added in proof. With regard to Ref. 16, it is perhaps necessary to point out that the term $B(E2)_{\mathrm{ex}}$ used there is equal to $\frac{1}{2}(5/16\pi)\Lambda(E2)$ in terms of the notation used here, the factor $\frac{1}{2}$ arising from the fact that the one symbol refers to excitation and the other to emission. Stelson and McGowan have kindly drawn out attention to a more detailed discussion of their experiments in Nucl. Phys. **16**, 92 (1960), where, incidentally, they also suggest that the discrepancy between the experimental and theoretical values for the $E2$-decay rate may be due to collective motion.

The author is greatly indebted to C. M. Vincent for some very helpful discussions. He also wishes to thank A. M. Starfield for valuable assistance in computer programming.

[17] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[18] D. C. Choudhury, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd **28**, No. 4 (1954).
[19] K. W. Ford and C. Levinson, Phys. Rev. **100**, 1 (1955).
[20] W. W. True, Phys. Rev. **101**, 1342 (1956).
[21] B. Mottelson, in *Proceedings of the International School of Physics "Enrico Fermi" (Nuclear Spectroscopy)* (Academic Press Inc., New York, 1960).