Radiative Corrections to Charged Pion Decays Mediated by Vector Bosons*

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(Received 22 April 1963)

The lowest order electromagnetic corrections to the ratio $\Gamma(\pi\to e+\nu)/\Gamma(\pi\to\mu+\nu')$ have been calculated in an intermediate vector boson theory. The results indicate that the ratio is relatively insensitive to the value of the boson's mass. The ratio agrees with the experimentally determined value.

I. INTRODUCTION

THE universal V-A Fermi theory[1] has proved quite successful as a description of the weak elementary particle interactions. Perhaps the most dramatic prediction of this theory is the value of the ratio R of the rates of the charged pion decay modes, $\pi\to e+\nu$ and $\pi\to\mu+\nu'$. The comparison of the predicted ratio with the results of a sensitive experiment necessitates the calculation of the radiative corrections to the decay modes. These corrections have been determined by a number of authors.[2] The results depend upon how the inner-bremsstrahlung contribution is taken into account. The experimental value of R has been determined by Anderson *et al.*,[3] and it agrees with the corrected theoretical values. In this paper we determine the sensitivity of R to the presence of a charged, weakly interacting intermediate vector boson (IVB). We first describe the form of the interactions to be considered and then determine the virtual photon and inner-bremsstrahlung corrections to the pion decay modes.

II. INTERACTION LAGRANGIAN

The interaction of a spin-one particle with the electromagnetic field can include the usual minimal term, a magnetic moment term, and an electric quadrupole term.[4] In view of the fact that the IVB has only electromagnetic and weak interactions, it has been suggested[5] that its electromagnetic coupling should be minimal, in analogy to the charged leptons. This is the type of electromagnetic interaction that was assumed for the IVB when its effects upon β and μ decay were calculated.[6] It was pointed out in those calculations that if the small momentum transfers are ignored, the contribution to $\Gamma_\mu/\Gamma_{O^{14}}$ is cutoff-independent. It was further stated that the momentum-transfer corrections introduce a quadratic divergence into the expression for the above ratio. These momentum-transfer contributions have been re-examined, and it is found that the cutoff dependence is only logarithmic for minimal electromagnetic coupling. However, if the IVB field W^μ is given a magnetic moment interaction $ie(\frac{1}{2}\Gamma)F_{\mu\nu}\times(W^\mu W^{\nu*}-W^\nu W^{\mu*})$, a quadratic divergence is introduced by the momentum-transfer corrections.

The cutoff dependence can be determined by examining the boson's electromagnetic self-energy parts after mass renormalization has been carried out. With only the minimal electromagnetic interaction, the quadratic divergence in the self-energy part is of the form $(e^2\Lambda^2/m^2)[(k^2+m^2)g_{\alpha\beta}-k_\alpha k_\beta]$, where m is the IVB mass, Λ is an ultraviolet cutoff, and k_α is the boson's four-momentum. The bracketed expression is the inverse propagator for the IVB. Hence, this divergence will be removed by the redefinition of the weak coupling constant. This coupling constant redefinition is carried out when any ratio of processes is computed. However, if we introduce a magnetic moment interaction, the IVB self-energy part includes a term

$$(e^2\Gamma^2\Lambda^2/m^2)\{g_{\alpha\beta}(k^2+m^2)[-13+15\ln(\Lambda/m)] +k_\alpha k_\beta[7-6\ln(\Lambda/m)]\}.$$

This term is not proportional to the reciprocal boson propagator. Thus, it is not removed by a redefinition of the weak coupling constant.

These considerations suggest that we use a minimal electromagnetic interaction for the IVB when calculating the radiative corrections to the pion-decay modes. This interaction is given in Eq. (2) of Ref. 6.

The decay of a charged pion into a pair of leptons is given by the phenomenological coupling

$$L_1=ia\partial_\mu\phi_\pi\bar{\psi}_l\gamma^\mu(1+i\gamma_5)\psi_\nu+\text{H.c.},\qquad(1)$$

where l represents either an electron or a muon and a is a weak coupling constant of dimension (length).[1] We assume that both the electron and muon neutrinos have mass zero. L_1 reduces the effects of the strong interactions of the pion to an effective local pion-lepton coupling. This is a reasonable assumption in view of the relatively low energies involved in the decay processes.[7]

To introduce the IVB we replace L_1 by

$$L_2=if\partial_\mu\phi_\pi W^{\mu*}+\text{H.c.}\qquad(2)$$

* Supported in part by the U. S. National Science Foundation. Part of this work was completed while the author was at the Johns Hopkins University, Baltimore, Maryland.

[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); E. C. G. Sudarshan and R. E. Marshak, *ibid.* **109**, 1860 (1958).

[2] S. M. Berman, Phys. Rev. Letters **1**, 468 (1958); T. Kinoshita, *ibid.* **2**, 477 (1959); Ya Smorodinskii and Hu Shik-K'e, Zh. Eksperim. i Teor. Fiz. **41**, 612 (1961) [translation: Soviet Phys.—JETP **14**, 438 (1962)].

[3] H. L. Anderson, T. Fujii, R. H. Miller, and L. Tau, Phys. Rev. **119**, 2050 (1960).

[4] S. A. Bludman and J. A. Young, Phys. Rev. **126**, 303 (1962).

[5] P. Meyer and G. Salzman, Nuovo Cimento **14**, 1310 (1959).

[6] R. A. Shaffer, Phys. Rev. **128**, 1452 (1962).

[7] See the first paper of Ref. 2.

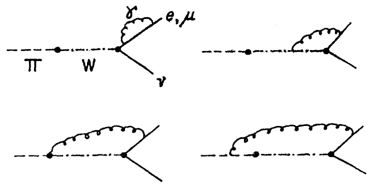

FIG. 1. Feynman diagrams that contribute to $\Gamma(\pi\rightarrow e+\nu)/\Gamma(\pi\rightarrow\mu+\nu')$.

and

$$L_3 = igW^{\mu}\bar{\psi}_l\gamma_\mu(1+i\gamma_5)\psi_\nu + \text{H.c.}, \qquad (3)$$

where f and g are semiweak coupling constants with

$$fg/m^2 = ai. \qquad (4)$$

With this definition of the coupling constant there is no nonradiative contribution to $\pi\rightarrow l+\nu$ from the IVB's propagator.[8] The semiweak and electromagnetic IVB interactions are not renormalizable. However, if the same cutoff is used in all expressions involving virtual photons, the ratio R is finite.

III. VIRTUAL PHOTON CONTRIBUTIONS

The lowest order virtual photon corrections to R are contributed by the Feynman diagrams shown in Fig. 1. We evaluate the Feynman integrals, square matrix elements, extract $O(e^2)$ contributions, and sum over final states. Assuming $m_\pi^2 \ll m^2$, we obtain

$$\begin{aligned} R_{\text{virt}} = R_0(\alpha/2\pi)[&\ln(M_\mu/M_e) + 2\mu(1-\mu)^{-1}\ln\mu - 2 \\ &\times\ln(M_e/m_\pi)\ln(\epsilon^2/M_e m_\pi) + (1+\mu)(1-\mu)^{-1} \\ &\times\ln\mu\ln(\epsilon^2/M_\mu m_\pi) - (m_\pi^2/m^2)\ln(M_\mu/M_e)], \end{aligned} \qquad (5)$$

where R_0 is the uncorrected ratio given by

$$R_0 = (M_e/M_\mu)^2[(m_\pi^2 - M_e^2)/(m_\pi^2 - M_\mu^2)]^2, \qquad (6)$$

$\alpha \cong 1/137$, $\mu = M_\mu^2/m_\pi^2$, and ϵ is a photon mass introduced to prevent an infrared divergence.

A finite IVB mass introduces the term $-(m_\pi^2/m^2)\times\ln(M_\mu/M_e)$ into Eq. (5). Since we expect $m > m_{K\text{meson}}$, the contribution to R is negligible. Thus, the virtual photon contribution to R is the same as that obtained with a local pion-lepton coupling.[9]

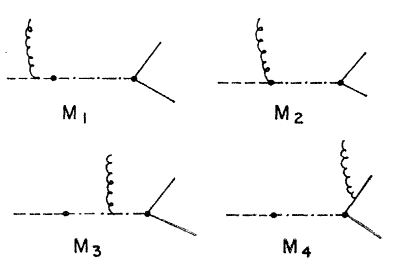

FIG. 2. Feynman diagrams that contribute to $\pi\rightarrow l+\nu+\gamma$ labeled by the corresponding matrix elements.

[8] See, for example, S. M. Berman, A. Ghani, and R. A. Salmeron, Nuovo Cimento **25**, 685 (1962).
[9] See Eq. (7) of the second paper in Ref. 2.

IV. INNER BREMSSTRAHLUNG

In this section we consider the diagrams shown in Fig. 2. For a pion at rest the contribution of M_1 is zero. The contribution of M_4 is independent of the IVB's mass. Squaring matrix elements, summing over final states, and assuming $m_\pi^2 \ll m^2$, we obtain for the total inner-bremsstrahlung contribution to $\pi\rightarrow\mu+\nu'$

$$\begin{aligned} P_{\text{IB}} = P_0(\alpha/\pi)\{&[(1+\mu)(1-\mu)^{-1}\ln\mu + 2] \\ &\times[\ln(\epsilon/m_\pi) - \ln(1-\mu) - \tfrac{1}{4}\ln\mu + \tfrac{3}{4}] \\ &-\mu(10-7\mu)[4(1-\mu)^2]^{-1} \\ &\times\ln\mu - 2(1+\mu)(1-\mu)^{-1}L(1-\mu) \\ &+(15-21\mu)[8(1-\mu)]^{-1} \\ &+(2\mu^2+5\mu-7)m_\pi^2[18m^2(\mu-1)]^{-1} \\ &\quad+m_\pi^2(1-3\mu^2)[6m^2(\mu-1)^2]^{-1}\ln\mu\}, \end{aligned} \qquad (7)$$

where P_0 is the uncorrected spectrum for $\pi\rightarrow\mu+\nu'$ and $L(1-\mu)$ is the Spence function $-\int_0^{1-\mu}[\ln(1-x)/x]dx$. We see that again the IVB contribution is small. For $\mu\rightarrow e$ it amounts to a less than 0.1% correction to P_0. For $m\rightarrow\infty$ we obtain Eq. (4) of Ref. 9.

We are also interested in the probability of the emission of a charged lepton with energy near the two-body decay energy E_0. To determine this probability we first calculate the probability of the emission of a lepton with energy less than $E_0 - \Delta E$, where $\Delta E \ll E_0$. We obtain

$$\begin{aligned} P_{\text{IB}}(\Delta E) = -P_0(\alpha/\pi)\{&[(1+\mu)(1-\mu)^{-1}\ln\mu + 2] \\ &\times[\ln(m_\pi/2\Delta E) + 2\ln(1-\mu) - \tfrac{3}{4}] \\ &+\mu(10-7\mu)[4(1-\mu)^2]^{-1}\ln\mu \\ &+2(1+\mu)(1-\mu)^{-1}L(1-\mu) - (15-21\mu) \\ &\times[8(1-\mu)]^{-1} - (2\mu^2+5\mu-7)m_\pi^2 \\ &\times[18m^2(\mu-1)]^{-1} - (1-3\mu^2)m_\pi^2 \\ &\quad\times[6m^2(\mu-1)^2]^{-1}\ln\mu\}. \end{aligned} \qquad (8)$$

We have the same small contribution from the IVB. Again, for $m\rightarrow\infty$ we obtain Kinoshita's result. The probability of the emission of a charged lepton with energy near E_0 is now given by

$$\begin{aligned} P_{\text{IB}} - P_{\text{IB}}(\Delta E) = P_0(\alpha/\pi)&[(1+\mu)(1-\mu)^{-1}\ln\mu + 2] \\ &\times\ln[\epsilon(1-\mu)/(2\Delta E\mu^{1/4})]. \end{aligned} \qquad (9)$$

This expression is independent of the IVB mass.

V. THE RATIO OF THE Π DECAY MODES

From the results in Secs. III and IV, we see that the IVB's effect upon the charged pion decay modes is very small, if the boson's mass is greater than the mass of a K meson. The contributions of the lowest order radiative corrections to R for a direct pion-lepton coupling are given in Refs. 7 and 9. These corrections are quite large for the cases in which ΔE is $O(M_e)$. The experiment of Anderson *et al.* confirms the existence of the radiative corrections. However, the uncertainty in the experimental results is about 6%, and we see that it would require an enormous increase in experimental sensitivity to detect the presence of an IVB by observing the pion decay modes.

It seems that the existence of an IVB will be determined by examining certain weak and electromagnetic production processes.[10] The possible resolution of the O^{14}-μ coupling constant discrepancy in a universal V-A theory and the introduction of structure into weak scattering processes to preserve unitarity[11] continue to be the chief theoretical justifications for the conjectured existence of an IVB.

ACKNOWLEDGMENTS

The author would like to thank Professor G. Feldman for suggesting this problem and Professor W. G. Holladay for many helpful discussions.

[10] T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960), and Ref. 4.

[11] T. D. Lee, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1961), p. 567.

PHYSICAL REVIEW VOLUME 131, NUMBER 5 1 SEPTEMBER 1963

Forward and Backward "Selection Rule" in a Class of Inelastic Collisions

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(Received 25 April 1963)

A rigorous rule which forbids forward and backward scattering is established for a certain class of inelastic collisions. The rule depends only on conservation of angular momentum and parity, and is therefore true, independent of the interaction and of the mechanism of the reaction. The class of collisions (called, in what follows, parity unfavored) consists of two particles A,B of spin zero colliding to produce a particle C of spin zero and D of spin S with the intrinsic parities P_A, P_B, P_C, and P_D satisfying the condition $P_AP_BP_CP_D=(-1)^{S+1}$. It is also remarked that in a parity-unfavored reaction the particle with spin S which is produced is always aligned with respect to the direction of the incident beam. Some applications in elementary-particle interactions and low-energy nuclear reactions are briefly discussed.

THE purpose of the present note is to point out the existence of a rigorous selection rule, due only to parity and angular momentum conservation, in a restricted but important class of collisions. The rule is as follows: Consider a reaction where two particles A and B of spin zero collide to produce a particle C of spin zero and a particle D of spin S. Assume that parity is conserved in the process and that the product of the intrinsic parities P_AP_B in the initial state is equal to $(-1)^{S+1}P_CP_D$, where P_C and P_D are the intrinsic parities of the final particles; or assume equivalently that

$$P_AP_BP_CP_D=(-1)^{S+1}. \tag{1}$$

Then, independent of the interaction and of the mechanism of the reaction, the angular distribution vanishes in the forward and backward directions. In the above, by particle we mean any elementary or composite system. If we denote as "parity unfavored" a reaction of the kind described above for which (1) is satisfied, the rule may be expressed simply as: "Parity-unfavored reactions are forbidden in the forward and backward directions."

The proof of the rule is straightforward: Consider the partial wave with orbital angular momentum l in the initial state; since the initial particles both have spin zero, l is also the total angular momentum. As a result of the collision, this ingoing wave gives rise to outgoing waves having orbital angular momentum l'. Due to the conservation of parity and of angular momentum, l' must satisfy the two conditions:

$$|l-S|\leq l'\leq l+S, \tag{2}$$

$$(-1)^l=(-1)^{l'+S+1}, \tag{3}$$

the second of which simply expresses the fact that for even l,l' has to be even or odd depending on whether S is odd or even. This is due to parity conservation and to the assumed relation between the initial and final intrinsic parities. Therefore, with the z axis taken in the direction of the incident beam, the angular part of the outgoing wave l' corresponding to the ingoing wave l is written as follows:

$$\psi_{l'}(\theta,\varphi)\sim\sum_{m'} C_{l'S}(l0|m',-m')Y_{l',m'}(\theta,\varphi)\chi_{S,-m'}, \tag{4}$$

where $\chi_{S,m}$ are the spin functions and $C_{l'S}(l0|m',-m')$ are the Clebsch-Gordan coefficients. Now, since $Y_{lm}(0,\varphi)=Y_{lm}(\pi,\varphi)=0$, for $m\neq0$, the only term in (4) which gives rise to a nonvanishing contribution in the forward or backward direction is that corresponding to