experimental results is about 6%, and we see that it would require an enormous increase in experimental sensitivity to detect the presence of an IVB by observing the pion decay modes.

It seems that the existence of an IVB will be determined by examining certain weak and electromagnetic production processes.[10] The possible resolution of the O^{14}-μ coupling constant discrepancy in a universal V-A theory and the introduction of structure into weak scattering processes to preserve unitarity[11] continue to be the chief theoretical justifications for the conjectured existence of an IVB.

[10] T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960), and Ref. 4.

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[11] T. D. Lee, in *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1961), p. 567.

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Forward and Backward "Selection Rule" in a Class of Inelastic Collisions

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A rigorous rule which forbids forward and backward scattering is established for a certain class of inelastic collisions. The rule depends only on conservation of angular momentum and parity, and is therefore true, independent of the interaction and of the mechanism of the reaction. The class of collisions (called, in what follows, parity unfavored) consists of two particles A,B of spin zero colliding to produce a particle C of spin zero and D of spin S with the intrinsic parities P_A, P_B, P_C, and P_D satisfying the condition $P_AP_BP_CP_D=(-1)^{S+1}$. It is also remarked that in a parity-unfavored reaction the particle with spin S which is produced is always aligned with respect to the direction of the incident beam. Some applications in elementary-particle interactions and low-energy nuclear reactions are briefly discussed.

THE purpose of the present note is to point out the existence of a rigorous selection rule, due only to parity and angular momentum conservation, in a restricted but important class of collisions. The rule is as follows: Consider a reaction where two particles A and B of spin zero collide to produce a particle C of spin zero and a particle D of spin S. Assume that parity is conserved in the process and that the product of the intrinsic parities P_AP_B in the initial state is equal to $(-1)^{S+1}P_CP_D$, where P_C and P_D are the intrinsic parities of the final particles; or assume equivalently that

$$P_AP_BP_CP_D=(-1)^{S+1}. \tag{1}$$

Then, independent of the interaction and of the mechanism of the reaction, the angular distribution vanishes in the forward and backward directions. In the above, by particle we mean any elementary or composite system. If we denote as "parity unfavored" a reaction of the kind described above for which (1) is satisfied, the rule may be expressed simply as: "Parity-unfavored reactions are forbidden in the forward and backward directions."

The proof of the rule is straightforward: Consider the partial wave with orbital angular momentum l in the initial state; since the initial particles both have spin zero, l is also the total angular momentum. As a result of the collision, this ingoing wave gives rise to outgoing waves having orbital angular momentum l'. Due to the conservation of parity and of angular momentum, l' must satisfy the two conditions:

$$|l-S|\leq l'\leq l+S, \tag{2}$$

$$(-1)^l=(-1)^{l'+S+1}, \tag{3}$$

the second of which simply expresses the fact that for even l,l' has to be even or odd depending on whether S is odd or even. This is due to parity conservation and to the assumed relation between the initial and final intrinsic parities. Therefore, with the z axis taken in the direction of the incident beam, the angular part of the outgoing wave l' corresponding to the ingoing wave l is written as follows:

$$\psi_{l'}(\theta,\varphi)\sim\sum_{m'} C_{l'S}(l0|m',-m')Y_{l',m'}(\theta,\varphi)\chi_{S,-m'}, \tag{4}$$

where $\chi_{S,m}$ are the spin functions and $C_{l'S}(l0|m',-m')$ are the Clebsch-Gordan coefficients. Now, since $Y_{lm}(0,\varphi)=Y_{lm}(\pi,\varphi)=0$, for $m\neq0$, the only term in (4) which gives rise to a nonvanishing contribution in the forward or backward direction is that corresponding to

$m'=0$. But, as is well known,[1]

$$C_{l'S}(l0|0,0)=0, \tag{5}$$

if $l'+l+S$ is odd, which is precisely the case in view of (3); therefore, $\psi_{l'}$ vanishes (like θ) in the forward or backward direction.

To clarify the meaning of the rule, assume first that the two ingoing and two outgoing particles all have the same intrinsic parity, or more generally, that $P_AP_BP_CP_D=1$. Then the rule asserts that no forward or backward scattering is possible for $S=1, 3, 5, \cdots$.[2] If, on the other hand, there is a change of intrinsic parity from the initial to the final state (that is, $P_AP_BP_CP_D=-1$), then no forward or backward scattering is possible for $S=0, 2, 4, \cdots$.

Before proceeding to indicate a few possible applications of the rule, the following remarks are necessary:

(1) It is, of course, essential to give an order of magnitude for the opening of the forward (and backward) cone inside which the angular distribution is expected to have a θ^2 behavior. This is easily done for short-range interactions [Coulomb interactions will be considered below under (2)]. The behavior of the scattering amplitude as a function of θ for small θ for an outgoing wave with orbital angular momentum l' can be established by remarking that $Y_{l'm'}$ has the following θ dependence[3] for small θ:

$$Y_{l'm'}\sim(\sin\theta)^{|m'|}\times\left\{1-\frac{(l'-|m'|)(l'+|m'|+1)}{|m'|+1}\frac{\theta^2}{4}+\cdots\right\}. \tag{6}$$

In the sum (4) over m', each $Y_{l'm'}$ can, therefore, be replaced for small θ by (6); for small θ the terms with $|m'|=1$ are the dominant ones; such terms have a linear dependence on θ if

$$\theta<(8/l'(l'+2))^{1/2}\cong 1/l'.$$

The condition of linearity in θ of the scattering amplitude is, therefore, for an outgoing wave with orbital angular momentum l':

$$\theta<l'^{-1}.$$

For an interaction with range a, the maximum l' is $\sim pa$, where p is the momentum of the outgoing wave; the scattering amplitude has, therefore, a linear dependence on θ if

$$\theta\leq 1/pa. \tag{7}$$

The same expansion (6) and, therefore, the same relation (7) holds in the backward cone by the simple replacement of θ with $\pi-\theta$.

(2) One can raise the question of whether the singularity of the forward scattering amplitude in the Coulomb case may not spoil the rule. To answer this question note first that, for inelastic scattering, the Coulomb scattering amplitude has no forward singularity, so that the rule also holds in this case. However, for the forward scattering and, in some cases, also for the backward scattering, the Coulomb interaction (here, by Coulomb we mean all those interactions due only to the exchange of virtual photons) may well reduce the cone in which the scattering amplitude is linear in θ below the value (7). A general treatment of this problem will not be given here; we only remark at this point that the Coulomb interaction is usually amenable to a more or less accurate theoretical treatment. One can, therefore, predict, by a separate calculation in each case of interest, the order of magnitude of the forward and backward cones of linearity of the Coulomb amplitude; and, in many cases, one can also calculate the detailed behavior and magnitude of the Coulomb amplitude outside the region of θ linearity. It, therefore, seems possible, for parity-unfavored reactions, to predict generally the angular distribution in the entire forward or backward cones defined by (7) also when the Coulomb interaction is taken into account.

(3) The same argument that has been used in proving the rule also implies that, independent of the direction of production, the particle with spin S that is produced in a parity-unfavored reaction cannot be formed in a state of spin with zero projection in the direction of the incident beam.

In fact, using the previous notation, we have that, due to (5), no $m'=0$ substate can be formed in the reaction. Since, in addition, by using the property

$$C_{J_1J_2}(J0|m',-m')=(-1)^{J_1+J_2-J}C_{J_1J_2}(J0|-m',m')$$

of the Clebsch-Gordan coefficients, the amplitude for production of the $\chi_{S,-m'}$ spin state is seen to differ by only an $m'l'$-independent phase factor from that for production of $\chi_{S,m'}$, we have the following result: Particles of spin S produced in a parity-unfavored reaction have their spin aligned along the direction of the incident beam.

It is obvious that for a particle of spin 1 the alignment is the maximum one possible independent of the direction. The existence of such an alignment may give rise, of course, to typical angular correlations if the particle of spin S subsequently decays.

We shall at this point give a few examples of parity-unfavored reactions where the rule can be applied usefully; we shall list problems of different nature both in elementary-particle and low-energy nuclear physics.

(1) Consider the coherent production of a K^{*+} resonance (888 MeV) from K^+ on some spin-zero nucleus

[1] Compare, for example, A. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957).

[2] If $P_AP_BP_CP_D=+1$ and $S=1$ the rule can be simply proved by remarking that the most general transition amplitude must necessarily have the form $\mathbf{k}\times\mathbf{k}'\cdot\mathbf{S}$, where $\mathbf{S}$ is a pseudovector operator effecting a $0^+\rightarrow 1^+$ transition and $\mathbf{k}$, $\mathbf{k}'$ are the momenta of the incident and final particles in the center-of-mass system.

[3] Compare, for example, W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Chelsea Publishing Company, New York, 1949), p. 54.

(e.g. He^4) (by coherent we mean, of course, that the nucleus remains in its ground state). Then, if the spin of K^+ is one, the rule predicts that no forward production is possible.[4] A similar statement holds for the production of a ρ resonance by pions. The two processes in question can be used, perhaps, to determine, via the peripheral approximation, the $K^*K\omega_0$ and $\rho\pi\omega_0$ coupling constants, respectively; the rule thus allows one to determine, in the formulas to be used, a model-independent feature (that is, the forward vanishing of the cross section) from features which are characteristic of the peripheral approximation.[5]

It may be added that in the coherent production of two pions by one pion on a spin-zero nucleus an even more general result holds: Because of the conservation of the isotopic spin, the two pions are necessarily produced (neglecting Coulomb effects) in a $T=1$ state. The total angular momentum of the two pions in a $T=1$ state with respect to their center-of-mass system is necessarily odd; therefore, the parity of this system is odd. The two pions, in their center-of-mass system, can be considered as a "particle" in the sense of the rule. The reaction is, therefore, an unfavored one and the rule applies. This means that the two pions can never be produced in a configuration in which their resultant total momentum is in the same direction as that of the incident beam.

(2) It is still important, in our opinion, to improve the accuracy with which the conservation of parity is established in the strong interactions of the strange particles; such an accuracy is presently at most of the order of 10%.[6] An experiment which, perhaps, can be performed with better accuracy is that of measuring the angular distribution in the inelastic scattering of a K^+ meson from some 0^+ nucleus with the excitation of a 1^+, 2^-, of 3^+ level; or, equivalently from some 1^+, 2^-, or 3^+ nucleus with the excitation of a 0^+ level. According to the rule, if parity is conserved, the forward and backward cross sections must vanish; by measuring the angular distribution of the inelastically scattered K^+ near the forward or (preferably) backward direction one should be able to obtain a reasonably accurate determination of the reflection invariance of the $\bar{K}K\bar{N}N$ interaction.[7] Appropriate targets for this purpose can be, e.g., Li^6(g.s. $\rightarrow$ 3.56-MeV level), C^{12}(g.s. $\rightarrow$ 15.1-MeV level), and N^{14}(g.s. $\rightarrow$ 2.31-MeV level).[8]

(3) As a final example, we mention the case of nuclear excitation by α particles or other spin-zero projectiles. Consider the inelastic scattering of α particles on a nucleus with a ground-state spin zero with the excitation of some 1^+ or 2^- or $3^+ \cdots$ level. The angular distribution of the inelastically scattered α particles then vanishes near the forward or backward direction independent of the mechanism of the reaction (direct reaction or compound nucleus formation). In addition, for any direction of scattering, the excited nucleus is aligned.[9]

Note added in proof. In a paper by Almqvist *et al.* [Phys. Rev. **130**, 1141 (1963)] on the reaction $C^{12}+C^{12} \rightarrow Ne^{20}+He^4$, which appeared during the publication of this note, reference is made to a letter by A. Litherland in Can. J. Phys. **39**, 1245 (1961) where the present rule is stated and applied to nuclear reactions. We are sorry for not having known before Litherland's note, but we hope that the present discussion of the use of the rule in elementary particle reactions may be useful.

We also add that Dr. D. Lichtenberg has kindly sent us the manuscript of a talk read at the Ohio University Conference on 26 April 1963, in which he has dealt with related questions. Finally, it must be mentioned that the K-He^4 reaction was discussed, in connection with the K^* spin determination, by D. O. Caldwell in Phys. Rev. Letters **7**, 259 (1961).

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[4] If the spin of K^* is zero, the transition is, of course, absolutely forbidden in all directions. Quite apart from the rule, the study of this process, if feasible, might be of interest to confirm the spin-one assignment for the K^*.

[5] The structure of the formulas is similar to that of the π^0 coherent photoproduction (Primakoff effect); compare, C. Chiuderi and G. Morpurgo, Nuovo Cimento **19**, 497 (1961). Note, however, that there the vanishing of the forward cross section is simply due to the conservation of the z component of the angular momentum; this is because the photon has only two states of spin.

[6] Compare, for a discussion and references, G. Morpurgo, Ann. Rev. Nuc. Sci. **11**, 41 (1961).

[7] Notice, for example, that an interaction mediated by pions with a vertex of the type $\bar{K}K\pi$ is automatically excluded, if we postulate time-reversal invariance. One can, however, construct interactions (for example, with intermediate hyperons) which are T invariant but not P and C invariant.

[8] Of course, the most typical experiment of this kind should consist of studying the inelastic scattering of K^+ from a 0^+ ground state into a 0^- level of some nucleus. It is, however, easy to see that in the only nucleus O^{16} which has such levels (apart from C^{14} which is unstable) the conditions are not very appropriate for an experiment. The author thanks Professor R. Malvano for a discussion of this point.

[9] It is of some interest to see the connection between the present rule and a rule given by A. Bohr [Nucl. Phys. **10**, 486 (1959)]. When applied to a collision between two particles of spin zero to give a particle of spin zero and a particle of spin S, Bohr's rule states that the particle of spin S is polarized with respect to the direction $\mathbf{n}$ of the *normal to the collision plane*; more precisely, calling M the projection of the spin along $\mathbf{n}$, M has only even or odd values depending on whether $P_AP_BP_CP_D=+1$ or -1. The connection with the present rule is established through the following points: (a) Express the spin functions referring to the direction $\mathbf{n}$ (call them η_{SM}) through our spin functions χ_{Sm}, where m refers to the direction z of the incident beam; choosing the x axis along $\mathbf{n}$ and making use of the standard notation, the connection is given by $\eta_{SM}=\sum_m d_{mM}{}^{(S)}(\pi/2)\chi_{Sm}$. (b) Use the fact that $d_{0M}{}^{(S)}(\pi/2)=0$ if $S+M$ is odd. (c) Note that it follows from what we have said above that $S+M$ is odd when $P_AP_BP_CP_D=(-1)^{S+1}$. (d) Note that, therefore, when $P_AP_BP_CP_D=(-1)^{S+1}$, no term with $m=0$ appears in the expansion of η_{SM} in terms of the χ_{Sm}. (e) Our rule then follows by using only the conservation of the z component of the angular momentum.