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Baryon-Baryon Interactions and the Eightfold Way*

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The deuteron is shown to belong to a ten-dimensional irreducible representation of the group $SU(3)$ in the symmetry scheme called the eightfold way. The baryon-baryon states, which together with the deuteron comprise this supermultiplet, are studied. Data are discussed which favor the existence of some of these states, and the masses of the remaining states are estimated.

I. INTRODUCTION

RECENTLY, Gell-Mann[1] and Ne'eman[2] have independently proposed a scheme, called the eightfold way, in which the eight baryons (N, Λ, Σ, and Ξ) are assumed to form a supermultiplet, degenerate in the limit of a certain symmetry, and split into the familiar isotopic multiplets by a dynamical interaction. The symmetry is called unitary symmetry and the relevant group is the group of unitary unimodular transformations in three dimensions, which is denoted by $SU(3)$. The eight baryons belong to the regular (eight-dimensional) representation of $SU(3)$. All the known pseudoscalar mesons, vector mesons, and meson-baryon resonances have been tentatively identified with various representations of $SU(3)$.[3]

In this note, we examine the role of the deuteron in the eightfold way and find it must belong to a ten-dimensional representation of $SU(3)$. In the limit of exact unitary symmetry, the implication is that all ten of the baryon-baryon states which comprise the supermultiplet are similarly bound. Since it is clear that unitary symmetry is not exact in the physical world, some of these might not occur as actual bound states. However, unless the symmetry is broken so badly that it is meaningless, one might expect these states to be nearly bound, or resonant.[4] As will be discussed below, there is some evidence to support this point of view.

Since the symmetry of the eightfold way seems to be broken in a particularly simple and definite way, the mass splittings in the supermultiplets are related. Using this fact and certain existing data, the masses of the heretofore unobserved constituents of this deuteron decuplet are estimated. It is suggested that these be looked for as a test of the applicability of unitary symmetry and the eightfold way to the study of the hyperon and cascade interactions.

II. BARYON-BARYON STATES

In the eightfold way, the eight baryons comprise an eight-dimensional representation of the special unitary group $SU(3)$, which corresponds to a tensor having one upper and one lower index. These indexes range from 1 to 3, and their contraction vanishes. This baryon supermultiplet can be represented by a 3×3 traceless matrix as follows:

$$\psi_\alpha{}^\beta=\begin{bmatrix} -(\frac{2}{3})^{1/2}\Lambda & p & n \\ \Xi^- & (1/\sqrt{6})\Lambda+(1/\sqrt{2})\Sigma^0 & \Sigma^- \\ \Xi^0 & \Sigma^+ & (1/\sqrt{6})\Lambda-(1/\sqrt{2})\Sigma^0 \end{bmatrix}. \quad (1)$$

The possible baryon-baryon states are given by the Kronecker product of this representation with itself, i.e., by the product $\psi_\alpha{}^\beta\otimes\psi_\mu{}^\nu$. If we denote this product by $8\otimes8$, we find it can be decomposed into irreducible representations of $SU(3)$, as follows:

$$8\otimes8=1\oplus8\oplus8\oplus10\oplus\overline{10}\oplus27. \quad (2)$$

Here we have used the dimension to denote the representation and the $\overline{10}$ is the adjoint of the **10**. The **8** appears twice and it is conventional to choose linear combinations such that one is symmetric and the other is antisymmetric. The hypercharges (Y) and isotopic spins (I) of the states comprising the irreducible representations appearing in the decomposition of the product $8\otimes8$ are given in Table I.

Inspection of Table I shows the deuteron, which has $Y=2$ and $I=0$, must belong to the $\overline{10}$, since no other representation contained in the product $8\otimes8$ admits such a state. The baryon-baryon states comprising the $\overline{10}$ can be expanded in terms of the single baryon states $\psi_\alpha{}^\beta$ using the Wigner coefficients for $SU(3)$. We effect this expansion by noting that the $\overline{10}$ corresponds to a

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962) and California Institute of Technology Synchrotron Laboratory Report, CTSL-20 (unpublished).
[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] For a summary see S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).
[4] We use "resonance" here in its loosest sense to denote a relative enhancement of an interaction at a reasonably well-defined energy.

completely symmetric tensor having no upper indexes and three lower indexes.

One can construct only one such tensor that is bilinear in $\psi_\alpha{}^\beta$. We denote this tensor by $\chi_{\alpha\beta\gamma}$ and find it to be

$$\chi_{\alpha\beta\gamma}=(\psi_\alpha{}^\mu\psi_\beta{}^\nu+\psi_\beta{}^\mu\psi_\alpha{}^\nu)\epsilon_{\mu\nu\gamma}+(\psi_\alpha{}^\mu\psi_\gamma{}^\nu+\psi_\gamma{}^\mu\psi_\alpha{}^\nu)\epsilon_{\mu\nu\beta}+(\psi_\beta{}^\mu\psi_\gamma{}^\nu+\psi_\gamma{}^\mu\psi_\beta{}^\nu)\epsilon_{\mu\nu\alpha}, \quad (3)$$

where $\epsilon_{\alpha\beta\gamma}$ is the completely antisymmetric tensor density with $\epsilon_{123}=+1$, and repeated indexes are summed from 1 to 3. If one puts in the baryon states $\psi_\alpha{}^\beta$, performs the summations, and normalizes the resulting wave functions, one obtains

$$\chi_{111}=|Y=2,I=0,I_3=0\rangle=\frac{1}{\sqrt{2}}(pn-np),$$

$$\chi_{113}=|Y=1,I=\tfrac{1}{2},I_3=\tfrac{1}{2}\rangle=\frac{1}{(12)^{1/2}}[\sqrt{3}(p\Lambda-\Lambda p)+\Sigma^0p-p\Sigma^0+\sqrt{2}(\Sigma^+n-n\Sigma^+)],$$

$$\chi_{112}=|Y=1,I=\tfrac{1}{2},I_3=-\tfrac{1}{2}\rangle=\frac{1}{(12)^{1/2}}[\sqrt{3}(\Lambda n-n\Lambda)+\Sigma^0n-n\Sigma^0+\sqrt{2}(p\Sigma^--\Sigma^-p)],$$

$$\chi_{133}=|Y=0,I=1,I_3=1\rangle=\frac{1}{(12)^{1/2}}[\sqrt{3}(\Sigma^+\Lambda-\Lambda\Sigma^+)+\Sigma^0\Sigma^+-\Sigma^+\Sigma^0+\sqrt{2}(\Xi^0p-p\Xi^0)],$$

$$\chi_{123}=|Y=0,I=1,I_3=0\rangle=\frac{1}{(12)^{1/2}}[\sqrt{3}(\Sigma^0\Lambda-\Lambda\Sigma^0)+\Sigma^+\Sigma^--\Sigma^-\Sigma^++n\Xi^0-\Xi^0n+\Xi^-p-p\Xi^-],$$

$$\chi_{122}=|Y=0,I=1,I_3=-1\rangle=\frac{1}{(12)^{1/2}}[\sqrt{3}(\Lambda\Sigma^--\Sigma^-\Lambda)+\Sigma^0\Sigma^--\Sigma^-\Sigma^0+\sqrt{2}(n\Xi^--\Xi^-n)],$$

$$\chi_{333}=|Y=-1,I=\tfrac{3}{2},I_3=\tfrac{3}{2}\rangle=\frac{1}{\sqrt{2}}(\Xi^0\Sigma^+-\Sigma^+\Xi^0),$$

$$\chi_{233}=|Y=-1,I=\tfrac{3}{2},I_3=\tfrac{1}{2}\rangle=\frac{1}{\sqrt{6}}[\sqrt{2}(\Xi^0\Sigma^0-\Sigma^0\Xi^0)+\Xi^-\Sigma^+-\Sigma^+\Xi^-],$$

$$\chi_{223}=|Y=-1,I=\tfrac{3}{2},I_3=-\tfrac{1}{2}\rangle=\frac{1}{\sqrt{6}}[\sqrt{2}(\Xi^-\Sigma^0-\Sigma^0\Xi^-)+\Sigma^-\Xi^0-\Xi^0\Sigma^-],$$

$$\chi_{222}=|Y=-1,I=\tfrac{3}{2},I_3=-\tfrac{3}{2}\rangle=\frac{1}{\sqrt{2}}(\Sigma^-\Xi^--\Xi^-\Sigma^-). \quad (4)$$

TABLE I. Hypercharges and isotopic spins of the constituents of the irreducible representations contained in the Kronecker product $8\otimes8$.

Representation	(Y,I)
1	(0,0)
8	$(1,\frac{1}{2})$ (0,0) (0,1) $(-1,\frac{1}{2})$
10	$(1,\frac{3}{2})$ (0,1) $(-1,\frac{1}{2})$ $(-2,0)$
$\overline{\mathbf{10}}$	(2,0) $(1,\frac{1}{2})$ (0,1) $(-1,\frac{3}{2})$
27	(2,1) $(1,\frac{1}{2})$ $(1,\frac{3}{2})$ (0,0) (0,1) (0,2) $(-1,\frac{3}{2})$ $(-1,\frac{1}{2})$ $(-2,1)$

The first of these (χ_{111}) clearly corresponds to the channel in which the deuteron occurs as a bound state. In the limit of exact unitary symmetry, in which the eight baryons are degenerate, the forces that bind the deuteron are the same in each of the ten baryon-baryon states $\chi_{\alpha\beta\gamma}$. In this limit they are all bound states with the same spin and parity (1^+) and with the same binding energy (2.2 MeV) as the deuteron. Clearly, unitary symmetry is not exact in the physical world. However, if the symmetry is not broken so badly that the classification is no longer useful, one might expect nearly bound, or resonant, states to occur in these baryon-baryon channels. The experimental data bearing on this point of view will be discussed next.

III. COMPARISON WITH EXPERIMENT

The experimental information concerning the interactions in the states $\chi_{\alpha\beta\gamma}$ decreases rapidly with the strangenesses of the baryons involved, being virtually nonexistent for the $Y=0$ triplet and the $Y=-1$ quadruplet. The $Y=1$ doublet does not seem to be bound.[5] However, there is some evidence that the hyperon-nucleon interaction is quite strong near the ΣN threshold in the state with $I=\frac{1}{2}$ and spin and parity 1^+. This can be interpreted as a manifestation of a resonant hyperon-nucleon interaction in the states χ_{112} and χ_{113} around 2130 MeV.

The evidence comes from K^- mesons stopping in deuterium[6] and Σ^- capture on protons.[7] In the former experiments one finds that the kinetic-energy spectrum of the final pion in the reaction $K^-+d\rightarrow\Lambda+p+\pi^-$ shows two peaks, one at 147 MeV and one at 92 MeV. The higher peak has been interpreted as being caused by the K^- meson interacting directly with the neutron in the deuteron to produce the Λ and π^-, while the proton is a spectator. The lower peak at 97 MeV corresponds to the energy expected if the K^- meson interacted directly with one nucleon to produce a Σ and π^-,

[5] R. H. Dalitz, in *Proceedings of the Rutherford Jubilee International Conference* (Heywood and Company Ltd., London, 1961), p. 103.

[6] L. Alvarez, in *Proceedings of the Ninth International Annual Conference on High Energy Physics* (Academy of Sciences, U.S.S.R., Moscow, 1960), Vol. I, p. 471; O. I. Dahl, N. Horowitz, D. H. Miller, J. J. Murray, and P. G. White, Phys. Rev. Letters **6**, 142 (1961); and D. H. Miller (private communication).

[7] R. R. Ross, Bull. Am. Phys. Soc. **3**, 335 (1958).

while the other nucleon was merely a spectator. Since the observed final state contains a Λ rather than a Σ, a two-step process is suggested in which the K^- is first absorbed by one of the nucleons to produce a Σ and π^-, after which the rather low-energy Σ interacts with the other nucleon and converts into a Λ. Analyses[8,9] based on this model and phenomenological interactions are in agreement with the data in this region of the pion-energy spectrum. These calculations suggest the conversion reaction $\Sigma+N\rightarrow\Lambda+N$ occurs primarily in the S state and is quite strong. This result agrees qualitatively with the prediction of the eightfold way that the hyperon-nucleon interaction in the states χ_{112} and χ_{113} should be strong in analogy to the nucleon-nucleon forces which bind the deuteron.

In the Σ^-p capture data, one also finds a considerable amount of $\Sigma^0\Lambda$ conversion in the ${}^3S_1$ state. In capture at rest one observes the reactions $\Sigma^-+p\rightarrow\Sigma^0+n$ and $\Sigma^-+p\rightarrow\Lambda+n$ with a branching ratio[7] $R=\Lambda/(\Lambda+\Sigma^0)=0.67\pm0.05$. To estimate this branching ratio it is reasonable to assume the conversion takes place from the S state, since the capture occurs at rest. However, there is still a contribution from the ${}^1S_0$ state which is not related by the eightfold way to the hyperon-nucleon interaction in the $\overline{\mathbf{10}}$. Consequently, let us neglect this contribution and estimate the branching ratio, R, assuming the $\Sigma\Lambda$ conversion occurs predominantly from ${}^3S_1$ state represented by χ_{112}. Including phase-space factors, one then finds $R=0.70$, which compares favorably with the experimental value. Although this crude estimate cannot be taken too seriously, it is a qualitative indication that the hyperon-nucleon interaction in the ${}^3S_1$ state χ_{112} of the baryon-baryon $\overline{\mathbf{10}}$ is relatively strong in analogy to the interaction responsible for the deuteron's binding energy, as required by the eightfold way. This conclusion is extended to the state χ_{113} simply by isotopic spin invariance.

Unfortunately, there is insufficient data regarding the hyperon-hyperon, nucleon-cascade, and hyperon-cascade interactions to discuss the $Y=0$ triplet and the $Y=-1$ quadruplet occurring in baryon-baryon $\overline{\mathbf{10}}$. In the next section we will attempt to estimate where resonant interactions in these states are most likely to occur.

[8] R. Karplus and L. Rodberg, Phys. Rev. **115**, 1058 (1959).
[9] T. Kotani and M. Ross, Nuovo Cimento **14**, 1282 (1959).

IV. PREDICTIONS

Since the symmetry of the eightfold way seems to be broken in a particularly simple manner, there exists a relation among the masses within a given supermultiplet. In particular, the squares of the masses[10] of the baryon-baryon $\overline{\mathbf{10}}$ are equally spaced, according to the Gell-Mann–Okubo[11] mass formula. Therefore, one has the relation

$$M^2=M_0^2(1+\alpha Y)\,, \tag{5}$$

where M_0 and α are parameters that one can evaluate once two of the masses are known. Using the deuteron mass of 1876 MeV for the $Y=2$ state and 2130 MeV for the $Y=1$ state, as suggested by the K^-d and Σ^-p capture data, one finds these parameters to be $M_0=2357$ MeV and $\alpha=-0.1832$. Using these values, the masses of the $Y=0$, $I=1$ and $Y=-1$, $I=\frac{3}{2}$ baryon-baryon resonances are predicted to be 2357 MeV and 2564 MeV, respectively. Comparing these masses with the masses of their constituents, one finds they are far from being bound states. Nevertheless, they can manifest themselves as important final-state interactions in certain production reactions. For example, the production reactions $p+p\rightarrow\Sigma^++\Lambda+K+K^0$, $p+p\rightarrow p+\Xi^-+K^++K^+$, and $\Sigma^-+p\rightarrow\Lambda+\Sigma^-+K^+$ should show an enhancement of events with the final two hyperons in the 1^+ state when their center-of-mass energy is near 2357 MeV.

Confirmation of the existence of the $Y=0$ triplet and the $Y=1$ quadruplet would support the use of the eightfold way as a theoretical means of relating the virtually unknown hyperon and cascade interactions to the relatively well-known nucleon interactions.

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[10] It is not clear that the masses themselves rather than their squares should not be equally spaced, even though the states are formally bosons. However, the numerical results are not much different if the masses rather than their squares are used in Eq. (6). In this case one finds $M_0=2384$ MeV and $\alpha=-0.1065$. Then the $Y=0$ triplet is at 2384 MeV and the $Y=-1$ quadruplet is at 2638 MeV.
[11] Ref. 1 and S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).