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One-Dimensional Theory of the Parasitic Paramagnetism Term in the Approach to Saturation

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By assuming a discontinuity in the spin direction at a defect, Arrott showed that for high density of defects, the approach to magnetic saturation should contain a parasitic paramagnetism term, as is observed experimentally. It is shown here that the artificial assumption of a discontinuity is not necessary to get these results. Using the same one-dimensional model of Arrott and the usual theory for the magnetic hardness term, that implies unsmooth but continuous functions, the parasitic paramagnetism term is also obtained for the same limit of high density of defects, in regions where the nearest neighbors to each defect are defects of opposite sign.

EXPERIMENTAL values of the magnetization M of a ferromagnetic material in a field H much larger than the coercive force are often analyzed as[1]

$$M=M_s-a/H-b/H^2+cH\,, \qquad (1)$$

where M_s is the saturation magnetization and a, b, and c are constants. The term b/H^2 has been calculated in terms of anisotropy,[2] yielding results in satisfactory agreement with experiment. The term a/H, known as the magnetic hardness, has been calculated by Brown,[3] in terms of high localized forces acting on the spins at crystalline defects. When the stress field at dislocation is considered,[4] one obtains contributions both to the magnetic hardness and to the b/H^2 term.

The last term in (1), the so-called parasitic paramagnetism cH, has never been fully accounted for. Holstein and Primakoff[5] derived a term proportional to $H^{1/2}$ (which they argue is hardly distinguishable experimentally from the parasitic paramagnetism in the presence of the other terms), with numerically plausible results for c. Recently, Arrott[6] has modified Brown's treatment of the magnetic hardness and showed that when the defects are close to each other (Brown[3] considered only widely separated defects), the a/H term is replaced by a cH term. The parasitic paramagnetism thus originates from those regions in the material in which there is a high density of defects.

Arrott assumes a one-dimensional model in which there is a finite discontinuity between adjacent layers of atoms at the defect. He obtains the magnitude of this discontinuity by minimizing the sum of exchange energy (which tends to keep neighboring spins aligned) and Dzialoshinskii-Moriya interaction energy (which tends to make them perpendicular to each other) at the defect, plus the exchange and self-magnetostatic energies in the rest of the material, already minimized with the discontinuity as a parameter. Now, although the results seem attractive, we consider this model objectionable on two grounds. In the first place, it is difficult to accept discontinuity as the boundary condition for the differential equation associated with the minimization of exchange and self-magnetostatic energies, when a discontinuity implies an infinite exchange energy of this model of a continuous material. Even on a microscopic scale, it has already been remarked[7] that a discontinuity in the spin direction means a certain pair of spins has an exchange energy larger by orders of magnitude than that of the other pairs, and this cannot be conceived as a minimum of energy. Although the Dzialoshinskii-Moriya interaction acts just on the spins at the defect, the exchange interaction makes the disturbance gradual over a wide region, so that the most reasonable form will be a discontinuity in the *derivative*, as assumed by Brown. Secondly, the one-dimensional picture of planes of defects is too crude a picture anyway. Brown obtains the mangetic-hardness term for two-dimensional study of line defects, and an experimentally unobserved $H^{-3/2}$ term for the one-dimensional approach. Arrott obtains $H^{-1/2}$ in one dimension for widely separated defects, and does not try the two-dimensional problem. He can only hope his treatment will also lead to the $1/H$ term in two dimensions, since this agrees with experiment, but in view of the large difference in the results in one dimension, retaining the right form of magnetic hardness in two dimensions seems highly improbable.

It is the purpose of this paper to show that in the limit of closely spaced defects assumed by Arrott one can obtain a term proportional to H also from Brown's approach, retaining Brown's result for the limit of widely spaced defects. This will account for the parasitic paramagnetism, without the objectionable discontinuity, and without losing the magnetic-hardness term.

The results reported in the following can also be obtained for a periodic model, as in Arrott's treatment. However, this artificial assumption is not necessary, and will therefore not be adopted. It is still assumed that the small angle α of deviation of the magnetization from the direction of the field is a function of the coordinate y only, the defects being the planes $y=\text{const}$.

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Consider a particular defect, and let the origin be chosen so that this plane is $y=0$. If its nearest-neighbor defect on the positive y axis is of opposite sign, the function α should pass from positive to negative values somewhere between these defects. There should thus be a certain positive value of y at which $\alpha=0$. Similarly, if the nearest defect on the other side has also the opposite sign to that of the defect at $y=0$, there is also a negative value of y for which $\alpha=0$. Denoting these values of y by $-\frac{1}{2}L_1$ and $\frac{1}{2}L_2$, respectively,

$$\alpha(-\tfrac{1}{2}L_1)=\alpha(\tfrac{1}{2}L_2)=0. \qquad (2)$$

In the region $-\frac{1}{2}L_1\leqslant y\leqslant\frac{1}{2}L_2$ there is just one defect, at $y=0$. At this point, according to Brown,[3] α should be continuous, with a discontinuity of magnitude F in its derivative. These conditions, and (2) should therefore serve as boundary conditions for the differential equation for the function $\alpha(y)$ which minimizes the energy, and which is according to Brown[3]

$$d^2\alpha/dy^2-\lambda^2\alpha=0, \qquad (3)$$

with

$$\lambda^2=HM_s/C, \qquad (4)$$

where C is the exchange constant (Brown's λ is Arrott's $1/\rho_0$). The extra term of energy, which Arrott claims that Brown did not consider, vanishes for continuous function.

The boundary conditions determine uniquely the solution of (3), yielding an appropriate combination of the exponentials of $\pm\lambda y$. Using this solution in the general expression for the contribution of the region $-\frac{1}{2}L_1\leqslant y\leqslant\frac{1}{2}L_2$ to the deviation of magnetization from saturation,[3]

$$\frac{\Delta M}{M_s}=\tfrac{1}{2}\langle\alpha^2\rangle_{\text{AV}}=\frac{1}{L_1+L_2}\int_{-\frac{1}{2}L_1}^{\frac{1}{2}L_2}\alpha^2dy, \qquad (5)$$

one obtains finally

$$\Delta M/M_s=F^2[(\sinh\lambda L_1-\lambda L_1)(\cosh\lambda L_2-1)+(\sinh\lambda L_2-\lambda L_2)(\cosh\lambda L_1-1)]/\Omega, \qquad (6a)$$

$$\Omega=4\lambda^3(L_1+L_2)[\cosh\lambda(L_1+L_2)-1], \qquad (6b)$$

which implies in the limit $\lambda L_1\ll1$, $\lambda L_2\ll1$,

$$\frac{\Delta M}{M_s}\approx\frac{F^2L_1^2L_2^2}{24(L_1+L_2)^2}\left[1-\frac{\lambda^2}{30}(L_1^2+4L_1L_2+L_2^2)\right]. \qquad (7)$$

The other extreme, $\lambda L_1\gg1$, $\lambda L_2\gg1$, has already been studied by Brown,[3] and one actually obtains his result by using this limit in (6).

Since according to (4), λ^2 is proportional to H, the relation (7) implies a term proportional to H, i.e., a parasitic paramagnetism, with apparent reduction in the saturation magnetization value. The result of Arrott thus follows without the assumption of discontinuity. Moreover, the magnetic-hardness term will certainly be retained in the two-dimensional study of widely separated line defects.

Equation (6) has been obtained with the assumption that the nearest neighbors of each defect are of opposite signs. The case when they have the same sign can be studied by equating $d\alpha/dy$ to zero rather than α, somewhere between the defects. The calculation is similar and it yields Brown's $H^{-3/2}$ term for widely separated defects and a H^{-2} term for closely spaced defects. It can thus be concluded that in regions of the material where there are clusters of defects, one obtains a contribution to the anisotropy term for nearest neighbors of the same sign, and parasitic paramagnetism where the signs are reversed. Preliminary estimation for the case of the line rather than plane defects, indicate the same results.

The assumption that dislocations are arranged in pairs of opposite signs, has been used[4] in a more detailed study of stresses at dislocations. It has been argued there that, on the average, the number of positive defects should equal that of negative ones, since there is no preference for any sign. Still, it is not conceivable that in regions of high density of defects, they are so arranged that everyone of them has nearest neighbors of opposite sign. It is therefore not possible to compare quantitatively to experiment before some estimation is made for the statistical distribution of these defects.