Electromagnetic Induction of Neutral Lepton Currents

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We have considered the induction of neutral lepton currents in the weak interaction Lagrangian through the mediation of the electromagnetic field. These considerations are within the framework of the conventional picture in which neutral lepton currents are conspicuous by their absence. Our motivation, in fact, derives from the need to establish the relevance, or irrelevance, of any conceivable physical observation of these currents to a theory of weak interactions. The couplings associated with both the vector and axial vector lepton currents are investigated in a simple model. We also consider some physical processes that may be expected to carry the signature of neutral lepton currents.

1. INTRODUCTION

THE necessity for a systematic investigation of higher order effects in weak interactions has recently been underlined by Pais as well as Feinberg and Gürsey.[1,2] Such an investigation is essential if one is to decide whether the currently known Lagrangian has a purely phenomenological significance or if it admits of interpretation as a primary Lagrangian needed to construct a field theory of weak interactions. Assuming that this study will find quantitative fulfillment, one of the first predictions will undoubtedly be on the strength of the coupling associated with neutral lepton currents. As is well known, these currents are conspicuous by their absence in first order, even though their counterparts involving the strongly interacting fields are more or less forced in by the $|\Delta\mathbf{T}|=\frac{1}{2}$ selection rule. However, the presence of any strangeness-preserving neutral current in the primary Lagrangian is sufficient to generate neutral lepton currents through the mediation of the electromagnetic field. Thus, a current of the form $\bar{p}\gamma_\mu p$ could generate a current of the form $\bar{l}\gamma_\mu l$ on account of the virtual sequence $\bar{p}+p\rightarrow\gamma\rightarrow l+\bar{l}$. The question then arises as to whether the structure of these electromagnetically induced currents and the magnitude of their coupling is such as to still leave some room for the detection of purely weak effects. Consideration of this question constitutes the primary motivation for the present work.

In this note we shall confine ourselves to the case of charged leptons. The only significant coupling of the induced neutral lepton currents is, therefore, that to the strangeness-changing current; any manifestations of a coupling to the strangeness-preserving current will be masked by purely electromagnetic effects.[1] For this reason only the former coupling is mentioned in Sec. 2, where we enumerate some general features in a simple model. The vector and axial vector parts are then considered separately in Secs. 3 and 4, respectively. In Sec. 5 we discuss some relevant physical processes giving order of magnitude estimates for rates, etc. (It may perhaps be redundant to warn the reader against taking these estimates much too seriously.) The conclusions are summarized in Sec. 6.

2. INDUCED CURRENTS IN A SIMPLE MODEL

Without loss of generality we may assume that the weak interactions are mediated by intermediate vector bosons. Then, as is well known, the acceptance of a kinematical $|\Delta\mathbf{T}|=\frac{1}{2}$ rule forces us to postulate neutral, as well as charged, bosons. The absence of any neutral lepton currents in the primary Lagrangian tells us, however, that

$$g_0{}^V=g_0{}^A=0\,, \tag{2.1}$$

where $g_0{}^V$ ($g_0{}^A$) is the coupling of neutral bosons to the vector (axial vector) part of the neutral lepton current.

We first address ourselves to the task of calculating $g_0{}^V$ and $g_0{}^A$ in the presence of the electromagnetic field. Our purpose is twofold: We hope (i) to gain some insight into the structure of the induced current and the magnitude of its coupling and (ii) to evaluate some physical amplitudes which can be expressed in terms of these parameters.

The above coupling constants, or their known multiples, can be defined in terms of the invariant form factors which occur in the matrix element

$$M_\mu(P^2)=\langle l\bar{l}|J_\mu{}^0|0\rangle\,. \tag{2.2}$$

Here $J_\mu{}^0$ is the strangeness-preserving neutral current associated with the strongly interacting particles and P is the four-momentum of the $l\bar{l}$ state. For the sake of convenience, we shall work directly in terms of the values of these form factors at $P^2=0$, rather than express them in terms of $g_0{}^V$ and $g_0{}^A$.

We proceed to examine M_μ. This is most conveniently done within the framework of dispersion theory, since a class of renormalization difficulties is thereby avoided. The dynamical singularities of M_μ emerge as usual from the unitarity condition, viz.,

$$A[M_\mu{}^V]=\tfrac{1}{2}\textstyle\sum_n \langle l\bar{l}|T|n\rangle\langle n|J_\mu{}^V|0\rangle(2\pi)^4 \times\delta^4(P_n-p-\bar{p})\,, \tag{2.3}$$

$$A[M_\mu{}^A]=\tfrac{1}{2}\textstyle\sum_m \langle l\bar{l}|T|m\rangle\langle|J_\mu{}^A|0\rangle(2\pi)^4 \times\delta^4(P_m-p-\bar{p})\,, \tag{2.4}$$

[1] A. Pais, Notes of lectures given at the IAEA Seminar, Trieste, Italy (unpublished).
[2] G. Feinberg and F. Gürsey, Phys. Rev. 128, 378 (1962).

where we have split the current into its vector and axial vector parts and written the unitarity conditions separately. The operator T involves only electromagnetic and strong interactions; the symbol A indicates the absorptive part.

The structure of the intermediate states is easily inferred by assuming that only first class currents exist in nature.[3] For such currents we have

$$GJ_\mu{}^V G^{-1}=J_\mu{}^V, \tag{2.5}$$

$$GJ_\mu{}^A G^{-1}=-J_\mu{}^A. \tag{2.6}$$

Thus, if we consider only pion states, the states n and m are characterized by even and odd numbers of pions, respectively. Furthermore, since J transforms as an isotopic vector, the state $|n\rangle$ is odd under charge conjugation, whereas $|m\rangle$ is even. Thus, $A[M_\mu{}^V]$ can be of order α, whereas $A[M_\mu{}^A]$ must of necessity be of order α^2. In the absence of any primary coupling the same holds true for $M_\mu{}^V$ and $M_\mu{}^A$, and, correspondingly, we expect that $g_0{}^A$ is smaller than $g_0{}^V$ by a factor of α.

The interaction induced in this model is thus of the form

$$S_\mu{}^0\bar{\psi}_l\gamma^\mu(a+b\gamma_5)\psi_l, \tag{2.7}$$

with $|b|/|a|\sim O(\alpha)$. Here $S_\mu{}^0$ is the neutral strangeness-changing current involving the strongly interacting fields.

3. THE VECTOR CURRENT

Using Lorentz invariance, one has the representation

$$\langle l\bar{l}|J_\mu{}^V|0\rangle=\left(\frac{1}{2\pi}\right)^3\left(\frac{M_l}{p_0}\right)\bar{u}(p)$$
$$\times[F_1{}^V(s)\gamma_\mu+F_2{}^V(s)i\sigma_{\mu\nu}(p+\bar{p})^\nu]v(\bar{p}), \tag{3.1}$$

with $s=(p+\bar{p})^2$, where p and $\bar{p}$ are the lepton four-momenta in the center-of-momentum frame and M_l the mass. The absorptive parts of $F_1{}^V$ and $F_2{}^V$ can be calculated in the canonical manner by making use of (2.3). Retaining only the lightest, viz., the 2π state, the amplitudes on the right-hand side of (2.3) can be expressed as

$$\langle\pi^+\pi^-|J_\mu{}^V|0\rangle=\frac{1}{(2\pi)^3(2q_0)}F(s)(\bar{q}-q)_\mu, \tag{3.2}$$

$$\langle l\bar{l}|T|\pi^+\pi^-\rangle=\frac{1}{(2\pi)^6}\left(\frac{M_l}{2p_0q_0}\right)\bar{u}(p)$$
$$\times[A(s,t)+B(s,t)\gamma\cdot(q-\bar{q})]v(\bar{p}), \tag{3.3}$$

where q, $\bar{q}$ are the π^+, π^- four-momenta, respectively, $t=(p-q)^2$.

It is clear that $F(s)$ has the phase of $\pi\pi$ scattering in the $T=J=1$ state and may be expressed as

$$F(s)=F(0)F_\pi(s), \tag{3.5}$$

where

$$F_\pi(s)=\exp\left[\frac{s}{\pi}\int_{4\mu^2}^\infty\frac{\delta(s')ds'}{s'(s'-s-i0)}\right]$$

is the electromagnetic form factor of the pion, δ being the $\pi\pi$ phase shift in the $T=J=1$ state. Substituting back into Eq. (2.3), we obtain,

$$\mathrm{Im}F_2{}^V(s)=-\frac{F(0)}{8\pi}F_\pi{}^*(s)\left(\frac{Q^3}{\sqrt{s}}\right)f(s), \tag{3.6}$$

$$\mathrm{Im}F_1{}^V(s)=2M_l\,\mathrm{Im}F_2{}^V(s)+\frac{F(0)}{6\pi}\frac{Q^3}{\sqrt{s}}F_\pi{}^*(s)g(s), \tag{3.7}$$

where $Q\equiv|\mathbf{q}|$ and

$$f(s)=\frac{1}{PQ}\left[A_1(s)+\frac{Q}{P}2M_lB_2(s)\right], \tag{3.8}$$

$$g(s)=B_0(s)-B_2(s), \tag{3.9}$$

with

$$A_l[B_l]=\frac{1}{2}\int_{-1}^{+1}A(s,t)[B(s,t)]P_l(\hat{q}\cdot\hat{p})d(\hat{q}\cdot\hat{p}).$$

It is evident from (3.6) and (3.7), as indeed from the general theorem of Fubini, Nambu, and Wataghin,[4] that, for $s>4\mu^2$, $f(s)$ as well as $g(s)$ should have the phase of $F_\pi(s)$. Furthermore, the only important singularities of these functions consist of a pole at $s=0$ arising from the one-photon Born term and the unitarity cut starting at $4\mu^2$. These properties serve to define the functions uniquely through integral equations of the form

$$\phi(s)=\frac{R_\phi}{s}+\frac{1}{\pi}\int_{4\mu^2}^\infty\frac{e^{-i\delta(s')}\sin\delta(s')\phi(s')ds'}{s'-s-i0}, \tag{3.10}$$

where $\phi\equiv f$ or g, and R is the appropriate residuum. Without further ado we quote the explicit solutions

$$f(s)=-\left(\frac{2e^2\kappa}{3M_l}\right)\frac{F_\pi(s)}{s}, \tag{3.11}$$

$$g(s)=+e^2(1-\kappa)F_\pi(s)/s. \tag{3.12}$$

Here e is the electric charge and κ the anomalous moment of the lepton; κ is retained only for the sake of completeness.

We now assume that $F_1{}^V(s)$ and $F_2{}^V(s)$ satisfy unsubtracted spectral representations; an assumption that can be justified *a posteriori*. In this manner we

[3] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[4] S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. **111**, 329 (1958) (Appendix II). See also W. Frazer and J. R. Fulco, *ibid.* **117**, 1609 (1960).

obtain

$$F_1{}^V(s)=\left(\frac{2\alpha F(0)}{3\pi}\right)\int_{4\mu^2}^{\infty}\frac{Q(s')^3}{\sqrt{s'}}\frac{|F_\pi(s')|^2}{s'(s'-s-i0)}ds',\qquad(3.13)$$

$$F_2{}^V(s)=\frac{\kappa}{2M_l}F_1{}^V(s).\qquad(3.14)$$

The calculation of the matrix element $M_\mu{}^V$ is thus reduced to a simple quadrature.

If one assumes that P-wave $\pi\pi$ scattering is dominated by a single resonance (ρ meson) then, as shown in the Appendix,

$$F_1{}^V(s)\cong\tfrac{1}{4}\alpha F(0)\frac{m_\rho{}^2}{(\gamma_{\rho\pi\pi}{}^2/4\pi)}\frac{F_\pi(s)-1}{s},\qquad(3.15)$$

with $\gamma_{\rho\pi\pi}{}^2/4\pi=3m_\rho{}^2\Gamma_\rho/(m_\rho{}^2-4\mu^2)^{3/2}$, where m_ρ is the mass and Γ_ρ the full width of the ρ meson. Notice the presence of $F_\pi(s)-1$ in (3.15); the absence of any pole in $F_1{}^V(s)$ at $s=0$ is, of course, implicit in (3.13) and follows generally from current conservation.

The magnitude of the vector neutral lepton-current coupling, relative to that for charged lepton currents, can be measured through the ratio

$$\frac{F_1{}^V(0)}{F(0)}\cong\frac{\pi\alpha}{\gamma_{\rho\pi\pi}{}^2}.\qquad(3.16)$$

4. THE AXIAL VECTOR CURRENT

We now proceed to examine the structure of $M_\mu{}^A$. The representation corresponding to (3.1), and in essentially the same notation is

$$\langle l\bar{l}|J_\mu{}^A|0\rangle=\frac{1}{(2\pi)^3}\left(\frac{M_l}{p_0}\right)\bar{u}(p)\times[F_1{}^A(s)\gamma_\mu\gamma_5+F_3{}^A(s)(p+\bar{p})_\mu\gamma_5]v(\bar{p}).\qquad(4.1)$$

The pion configurations $|m\rangle$ now lead to $|l\bar{l}\rangle$ through the mediation of at least two photons. Furthermore, the configurations contributing to $F_1{}^A(s)$ must carry the quantum numbers $T=1$, 1^{+-}, whereas $F_3{}^A(s)$ also receives contributions from $T=1$, 0^{--}. Thus, $F_3{}^A(s)$ is particularly easy to study since, at least for low s, it receives its major contribution from the one-pion intermediate state.

Introducing the parametrizations

$$\langle\pi^0|J_\mu{}^A|0\rangle=P_\mu\frac{f_\pi(s)}{(2\pi)^{3/2}(2P_0)^{1/2}},\qquad(4.2)$$

$$\langle l\bar{l}|T|\pi^0\rangle=-\left(\frac{M_l}{p_0}\right)\frac{1}{(2\pi)^{9/2}}\frac{1}{(2P_0)^{1/2}}\times F_{\pi l\bar{l}}(s)\bar{u}(p)\gamma_5 v(\bar{p}),\qquad(4.3)$$

P being the π^0 four-momentum, we obtain on substituting in (2.4),

$$\mathrm{Im}F_3{}^A(s)=-\pi f_\pi(s)F_{\pi l\bar{l}}(s)\delta(s-\mu^2).\qquad(4.4)$$

The absence of any primary coupling then implies

$$F_3{}^A(s)=\frac{f_\pi(\mu^2)F_{\pi l\bar{l}}(\mu^2)}{s-\mu^2}.\qquad(4.5)$$

One may calculate $F_{\pi l\bar{l}}(\mu^2)$ using, for example, the procedure of Drell.[5] This procedure gives

$$F_{\pi l\bar{l}}(\mu^2)=\frac{M_l}{2\pi\mu}\int_0^\infty\frac{ds\,\mathrm{Re}G(s)}{s-\mu^2-i0}\frac{s^{1/2}}{(s-4M_l{}^2)^{1/2}}\times\ln\frac{s^{1/2}+(s-4M_l{}^2)^{1/2}}{s^{1/2}-(s-4M_l{}^2)^{1/2}},\qquad(4.6)$$

where $G(s)$ is the dimensionless invariant form factor occurring in the decay $\pi^0\to2\gamma$.

Turning back to the pure pseudovector form factor, namely, $F_1{}^A(s)$; we remark that the occurrence of only "heavier" intermediate states enables one to assume that this form factor is slowly varying for small s. Thus, $F_1{}^A(s)\cong F_1{}^A(0)$. We can ascribe to $F_1{}^A(0)$ a reasonably meaningful *effective* value through a Goldberger-Treiman type formula,[6]

$$F_1{}^A(0)=\frac{\mu^2}{2M_l}F_3{}^A(0).\qquad(4.7)$$

The relative strength of the axial neutral lepton coupling can then be measured through the dimensionless ratio

$$a_l=\frac{F_1{}^A(0)}{\sqrt{2}g_A}=\frac{1}{G}\frac{M_N}{2M_l}F_{\pi l\bar{l}}(\mu^2).\qquad(4.8)$$

Here G is the πN coupling constant and we have used the usual Goldberger-Treiman formula[6] for f_π. For purpose of orientation as to order of magnitude,[7]

$$a_e\cong\frac{1}{2G}\frac{M_N}{M_e}\times4\times10^{-7},\qquad(4.9)$$

$$a_\mu\cong\frac{1}{2G}\frac{M_N}{M_\mu}\times2\times10^{-5}.\qquad(4.10)$$

Finally, the extent of parity nonconservation at the lepton "vertex" can be measured through the ratio

[5] S. D. Drell, Nuovo Cimento **11**, 694 (1959).

[6] See for example, J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960). This paper contains references to the original work.

[7] The very crude estimations of $F_{\pi^0 l\bar{l}}(\mu^2)$ quoted here and used in Sec. 5b were obtained by replacing $G(s)$ by $g_0\theta(\Lambda^2-s)$ and retaining only the leading term [which for large Λ would diverge as $(\log\Lambda)^2$]. One can determine g_0 from the π^0 decay width and we choose $\Lambda=2M_N$; cf. Drell, Ref. 5.

$|b|/|a|$ introduced in Sec. 2. We have

$$\frac{|b|}{|a|}=\frac{a_l F(0)}{F_1{}^V(0)}=\frac{a_l \gamma_{\rho\pi\pi}{}^2}{\pi\alpha}. \tag{4.11}$$

5. RELEVANT PHYSICAL PROCESSES

We proceed to consider some exotic physical processes that may be expected to carry the signature of neutral lepton currents.

A. The Process $W^0 \to l+\bar{l}$

Under the assumption that W^0 particles exist,[8] one can easily write down the partial decay width. An elementary calculation gives

$$\Gamma(W^0\to l+\bar{l})=\frac{|F_1{}^V(m_W{}^2)|^2}{12\pi}\left(1+\frac{2M_l{}^2}{m_W{}^2}\right)\times(m_W{}^2-4M_l{}^2)^{1/2} \tag{5.1}$$
$$\cong\frac{|F_1{}^V(m_W{}^2)|^2}{12\pi}m_W,$$

since $M_l{}^4 \ll m_W{}^4$. With $F_\pi(s)$ evaluated in the small-width approximation, one obtains the branching ratio

$$\frac{\Gamma(W^0\to l+\bar{l})}{\Gamma(W^0\to\pi^++\pi^-)}=\tfrac{1}{4}\alpha^2\frac{1}{(\gamma_{\rho\pi\pi}{}^2/4\pi)^2}\left(1-\frac{4\mu^2}{m_W{}^2}\right)^{-3/2}. \tag{5.2}$$

B. The Process $K_2{}^0 \to l+\bar{l}$

Dispersing in the $K_2{}^0$ mass and retaining the first available state (one pion), we obtain

$$\Gamma(K_2{}^0\to l+\bar{l})=\frac{1}{8\pi}(m_K{}^2-4M_l{}^2)^{1/2}\left|\frac{f_{K_2{}^0\pi}F_{\pi l\bar{l}}(\mu^2)}{m_K{}^2-\mu^2}\right|^2. \tag{5.3}$$

The unknown parameter $f_{K_2{}^0\pi}$ can be estimated from a variety of sources such as the $K_1{}^0$, $K_2{}^0$ mass difference, τ decay, etc. A generous upper limit turns out to be[9]

$$f_{K_2{}^0\pi^0}\lesssim 5.39\times10^{-8}M_N{}^2.$$

Hence,

$$\Gamma(K_2{}^0\to e+\bar{e})\lesssim 2.2\times10^{-4}\ \text{sec}^{-1}, \tag{5.4}$$

$$\Gamma(K_2{}^0\to \mu+\bar{\mu})\lesssim 0.7\ \text{sec}^{-1}. \tag{5.5}$$

C. The Process $K^+\to\pi^++l+l$

Historically this process was first considered by Dalitz[10] as an example of anomalous τ events. Since then a number of interesting observations have appeared in the literature,[11–13] with which we find ourselves in varying degrees of disagreement. In any case, our approach is sufficiently different to warrant a description.

We shall assume that two-pion intermediate states predominate in the channel $\bar{K}+\pi\to l+\bar{l}$. One can justify the neglect of 3π states by appealing to the weakness of the γ-3π vertex (which necessarily appears); the relevant arguments can be based either on a belief in the dominance of minimal pion-photon coupling or on experimental fact.[14]

Let us write the matrix element as the sum of three terms; the "dissection" being performed in the same way as in Ref. 13. We have

$$\langle l\bar{l}\pi^+|T|K^+\rangle=\frac{1}{(2\pi)^6}\cdot\frac{M_l}{(4m_KE_\pi p_0\bar{p}_0)^{1/2}}\times\bar{u}(p)(\gamma\cdot p_K+\gamma\cdot p_\pi)v(\bar{p})\frac{1}{s}A(s), \tag{5.6}$$

with

$$A(s)=\frac{f_{K^+\pi^+}(m_K{}^2)}{m_K{}^2-\mu^2}F_{\pi\pi}(s,m_K{}^2,\mu^2)-\frac{f_{K^+\pi^+}(\mu^2)}{m_K{}^2-\mu^2}F_{KK}(s,m_K{}^2,\mu^2)+A_{\text{ir}}(s), \tag{5.7}$$

$$s=(p_K-p_\pi)^2.$$

Here $F_{\pi\pi}(s,m_K{}^2,\mu^2)$ and $F_{KK}(s,m_K{}^2,\mu^2)$ are appropriate continuations of the pion and (isovector) K-meson electromagnetic form factors, and $A_{\text{ir}}(s)$ is the part of the matrix element not reducible in terms of two point vertices.

The calculations of Sec. 3 provide us with a simple approximate form for $A_{\text{ir}}(s)$, namely,

$$A_{\text{ir}}(s)=(\text{const})sG_+(s)F_1{}^V(s), \tag{5.8}$$

where $G_+(s)$ is the form factor which occurs in K_{e3} decay. Since gauge invariance requires that $A(0)=0$, we note that the above expression for $A_{\text{ir}}(s)$ is gauge-invariant by itself and that

$$A(s)=A_{\text{ir}}(s)+\frac{f_{K^+\pi^+}(\mu^2)}{m_K{}^2-\mu^2}\left[\frac{F_{\pi\pi}(s,m_K{}^2,\mu^2)}{F_{\pi\pi}(0,m_K{}^2,\mu^2)}-\frac{F_{KK}(s,m_K{}^2,\mu^2)}{F_{KK}(0,m_K{}^2,\mu^2)}\right]F_{KK}(0,m_K{}^2,\mu^2). \tag{5.9}$$

In the absence of any reliable procedure for calculating these doubly off-shell form factors, we content our-

[8] Following T. D. Lee and C. N. Yang [Phys. Rev. **119**, 1410 (1960)], we use the letter W to indicate the intermediate vector boson mediating the weak interactions.

[9] S. Oneda and Y. Tamkawa, Phys. Letters **2**, 243 (1959).

[10] R. H. Dalitz, Phys. Rev. **99**, 915 (1955).

[11] N. Cabbibo and E. Ferrari, Nuovo Cimento **18**, 928 (1960).

[12] L. B. Okun' and A. Rudik, Zh. Eksperim. i Teor. Fiz. **39**, 600 (1960) [translation: Soviet Phys.—JETP **12**, 422 (1961)].

[13] M. Baker and S. L. Glashow, Nuovo Cimento **25**, 857 (1962).

[14] B. R. Desai, Phys. Rev. **124**, 1248 (1961).

selves with the crude expression

$$F_{KK}\cong F_{\pi\pi}\cong m_\rho^2/(s-m_\rho^2). \tag{5.10}$$

Under the above approximations,

$$A(s)\cong A_{\rm ir}(s). \tag{5.11}$$

The spectrum is given by

$$\frac{\partial^2\Gamma}{\partial p_\pi \partial(\cos\theta)}=\frac{2m_K}{(2\pi)^3}\frac{x^2}{E_\pi}|G_+(s)F_1{}^V(s)|^2\xi^2[4\xi_0\eta_0-1+x^2]\times\frac{(m_K-E_\pi)^4}{[(1-x^2)^2-4y^2(1-x^2\cos^2\theta)]^{1/2}}, \tag{5.12}$$

where

$$\xi=\frac{-(1-x^2)x\cos\theta+[(1-x^2)^2-4y^2+4x^2y^2\cos^2\theta]^{1/2}}{2(1-x^2\cos^2\theta)},$$

$$\xi_0=1-\eta_0=(\xi^2+y^2)^{1/2},$$

$$x=\frac{|\mathbf{p}_\pi|}{m_K-E_\pi},\quad y=\frac{M_l}{m_K-E_\pi},\quad s=(m_K-E_\pi)^2(1-x^2),$$

and θ is the angle between the pion and the lepton. For electrons the result reduces to

$$\left[\frac{\partial^2\Gamma}{\partial p_\pi\partial(\cos\theta)}\right]_{l=e}=\frac{m_K}{2(2\pi)^3}\frac{x^4}{E_\pi}|G_+(s)F_1{}^V(s)|^2(1-x^2)^2\times(m_K-E_\pi)^4\frac{\sin^2\theta}{(1+x\cos\theta)^4}$$
$$\equiv\left|\frac{F_1{}^V(s)}{F(0)}\right|^2\left[\frac{\partial^2\Gamma}{\partial p_\pi\partial(\cos\theta)}\right]_{K_{e3}}. \tag{5.13}$$

We have written down the spectrum only for the sake of completeness; perhaps more important at the moment is the absolute decay rate. This can be immediately read off:

$$\frac{\Gamma(K^+\to\pi^++e+\bar{e})}{\Gamma(K_{e3}{}^+)}\cong\frac{|F_1{}^V(0)|^2}{|F(0)|^2}=\frac{\alpha^2}{16}\left(\frac{1}{\gamma_{\rho\pi\pi}{}^2/4\pi}\right)^2\cong1.64\times10^{-5} \tag{5.14}$$

for a ρ^0 width of 90 MeV. Since $K_{e3}{}^+$ decay accounts for about 5% of all K^+ decays, we expect a branching ratio $\sim10^{-6}$. This may be compared with the estimates of Cabbibo and Ferrari[11] ($\sim10^{-7}$) and of Baker and Glashow[13] ($\sim10^{-6}$). It is interesting to note that the result of these latter authors agrees with ours in order of magnitude, even if this agreement is largely fortuitous.

6. CONCLUSIONS

We have considered the induction of neutral lepton currents in the weak interaction Lagrangian through the mediation of the electromagnetic field. For purposes or orientation we chose a simple model in which these currents are generated by the electromagnetic coupling of W^0 to leptons. In this model it was possible to assess the couplings of both the vector and axial vector parts of the lepton current. These calculations are, of course, directly applicable to the process $W^0\to l+\bar{l}$ in case W^0 actually exists. In this connection we wish to append a word of caution concerning the use of expression (5.2), as well as other expressions in which the small-width approximation for $F_\pi(s)$ has been used to exhibit the results in a simple form. This approximation is valid only if $|s-m_\rho{}^2|\gg m_\rho\Gamma_\rho$. Thus, if the W^0 mass lay close to the ρ mass, Eq. (5.2) would be invalid. However, one can still use (5.1) to calculate the leptonic decay of W^0.

We have also considered the processes $K_2{}^0\to l+\bar{l}$ and $K^+\to\pi^++l+\bar{l}$. In view of the large number of K-decay events available, these reactions seem eminently suited for any search for neutral lepton currents. If the reaction rates turn out to be spectacularly larger than our estimations (which seems highly unlikely on the basis of present evidence), one would have some evidence for neutral lepton currents of nonelectromagnetic origin. The identification can be completed by polarization measurements. Of course, the degree of polarization should be very much larger than that obtained on the basis of 4.11. One may hope to avoid polarization measurements by looking at the spectrum in the latter process. For this purpose the electronic decay, favored by phase space, is by itself entirely useless.

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APPENDIX

We wish to consider the integral

$$I(s)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{Q(s')^3}{\sqrt{s'}}\frac{|F_\pi(s')|^2}{s'(s'-s-i\epsilon)}ds' \tag{A1}$$

in the approximation in which $\pi\pi$ scattering in the $T=J=1$ state is regarded as completely dominated by the ρ meson.

Let us define

$$A(s)=\frac{s^{1/2}}{Q^3}e^{i\delta}\sin\delta=-\left(\frac{8}{3}\right)\left(\frac{\gamma_{\rho\pi\pi}{}^2}{4\pi}\right)\Delta_F(s), \tag{A2}$$

where the ρ-meson propagator $\Delta_F{}^{\mu\nu}(s)$ defines $\Delta_F(s)$ through the relation

$$\Delta_F{}^{\mu\nu}(s)=\Delta_F(s)\left(g^{\mu\nu}-\frac{P_\rho{}^\mu P_\rho{}^\nu}{m_\rho{}^2}\right). \tag{A3}$$

In virtue of the assumption made above, the function $A(s)[F_\pi(s)]^{-1}$ is analytic in the whole s plane and thus reduces to a constant. Since $F_\pi(0)=1$, $\Delta_F(0)\cong -1/m_\rho{}^2$, we have

$$A(s)=A(0)F_\pi(s), \tag{A4}$$

where

$$A(0)=+\frac{8}{3m_\rho{}^2}\frac{\gamma_{\rho\pi\pi}{}^2}{4\pi}. \tag{A5}$$

Now unitarity tells us that

$$\frac{1}{2i}[A(s+i\epsilon)-A(s-i\epsilon)]=Q^3s^{-1/2}|A(s)|^2. \tag{A6}$$

Hence, with an obvious choice for the contour Γ,

$$\begin{aligned}I(s)&=\frac{1}{2\pi iA(0)}\int_\Gamma\frac{F_\pi(s')ds'}{s'(s'-s)}\\&=\frac{3m_\rho{}^2}{8}\frac{1}{(\gamma_{\rho\pi\pi}{}^2/4\pi)}\left[\frac{F_\pi(s)-1}{s}\right].\end{aligned} \tag{A7}$$

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Negative Pion Photoproduction from Bismuth Accompanied by Neutron Emission*

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The reaction $\gamma+\mathrm{Bi}^{209}\rightarrow\pi^-+\mathrm{Po}^{209-x}+xn$ has been investigated using 250-MeV bremsstrahlung. Alpha counting technique has been selected since it permits unambiguous identification of the produced radioactivity as coming from polonium. The effective cross section σ_x for the production of Po^{209-x} peaks at $x=2$. This result is in fair agreement with calculations based on an impulse approximation model which assumes (1) the π^- is photoproduced from a free neutron close to the surface, (2) the π^- is emitted without undergoing any collision, (3) the neutron emission results from the interaction of the recoiling proton with the Bi^{208} "spectator" nucleus, (4) this process can be treated as a (p,xn) reaction initiated by a free proton. The sum of the cross sections σ_x is $\sim$2.5 mb. This value is approximately 20 times that for π^- photoproduction from a free neutron, and is therefore compatible with a surface interaction model.

PHOTOPRODUCTION of negative pions from bismuth is an example of the possibility of using pion photoproduction for the investigation of problems in nuclear structure. This reaction yields several polonium isotopes via

$$\gamma+\mathrm{Bi}^{209}\rightarrow\pi^-+\mathrm{Po}^{209-x}+xn. \tag{1}$$

The process involves the emission of a π^- by one of the neutrons from the doubly magic Pb^{208} core. The residual proton is forced to leave the core, since the core is closed for protons. However, if polonium is to be formed, the proton must stay in the nucleus. The Po^{209} nucleus is de-excited via the emission of x neutrons ($x=0, 1, 2, \cdots$). An experiment has been performed to measure the cross sections for the reactions (1) and to investigate the probability distribution of x.

Six grams of anhydrous spectroscopically pure $\mathrm{BiCl_3}$ were irradiated for 8 h by 250-MeV bremsstrahlung from the University of Illinois 300-MeV betatron. The integrated gamma flux was measured with a calibrated ionization chamber and vibrating reed electrometer. The irradiated sample was dissolved in $6N$ HCl and the polonium extracted with tributyl phosphate in dibutylether.[1] The organic phase was washed with $6N$ HCl to decontaminate from bismuth, and the polonium stripped from the organic phase with concentrated $\mathrm{HNO_3}$. After evaporation of the $\mathrm{HNO_3}$ the polonium activity was taken up in $0.5N$ HCl and plated on a 1 cm² silver foil at 97°C,[2] adding a drop of 3% KCN solution every 20 min. The extraction and plating efficiencies are estimated to be better than 98%.[1,2] The entire process requires 3–4 h.

The plated polonium sample was placed at a distance of a few mm from a 3 cm² solid-state detector coupled into a low-noise, charge-sensitive preamplifier and a 512 channel analyzer. Only alpha particles were detected and the energy spectra were printed every hour for 2 days. The over-all energy spread of the system for each alpha group is less than 40 keV. The alpha counting technique has been selected since it permits unambiguous identification of the radioactivity as coming from polonium. It should be noted that

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