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Elastic Scattering of Deuterons by Beryllium*

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The differential cross section for the elastic scattering of deuterons by beryllium has been measured for bombarding energies from 0.4 to 1.8 MeV at the center-of-mass angles of 90°, 126°16′, and 163°30′. The ratios of the observed differential cross sections to the Rutherford cross sections were found to be slowly increasing functions of the bombarding energy. Over the energy range of the present experiment these ratios are compatible with the simple assumption that the scattering nuclei can be represented by nearly impenetrable charged spheres. Since no resonance structure was observed in this work, these results disagree with previously published work on the elastic scattering of deuterons by beryllium in which prominent scattering anomalies were reported at $E_d = 1.16$ and 1.35 MeV.

I. INTRODUCTION

BOTH direct reactions and the formation of B^{11} as a compound nucleus are expected to result from the bombardment of Be^9 by deuterons. For deuterons having energies between 0.400 and 1.800 MeV, the excitation of the B^{11} nucleus is between 16.15 and 17.29 MeV. In this region, two levels, lying at 16.77 and 16.93 MeV, respectively, have been inferred primarily from anomalies in the deuteron elastic-scattering cross section.[1] In addition, the results of an investigation of the reaction $Be^9(d,n)B^{10}$ indicated the existence of a level near 16.77 MeV.[2] A later investigation of this same reaction, however, indicated a smooth variation with energy of the neutron-production cross section and, therefore, cast doubt on the existence of compound-nucleus states in this energy region.[3]

Some information about other reactions induced by deuteron bombardment of beryllium also exists.[4] Unfortunately, the quantity and quality of these data have made it impossible to say with certainty whether or not there are well-defined excited states at 16.77 and 16.93 MeV in B^{11}. The present experiment was performed in an attempt to remove some of the uncertainty regarding the level structure in this high-excitation region of B^{11}.

II. EXPERIMENTAL EQUIPMENT AND PROCEDURE

The deuterons used as scattering projectiles in this experiment were accelerated by the 1.8-MV Van de Graaff generator of the Kellogg Radiation Laboratory. A deuteron beam, monoenergetic to about 0.1%, was obtained by passing the accelerated particles through a 1-m radius, 80° electrostatic analyzer. The momenta of the scattered deuterons and the nuclear reaction products were determined by using a double-focusing magnetic spectrometer[5] with an equilibrium orbit having a radius of 26.7 cm. The particles transmitted by the spectrometer were counted by using a silicon *p-n* junction solid-state detector, and the output of this detector was then processed by a charge-collecting preamplifier, an amplifier, a pulse-height analyzer, and appropriate counting circuits.

The scattering target was a small slab of beryllium; its dimensions were 3.8 cm × 0.64 cm × 0.16 cm. A mirror-like finish on the face of this target was obtained by polishing the surface with a high-speed buffing wheel.

The target holder was suspended from a rod coaxial with the center line of the cylindrical target chamber. This rod extended through the top of the target chamber so that the vertical position and angular orientation of the target could be varied at any time. In addition to holding the beryllium target, the target holder was built to hold a copper target which was used for energy and solid-angle calibrations.

The detection of the elastically scattered deuterons was complicated by the presence of protons, tritons, and singly and doubly charged alpha particles which were produced by competing reactions. Although all of the particles which pass through the magnetic spectrometer have the same momentum-to-charge ratio, the high-energy resolution provided by the solid-state detector made it possible to distinguish the deuterons from the other charged particles present. During the experiment, the number of elastically-scattered deuterons was determined by setting the window of a single-channel pulse-height analyzer so that the deuteron pulses were counted while pulses due to other particles were not.

As the energy of the incident deuterons was decreased, the height of the pulses produced in the detector by the scattered particles decreased until the output pulses began to overlap the noise pulses from the detector and preamplifier. When this situation existed, the entire pulse-height spectrum was recorded by a multichannel analyzer so that a reliable subtraction of the noise spectrum could be made.

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*Supported by the U. S. Office of Naval Research.

† Now assigned by the U. S. Army to Picatinny Arsenal, Dover, New Jersey.

[1] M. K. Jurić and S. D. Ćirilov, Bull. Inst. Nucl. Sci. "Boris Kidrich" Belgrade **6**, 45 (1956).

[2] A. I. Shpetnyi, Zh. Eksperem. i Teor. Fiz. **32**, 423 (1957) [translation: Soviet Phys.—JETP **5**, 357 (1957)].

[3] G. C. Neilson, W. K. Dawson, and J. T. Sample, Bull. Am. Phys. Soc. **3**, 323 (1958).

[4] See, for example, F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).

[5] C. W. Snyder, S. Rubin, W. A. Fowler, and C. C. Lauritsen, Rev. Sci. Instr. **21**, 852 (1950).

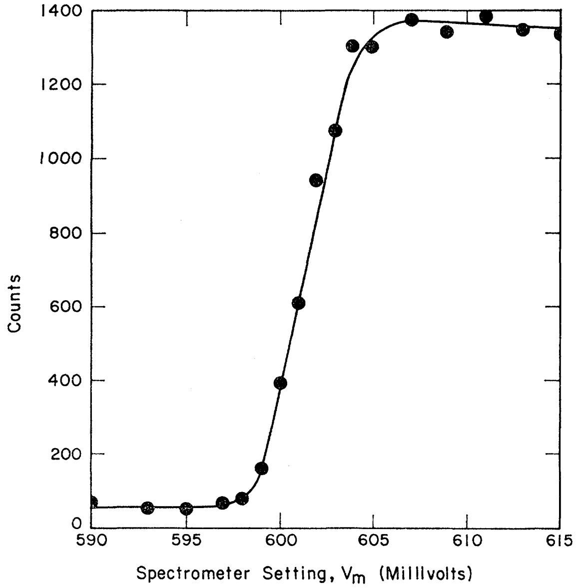

FIG. 1. A typical $Be^9(d,d)Be^9$ target profile. For this particular profile the energy of the deuterons incident on the target was 1.005 MeV and the magnetic spectrometer was positioned so that the laboratory scattering angle was 113° 26′. The abscissa scale is inversely proportional to the momentum of the scattered deuterons.

Figure 1 exhibits a typical thick-target "profile," which gives the number of scattered deuterons at a fixed bombarding energy as a function of the magnetic-spectrometer setting. The scattering cross sections were calculated from such thick-target data by using the expressions given in Refs. 5 and 6.

In order to convert the observed scattering yields to differential cross sections, it was necessary to know the acceptance solid angle of the magnetic spectrometer. This quantity was determined by scattering protons from copper and then assuming that the $Cu(p,p)Cu$ cross section was correctly given by the Rutherford formula. Several measurements of the proton-copper scattering cross section showed that its energy and angular variations are well described by the Rutherford law, and this gives one considerable confidence in this method of determining the spectrometer solid angle.

Several factors which could affect the observed number of scattered deuterons were considered. These included scaler dead-time, backscattering by the insensitive region at the face of the detector, and neutralization of the charge of a fraction of the deuterons emerging from the scattering target. Corrections for each effect were made where they were found not to be negligible. In addition, two other phenomena which could cause the scattering cross section to deviate from the Rutherford law even if no nuclear interaction occurred were examined theoretically. These were screening of the target nucleus by its electron cloud and effects resulting from the diffuse structure of the deuteron. Calculations indicated that these phenomena should have only a very small effect on the scattering cross section at the bombarding energies used in this experiment and would be undetectable at the level of precision of this work.

III. RESULTS

The results[7] of the measurements of the $Be^9(d,d)Be^9$ differential elastic scattering cross section at center-of-momentum angles, $\theta_c = 90°$, 125° 16′, and 163° 30′, are shown in Fig. 2. These angles were chosen since they are zeros in the Legendre polynomials P_1, P_2, and the maximum angle of the spectrometer, respectively. The data are expressed as the ratio of the observed cross section to the computed point-charge Rutherford cross section. For the measurements made at the center-of-mass angles of 90° and 125° 16′, the maximum particle momentum which could be analyzed by the magnetic spectrometer fixed the upper limits for the bombarding energies.

A detailed examination of the various sources of experimental uncertainty indicated that the fractional standard deviation for the absolute values of $d\sigma/d\sigma_R$ is about 6.7%. The largest contribution of this uncertainty comes from the uncertainty in the absolute value of the atomic stopping cross sections which were used in calculating the cross sections from the thick target yields.

At the lowest bombarding energies the charge neutralization correction and the background subtraction introduced additional uncertainty. These factors increase the uncertainty in $d\sigma/d\sigma_R$ to about 10.6% between the incident energies of 0.400 and 0.600 MeV for the two largest scattering angles.

IV. SCATTERING BY A CHARGED SPHERE

The most striking feature of the data shown in Fig. 2 is the absence of any rapid variation of $d\sigma/d\sigma_R$ with bombarding energy. The additional fact that the observed values of $d\sigma/d\sigma_R$ are approximately unity indicates that the Coulomb interaction is the dominant scattering mechanism. These two facts suggest that the $Be^9(d,d)Be^9$ interaction should be describable in terms of a relatively simple model in this energy region.

Scattering cross sections calculated on the assumption that the target nuclei were impenetrable charged spheres gave values for $d\sigma/d\sigma_R$ which increased with energy much more rapidly than the experimental results. In addition, such a nuclear model would imply that the associated total-reaction cross section is zero. This is clearly inconsistent with the available reaction

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[6] A. B. Brown, C. W. Snyder, W. A. Fowler, and C. C. Lauritsen, Phys. Rev. **82**, 159 (1951).

[7] More complete experimental details, as well as a discussion of the interpretation of the data, may be found in the author's Ph.D. thesis, California Institute of Technology, 1963 (unpublished).

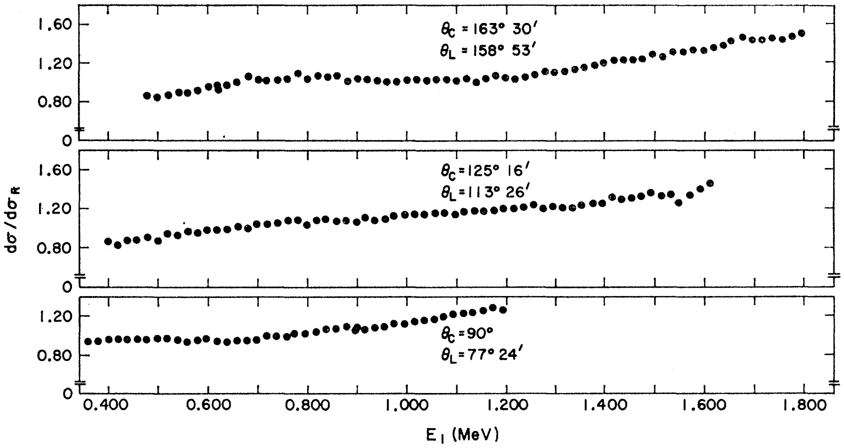

FIG. 2. The ratio of the experimental $Be^9(d,d)Be^9$ differential cross section to the Rutherford cross section.

data which indicate that a substantial total-reaction cross section exists.

A more realistic approach is to fix the boundary condition at the nuclear surface in such a way that attenuation of the amplitudes of at least some of the scattered partial waves is permitted. It is, perhaps, instructive to determine how much the amplitudes and phases of the scattered partial waves must differ from the values characteristic of a charged impenetrable sphere in order to reproduce the experimental data. Since scattering data are available at only three angles, and since the reaction data are also rather incomplete, it was decided to ignore the spin dependence of the scattering and reaction processes. Even without spin-dependent terms, the four available cross sections at each energy can only determine four parameters, for example, the real and imaginary parts of the s- and p-wave phase shifts.

It can readily be shown from the scattering amplitude given by Blatt and Weisskopf[8] that the ratio of differential cross sections $d\sigma/d\sigma_R$ is given by

$$\frac{d\sigma}{d\sigma_R}=1-\mathrm{Re}\left\{\frac{i\sin^2(\frac{1}{2}\theta)}{kZ}\exp[2i\eta\ln\sin(\tfrac{1}{2}\theta)]\right.$$

$$\left.\times\sum_{l=0}^{\infty}(2l+1)\exp(2i\psi_l)[1-\gamma_l\exp(-2i\sigma_l)]P_l(\cos\theta)\right\}$$

$$+\frac{\sin^4(\frac{1}{2}\theta)}{4k^2Z^2}\sum_{L=0}^{\infty}\sum_{l=0}^{\infty}\sum_{l'=|l-L|}^{l+L}(2l+1)(2l'+1)\exp(2i\psi_{l'}$$

$$-2i\psi_l)[(ll'00|ll'L0)]^2[1-\gamma_l\exp(-2i\sigma_l)]^*$$

$$\times[1-\gamma_{l'}\exp(-2i\sigma_{l'})]P_L(\cos\theta). \quad (1)$$

In this expression, Re specifies the real part, η is the usual Coulomb parameter $Z_1Z_2e^2/hv$, $Z=Z_1Z_0e^2/2Mv^2$ where M is the reduced mass of the system, $\hbar k=(2ME_{\mathrm{c.m.}})^{1/2}$, θ is the c.m. scattering angle, $(ll'00|ll'L0)$ is the usual Clebsch-Gordan coefficient, $\psi_l=\sigma_l-\sigma_0$ where the σ_l are the usual Coulomb phase shifts, the $\varphi_l=-\tan^{-1}(F_l/G_l)_{r=R}$ are the "hard-sphere" phase shifts, and the γ_l are the complex amplitudes of the scattered waves with orbital angular momentum l. In the analysis to follow the quantity $\xi_l=\sigma_l+\varphi_l$ will also be required.

It can also be shown that the total-reaction cross section is given by

$$\sigma_r=\frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)(1-|\gamma_l|^2).$$

Thus, a knowledge of the total-reaction cross section can give information about the magnitudes of the γ_l. The variation of the reaction cross section with energy was estimated from the available data[9–12]; the result is shown in Fig. 3. Because of the large number of energetically possible reactions, these data are incomplete. Therefore, this composite curve is only a lower estimate.

The relatively low incident energies used in this experiment suggest that absorption of only the s- and p-waves by the nucleus should be considered. Thus the γ_l for the d- and higher waves were assumed in all cases to have the impenetrable charged sphere value $\gamma_l=\exp[2i(\varphi_l+\sigma_l)]$, $l\geq 2$.

The first attempt to fit the experimental results was made for an incident energy of 0.500 MeV. Here it was assumed that only the s-wave channel would contribute appreciably to the reaction cross section and

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[8] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. 8. The expression on p. 336 from which Eq. (1) was derived contains an error in the sign of $A_{\mathrm{res}}{}^l$.

[9] D. De Jong, P. M. Endt, and J. G. Simons, Physica **18**, 676 (1952).

[10] J. A. Biggerstaff, Ph.D. thesis, University of Kentucky, Technical Report, The Kentucky Research Foundation, 1961 (unpublished).

[11] R. Bardes and G. E. Owen, Phys. Rev. **120**, 1369 (1960).

[12] G. C. Neilson, W. K. Dawson, F. A. Johnson, and J. T. Sample, Suffield Tech. Paper 176, Suffield Experiment Station, Ralston, Alberta, 1960 (unpublished).

that the amplitude γ_1 had the hard-sphere value. This determined $|\gamma_0|$ uniquely. The variation in the cross section due to changes in the phase of γ_0 and the interaction radius was examined and the best fit to the experimental data was determined by minimizing the quantity

$$\epsilon=\sum_{\theta}\frac{[\sigma_e(\theta)-\sigma_c(\theta)]^2}{\sigma_c(\theta)}, \tag{2}$$

where the subscripts refer to experimental and calculated values. The interaction radius which gave the best fit was 3.7 F, although the minimum in Eq. (2) was rather broad. For comparison, the expression

$$r=1.20(A_0{}^{1/3}+A_1{}^{1/3})$$

gives a rough estimate of 4.0 F for the sum of the radii of the target (0) and bombarding (1) particles. An interaction radius of 3.7 F was assumed for all partial waves at all energies covered by this work.

At 0.700 MeV a small amount of *p*-wave contribution to the reaction cross section was introduced. This required that the relative magnitudes of γ_0 and γ_1 also be determined by trial and error. Figure 4 shows an example of how changes in the phase of γ_0 affect $d\sigma/d\sigma_R$.

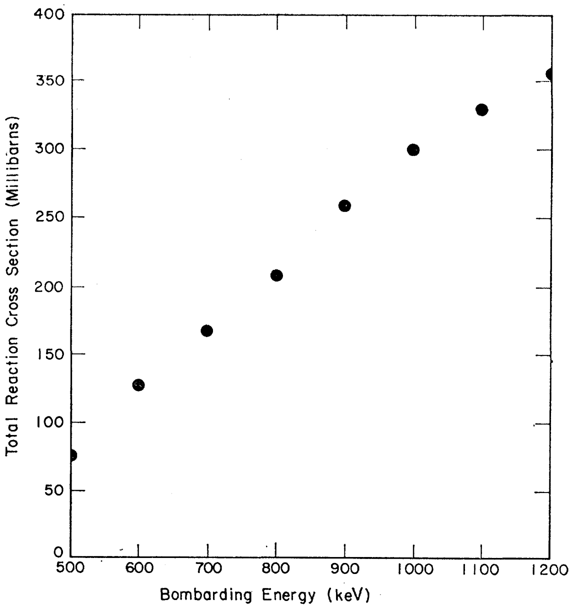

FIG. 3. Estimated total-reaction cross section for Be^9+d. These data were constructed from information found in the literature. The cross sections for several of the reaction channels have not been measured, so no contributions were included for these channels. Hence, the composite result shown here must be considered as a lower limit for the total-reaction cross section. The uncertainty can only be guessed, but +50 and −20% seem to be reasonable limits.

The values for the coefficients γ_0 and γ_1 which produce the best fit to the experimental data are listed in Table I. As mentioned earlier, for bombarding energies at 0.700 MeV and higher, the availability of four parameters, the magnitude and argument of both γ_0 and γ_1, should permit an exact fit of four cross sections having almost any values, although one does have the restriction that $|\gamma_l|\leq 1$. Thus, the significant fact learned from the charged-sphere calculations is not that a fit is possible, but rather that the parameters which give a fit vary in a reasonable way as the bombarding energy is changed. Undoubtedly, the unevenness in the variation of the parameters shown in Table I can be attributed to uncertainties in the scattering and reaction cross sections and the relative crudeness of the trial-and-error fitting procedure.

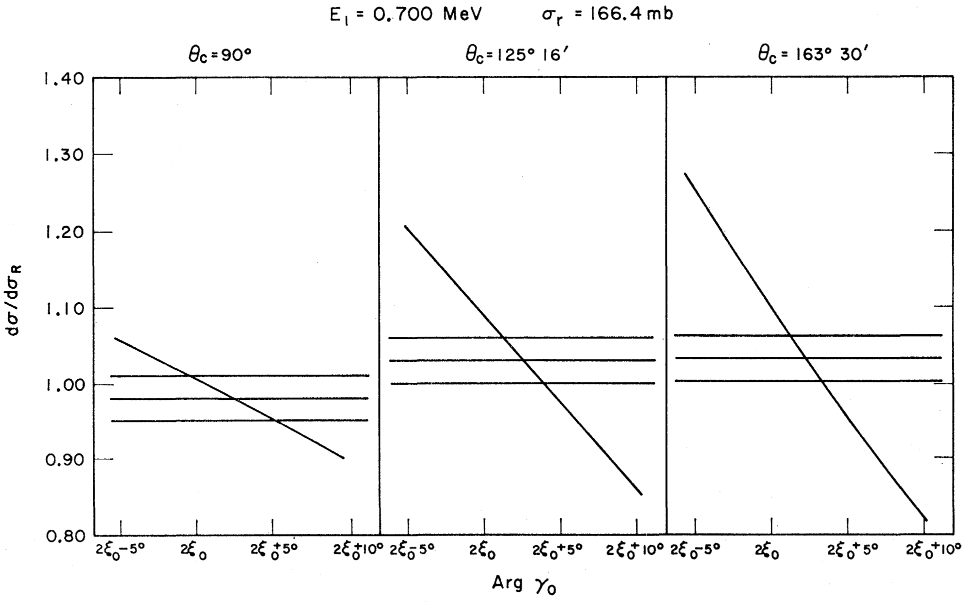

FIG. 4. The sloping lines show the calculated values of $d\sigma/d\sigma_R$ for various $\arg\gamma_0$. The phases of the complex amplitudes are expressed in terms of $\xi_l=\sigma_l+\varphi_l$. For these curves $\arg\gamma_l=2\xi_l+3°$. The three horizontal lines show the value of the experimental cross section and values which lie 3% above and below it, respectively.

TABLE I. Values of the complex partial-wave amplitudes which produce the best fit to the experimental cross sections. The phases of the amplitudes are expressed in terms of $\xi_l=\sigma_l+\varphi_l$. The quantity ϵ is defined by Eq. (2) and gives an indication of the goodness of fit.

E_1 in MeV	σ_r in mb	$\lvert\gamma_0\rvert$	$\lvert\gamma_1\rvert$	$\arg\gamma_0$	$\arg\gamma_1$	ϵ
0.500	74.3	0.9613	1.0000	$2\xi_0+6.5°$	$2\xi_1$	3.8×10^{-3}
0.600	129.3	0.9163	1.0000	$2\xi_0+4.3°$	$2\xi_1$	2.8×10^{-3}
0.700	166.4	0.9049	0.9900	$2\xi_0+2.5°$	$2\xi_1+3°$	25×10^{-6}
0.800	215.0	0.9064	0.9700	$2\xi_0+1.5°$	$2\xi_1$	28×10^{-6}
0.900	258.4	0.9100	0.9471	$2\xi_0+1.0°$	$2\xi_1$	61×10^{-6}
1.000	289.2	0.8800	0.9323	$2\xi_0+1.1°$	$2\xi_1+2°$	105×10^{-6}
1.100	328.5	0.9000	0.9062	$2\xi_0+0.5°$	$2\xi_1$	82×10^{-6}
1.200	354.7	0.8700	0.8874	$2\xi_0-0.7°$	$2\xi_1$	105×10^{-6}

V. DISCUSSION OF RESULTS

A comparison of the present results with those given by Jurić and Ćirilov[1] reveals that the two measurements are in disagreement concerning the existence of two states in B^{11} formed at incident-deuteron energies of 1.16 and 1.35 MeV and having widths of 70 and 120 keV, respectively. In the present work no evidence for resonances was found (Fig. 2). It seems possible that the poor energy resolution in the earlier work resulting from the use of photographic plates as detectors and a rather thick target would have made it difficult for these workers to distinguish the deuterons scattered by beryllium from those scattered by contaminants, such as carbon and oxygen, and from the abundant reaction products.

An unexpected feature of the excitation functions shown in Fig. 2 is the occurrence of cross sections which fall below the Rutherford values at low bombarding energies. This sub-Rutherford effect is interesting because similar results were observed in recent measurements of the $Li^7(d,d)Li^7$ scattering cross section.[13] One would expect the point-charge assumption of the Rutherford theory to be increasingly well satisfied as the energy of the projectile is decreased. Thus, the fact that the measured cross sections fall below the Rutherford values at low bombarding energies is not trivial.

It is possible that the scattering cross section actually is smaller than the Rutherford value at low energies because of absorption or some obscure effect peculiar to the structure of the deuteron. On the other hand, the measured values of the scattering cross section could be low because of some instrumental or other experimental difficulty for which an adequate correction has not been made. However, careful examination of various factors which could conceivably affect the experimental measurements has revealed no reason for doubting the validity of the measured scattering cross sections.

In the case of $Li^7(d,d)Li^7$, Ford[13] has concluded that the low values obtained for the low-energy cross sections are probably due to oxygen in the surface layers of the lithium target. Since this explanation seems to be untenable in the present experiment due to the very small amount of contaminant in the beryllium targets, the question of whether the low-energy cross sections are really significantly below the Rutherford values remains open.

Some of the Be^9+d reaction data cited previously suggest that there may be compound nuclear states in the energy region of the present experiment. These data are not necessarily incompatible with the absence of resonance-like elastic scattering anomalies. Because there are other low-barrier channels available for the breakup of Be^9+d compound nuclear states, the values of Γ_d/Γ for such resonances may be very small. The departure from Rutherford scattering expected from a compound nuclear state with $\Gamma_d/\Gamma \lesssim 1\%$ would be at most a few percent, and this could have escaped notice. The present experiment excludes the existence of compound nuclear states for which $\Gamma_d/\Gamma > 10\%$ in the excitation region from 16.15 to 17.29 MeV in B^{11}.

ACKNOWLEDGMENTS

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[13] J. L. C. Ford, Jr., Ph.D. thesis, California Institute of Technology, 1962 (to be published).