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Bootstrap Model of Nondegenerate Vector-Meson Octet*

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The bootstrap model of the V (vector) meson octet, in which these particles produce themselves as resonances in the P wave, two-particle states of the PS (pseudoscalar) meson octet, is generalized to include the effect of mass splitting. The three V-meson masses and five V-PS-PS interaction constants are considered as functions of the three PS-meson masses. Only terms linear in the deviations from degeneracy are included in the dispersion relations. The physically observed PS-meson masses lead to a calculated ρ/K^* mass ratio significantly smaller than one, while the calculated ratio of the mass of the isoscalar V meson to that of the K^* is either somewhat larger than or approximately equal to one. The mass splittings lead to deviations in the ratios of the interaction constants from the values corresponding to unitary symmetry, but the deviations are small enough so that unitary symmetry is satisfied approximately. The calculated $\rho\pi\pi$ and $K^*\pi K$ interaction constants are about 2 or $2\frac{1}{2}$ times larger than the experimental values.

I. INTRODUCTION

It has been shown by the author that if the V (vector) mesons ρ, K*, and either the φ or ω are regarded as degenerate resonances in the two-particle, P-wave states of the degenerate, PS (pseudoscalar) mesons π, K, and η, a simple bootstrap model predicts that the ratios of the V-PS-PS interaction constants are equal to the ratios predicted by the octet model of unitary symmetry.[1] In the present paper the degeneracy assumptions are removed. The bootstrap model leads to eight independent equations involving the masses and interaction constants, so that if the PS-meson masses are specified, one may solve the equations for the three V-meson masses and the five interaction constants $\gamma_{\varphi KK}$, $\gamma_{\rho KK}$, $\gamma_{\rho\pi\pi}$, $\gamma_{M\pi K}$, and $\gamma_{M\eta K}$, (where the symbol M represents the K^*). We will take the PS-meson masses from experiment.

There are three related purposes for this calculation. The first is to calculate the splitting of the V-meson degeneracy that results from the experimentally observed splitting of the PS degeneracy. The V-meson mass differences will be compared with experiment, providing a test of the bootstrap model.

The second purpose has to do with the physical interaction constants. In most comparisons of unitary symmetry with experiment only partial correction for the effects of mass differences is made. For example, it is often assumed that the ratios of the physical interaction constants or, equivalently, of the reduced widths of resonances, are given exactly by unitary symmetry. This assumption is certainly not correct, but one does not know how to improve it without using a dynamical theory.[2] The assumption of exact interaction symmetry seems particularly dangerous when applied to PS-meson interactions, since the ratios of the squares of the masses of the PS mesons are very different from unity. In the bootstrap model, one makes no such assumption, but actually calculates the physical interaction constants. In this model one can see whether or not the unitary symmetry of the interaction constants is preserved approximately when the large PS-meson mass splitting is included. Furthermore, the predicted deviations from exact symmetry may be tested experimentally.

The third purpose of this calculation is to clear up the question of the importance of the degeneracy assumptions in the results of Ref. 1. In a recent letter, Sakurai shows that if one assumes degenerate PS and V-meson octets, and assumes that the second-order, "bubble-diagram" mass corrections resulting from the V-PS-PS interactions are such as to preserve the degeneracy in both octets, one obtains four equations that imply that the ratios of the interaction constants must be those corresponding to unitary symmetry.[3] The implication is given that this prescription is essentially equivalent to the bootstrap prescription of Ref. 1, and that unitary symmetry is predicted in Ref. 1 only because both PS degeneracy and V degeneracy are assumed. Actually, the considerations of Sakurai are not equivalent to the bootstrap model, as may be seen from the fact that there are eight, independent, self-consistency equations in the bootstrap model. These equations may be used to determine not only the interaction ratios, but the magnitude of the interaction and all three V-meson masses separately. The V-meson degeneracy assumption of Ref. 1 is almost superfluous.[4]

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[1] R. H. Capps, Phys. Rev. Letters 10, 312 (1963).

[2] This problem is most famous in connection with the pseudoscalar meson-baryon-baryon coupling constants. One often hears the question, "Are the pseudoscalar coupling constants G^2 or the pseudovector coupling constants f^2 to be related by unitary symmetry?" This question is misleading since it is clear that the problem would exist even if only one set of coupling constants had been defined. The question of how the PS-meson mass differences affect interaction constants can never be answered without a dynamical theory. Furthermore, one cannot avoid the problem of mass differences by discussing amplitudes rather than interaction constants, unless he considers only energies that are very high compared to all the masses.

[3] J. J. Sakurai, Phys. Rev. Letters 10, 446 (1963). This letter may be divided into two parts. In the first part it is argued that the conclusions of Ref. 1 are obvious. In the second part (last half of the last paragraph) it is argued that the conclusions of Ref. 1 are not obvious. We are concerned here only with the first part of this letter.

[4] Since the self-consistency equations are nonlinear, it may be that if one relaxes the V-meson degeneracy assumption, he admits additional individual solutions. It is not known as yet whether, for

Furthermore, as is shown in Sec. III B of the present paper, the PS-meson degeneracy assumption is not necessary for the prediction of approximate unitary symmetry.

II. THE METHOD

A. Basic Equations

Experimentally, there appear to be four types of strongly interacting vector bosons, the ρ, K^* (denoted here by M), the ω and the φ.[5] The hypercharges and isotopic spins of these particles are, respectively, (0,1), $(\pm 1,\frac{1}{2})$, (0,0), and (0,0). Only one of the two isoscalar V mesons can occur as a resonance or bound state of two PS mesons in our model; this follows from the fact that there is only one P wave, $PS+PS$ state of the appropriate quantum numbers, the $K+\bar{K}$ state.[6] For definiteness, we denote the included isoscalar V meson by the symbol φ, although in reality this particle may correspond to the ω or to a linear combination of the φ and ω.

The dispersion technique used here is a modification of that used in earlier works.[1,7,8] The coupling of the $PS+PS$, P-wave scattering states to multiple-particle states or states involving other than PS mesons is neglected. The only forces considered are those resulting from the exchange of the V mesons ρ, M, and φ; the V-meson widths are neglected in computing the forces. The scattering amplitudes in Born approximation are then taken as the numerator function N in the expression $T=ND^{-1}$, and a once-subtracted dispersion relation is written for D. It is demanded that three resonance or bound-state poles develop, which may be identified with the ρ, M, and φ, and that no other such poles develop. The requirements that the masses and coupling constants of these resonances or bound states be equal to those assumed initially for the corresponding V mesons give rise to eight self-consistency equations for the five coupling constants and three V-meson masses. The φ meson is coupled only to the $K+\bar{K}$ state; this system yields two self-consistency equations. The ρ is coupled to the $\pi+\pi$ and $K+\bar{K}$ channels, and the M is coupled to the $\pi+K$ and $\eta+K$ channels; each of these two-channel systems gives rise to three self-consistency equations.

We define the P-wave amplitude T_{ij} between two $PS+PS$ states i and j in terms of elements of the unitary U-matrix by the equation,

$$T_{ij}=(U_{ij}-\delta_{ij})s^{1/2}(2iq_i^{3/2}q_j^{3/2})^{-1},$$

where s is the square of the total energy, and q_i and q_j are the magnitudes of the particle three-momenta in the states i and j. All quantities refer to the center-of-mass system, and $\hbar$ and c are taken as unity. We illustrate our method by considering one of the two-channel systems, denoting the channels by 1 and 2. The matrix ND^{-1} method is used.[9] We assume that the Born-approximation amplitudes N for the three processes $1\rightarrow 1$, $2\rightarrow 2$, and $1\rightleftarrows 2$ are proportional to a common function of energy $\beta(s)$, so that we may write

$$N_{ij}(s)=F_{ij}\beta(s), \tag{1}$$

where the F_{ij} are constants. [When the meson octets are not degenerate, the proportionality condition of Eq. (1) is an approximation. The accuracy of this approximation is discussed in Sec. III C.] The unitarity condition for the inverse amplitude T^{-1} is

$$\mathrm{Im}(T^{-1})_{ij}=-\delta_{ij}(q_i^3/s^{1/2})\theta_i(s),$$

where the function $\theta_i(s)$ is unity if $q_i^2\geqq 0$ and zero if $q_i^2<0$. If one solves the once-subtracted dispersion relation for the denominator matrix D, using the above unitarity condition, the resulting expressions for the amplitudes T_{ij} are,

$$T_{11}=|D(s)|^{-1}\beta(s)\{F_{11}+\alpha_2(s)[F_{12}^2-F_{11}F_{22}]\}, \tag{2a}$$

$$T_{12}=|D(s)|^{-1}\beta(s)F_{12}, \tag{2b}$$

$$|D(s)|=1-\alpha_1(s)F_{11}-\alpha_2(s)F_{22}-\alpha_1(s)\alpha_2(s)[F_{12}^2-F_{11}F_{22}], \tag{2c}$$

$$\alpha_i(s)=\frac{s-s_t}{\pi}\int_{q_i^2=0}^{\infty}\frac{ds'q_i'^3\beta(s')}{s'^{\frac{1}{2}}(s'-s_t)(s'-s-i\epsilon)}, \tag{2d}$$

where s_t is the value of s where the subtraction is made. The equation for T_{22} is obtained by reversing the subscripts 1 and 2 in Eq. (2a).

Following the procedure of Refs. 1 and 8, we define $\alpha_{i,r}$ to be the real part of α_i and $T_{ij,r}$ and $|D_r|$ to be the expressions for T_{ij} and $|D|$ that result if α_i is replaced by $\alpha_{i,r}$. The condition that a resonance or bound state occurs at the energy m_V^2 is

$$|D_r(m_V^2)|=0. \tag{3}$$

The interaction constants (reduced partial widths) of the resonance or bound state at m_V^2 are defined by the

fixed PS-meson masses, the requirement that each particle has at least one nonzero coupling constant defines a unique solution to the model.

[5] For experimental evidence concerning the φ meson, see P. Schlein, W. E. Slater, L. T. Smith, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **10**, 368 (1963); P. L. Connolly, E. L. Hart, K. W. Lai, G. London, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamoto, M. Goldberg, M. Gundzik, J. Leitner, and S. Lichtman, Phys. Rev. Letters **10**, 371 (1963).

[6] Our assumption that one of the isoscalar V mesons is not strongly coupled to the $K+\bar{K}$ state is common to the best-known theoretical models that accommodate two such V mesons. In a simple model of unitary symmetry, one of the two V mesons must be a unitary singlet, which has the wrong symmetry under hypercharge reflection to be coupled to a P-wave, $PS+PS$ state. In Sakurai's V-meson model [J. J. Sakuri, Ann. Phys. (N. Y.) **11**, 1 (1960)], the coupling of one of the V mesons is proportional to the baryon number, so that this particle is not coupled strongly to mesons. It should be noted that Sakurai's model and unitary symmetry are compatible.

[7] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).

[8] R. H. Capps, Phys. Rev. **131**, 1307 (1963).

[9] J. D. Bjorken, Phys. Rev. Letters **4**, 473 (1960).

equation,

$$\gamma_i\gamma_j/(4\pi)=\tfrac{3}{8}[(m_V^2-s)T_{ij,r}]_{s=m_V^2}, \tag{4}$$

which applies to all three processes $1\to 1$, $2\to 2$, and $1\rightleftarrows 2$. It can be shown that one of the four equations represented by Eqs. (3) and (4) is implied by the other three, so there are three independent self-consistency equations.

The application of Eq. (4) to the inelastic amplitude gives a simple equation, i.e.,

$$\frac{\gamma_1\gamma_2}{4\pi}=-\frac{3}{8}\left[\frac{F_{12}\beta(s)}{\partial|D_r|/\partial s}\right]_{s=m_V^2}. \tag{5}$$

Two other simple equations may be obtained by combining Eqs. (3) and (4),

$$\gamma_1\alpha_1(m_V^2)F_{12}=\gamma_2[1-\alpha_2(m_V^2)F_{22}], \tag{6a}$$

$$\gamma_2\alpha_2(m_V^2)F_{12}=\gamma_1[1-\alpha_1(m_V^2)F_{11}]. \tag{6b}$$

These last three equations are the self-consistency equations for either of the two-channel systems, expressed in a simple form.[10]

B. The Degeneracy Solution

It has been shown that if the *PS* mesons are degenerate and the *V* mesons are degenerate, and each of the six types of particles has at least one nonzero coupling constant, there is only one solution to the eight self-consistency equations.[1,11] We shall refer to this solution as the "degeneracy solution." The ratios of the interaction constants in this solution are those predicted by unitary symmetry, i.e.,

$$\gamma_{\varphi KK}^2:\gamma_{\rho\pi\pi}^2:\gamma_{\rho KK}^2:\gamma_{M\pi K}^2:\gamma_{M\eta K}^2=1:\tfrac{2}{3}:\tfrac{1}{3}:\tfrac{1}{2}:\tfrac{1}{2}, \tag{7}$$
$$\gamma_{\rho\pi\pi}\gamma_{\rho KK}>0.$$

The absolute magnitude of $\gamma_{\varphi KK}^2$ and the resonance energy are obtained if one chooses the subtraction energy at the end of the left-hand cut. A careful integration yields values slightly different from those given in Ref. 1, i.e.,

$$\gamma_{\varphi KK}^2/(4\pi)=2.58,\quad m_V^2/m_{PS}^2=5.89.$$

C. Approximations Necessary for Quasidegeneracy Solution

If large deviations from degeneracy are considered, the equations of the model become very complicated. In this paper, we will consider the simpler case of quasidegeneracy, i.e., we include in the equations only terms linear in the deviations of the squares of the masses and coupling constants from their values in the degeneracy solution. This method will shed no light on the problem of the number of different solutions to the bootstrap model, of course.[4]

[10] This choice of equations cannot be used if either γ_1 or γ_2 is zero.

[11] R. H. Capps (to be published).

The deviations from degeneracy affect the quantities in Eqs. (2), (5), and (6) in several ways. The quantities $F_{ij}\beta(m_V^2)$ and $\alpha_i(m_V^2)$ depend directly on the mass of the resonating *V* meson. In addition, the forces $F_{ij}\beta(m_V^2)$ are quadratic in the interaction constants γ_i, and depend on the masses of the exchanged *V* mesons and of the *PS* mesons. The integrals $\alpha_i(m_V^2)$ depend on the masses of the exchanged *V* mesons through $\beta(s')$ and the subtraction energy s_t. The $\alpha_i(m_V^2)$ depend on the *PS*-meson masses through $\beta(s')$, s_t, and the relation between q_i^2 and s.[12]

The proportionality condition of Eq. (1) is an approximation when degeneracy is not assumed. Some such approximation is necessary since, as discussed in Ref. 8, the scattering matrix T generally is not symmetric in an approximation to the N/D method when the numerator functions are not proportional. In the two-channel cases, we define $\beta(s)$ so that $F_{12}\beta(s)$ represents the Born approximation to the inelastic amplitude exactly. This is convenient since in both two-channel cases, only one *V* meson (the *M* meson) contributes to the force in the inelastic process. In order to describe the proportionality approximation used for the elastic amplitudes, we define $\mathfrak{N}_{ii}$ to be the exact expression in the Born approximation for the amplitude T_{ii}.[12] The Born-approximation amplitudes enter in the self-consistency relations both through their values at the resonance energy and through the dispersion integrals. Accordingly, we make two alternate proportionality approximations, defined below.

(I) The constant F_{ii} is chosen so that the ratio F_{ii}/F_{12} is equal to the ratio of the Born-approximation amplitudes at the resonance energy, i.e.,

$$F_{ii}/F_{12}=\mathfrak{N}_{ii}(m_V^2)/[F_{12}\beta(m_V^2)].$$

(II) The constant F_{ii} is chosen to give the correct dispersion integrals, i.e.,

$$F_{ii}/F_{12}=\alpha_{ii,r}(m_V^2)/\alpha_{0,r}(m_V^2),$$

where $\alpha_{0,r}$ is the value of $\alpha_{i,r}$ occurring in the degeneracy solution, and $\alpha_{ii,r}$ is the expression for $\alpha_{0,r}$ obtained when $\beta(s')$ is replaced by $\mathfrak{N}_{ii}(s')/F_{12}$.

In order to compare the above proportionality approximations, we consider the contribution of a particular vector meson to the Born approximation for a particular amplitude, and define δ_V and δ_Σ to be the deviations from the degeneracy-solution values of the square of the *V*-meson mass and the sum of the squares

[12] The numerical crossing coefficients in the Born-approximation amplitudes may be obtained from Ref. 1, while the energy dependence of the amplitudes and the relation between q_i^2 and s corresponding to nondegenerate *PS* mesons are given in Eqs. (6), (7), and (8) of Ref. 8. Eqs. (7) and (8) of Ref. 8 are not completely general in that they refer to an amplitude with a common particle in the initial and final states. However, in the quasidegeneracy approximation, a Born-approximation amplitude depends on the *PS*-meson masses only through the sum of the deviations of the squares of the masses of the four *PS* mesons involved in the amplitude, so these equations are sufficiently general for the present derivation.

of the masses of the four PS mesons involved in the amplitude. A numerical calculation shows that to first order the ratios of two such contributions at the resonance energy is

$$\mathfrak{N}_1(m_V^2)/\mathfrak{N}_2(m_V^2)=1-0.225(\delta_{V,1}-\delta_{V,2})\mu_0^{-2}+0.0628(\delta_{\Sigma,1}-\delta_{\Sigma,2})\mu_0^{-2},$$

where the indices 1 and 2 denote the two contributions and μ_0^2 denotes the square of the PS-meson mass in the degeneracy solution. On the other hand, the corresponding ratio of the dispersion integrals of these two contributions is

$$\alpha_{r,1}(m_V^2)/\alpha_{r,2}(m_V^2)=1-0.132(\delta_{V,1}-\delta_{V,2})\mu_0^{-2}+0.0274(\delta_{\Sigma,1}-\delta_{\Sigma,2})\mu_0^{-2}.$$

The differences between the corresponding coefficients in these two expressions represent an inaccuracy inherent in the method. Therefore, we compute all results separately with the two proportionality prescriptions I and II, and regard as meaningful only results common to the two prescriptions.

The choice of the subtraction energy also presents a tricky problem since, when the degeneracy assumption is removed, the ends of the left-hand cuts are in different places. We denote by Δ the deviation in s_t from the degeneracy-solution value. To first order in the deviations of the m^2, the end of the cut corresponding to the contribution of a particular V meson to a particular amplitude is $s_t=4\mu_0^2-m_0^2-\delta_V+\delta_\Sigma$, where $m_0^2=5.89\mu_0^2$ is the square of the degeneracy-solution value of the V-meson mass. Thus, if there were only one such contribution, an appropriate choice of Δ would be,

$$\Delta=-\delta_V+\delta_\Sigma. \tag{8}$$

In order to choose an appropriate Δ for each of the three sets of dispersion equations that generate the three resonances, one must take a suitable average over the contributing V and PS mesons.[13] We consider the following three alternate averages, denoted by A, B, and C.

(A) In each of the three cases, δ_V of Eq. (8) is averaged simply over the V mesons that contribute forces. In the two-channel cases, δ_Σ is set equal to the value appropriate to the inelastic process. The result of this average is

$$\begin{aligned}\Delta_\varphi&=-\tfrac{1}{2}(\delta_\varphi+\delta_\rho)+4\delta_K,\\ \Delta_\rho&=-\tfrac{1}{3}(\delta_\rho+\delta_\varphi+\delta_M)+2\delta_\pi+2\delta_K,\\ \Delta_M&=-\tfrac{1}{2}(\delta_M+\delta_\rho)+2\delta_K+\delta_\pi+\delta_\eta,\end{aligned} \tag{9a}$$

where Δ_i is the value of Δ used in the dispersion relations for the resonance i and δ_i is the deviation from the degeneracy-solution value of the square of the mass of the PS or V meson i, (i.e., $m_\pi^2=\mu_0^2+\delta_\pi$, $m_\rho^2=m_0^2+\delta_\rho$, etc.).

[13] We take such an average only for choosing subtraction energies, and not for computing any other factors in the equations.

Table I. Calculated V-meson masses and deviations in the coupling constants resulting from the physical, PS-meson input parameters $\delta_\pi=-0.92$ and $(\delta_\eta/\delta_\pi)=-\frac{1}{4}$. The symbols AI, AII, etc. refer to the subtraction and proportionality prescriptions defined in Sec. II C.

Case	A I	A II	B I	B II	C I	C II
δ_ρ	−3.88	−2.60	−3.86	−2.59	−2.98	−2.49
δ_M	−0.21	−0.54	0.28	−0.27	−1.25	−1.18
δ_φ	0.35	−0.39	0.38	−0.39	−0.75	−1.11
$\epsilon_{\varphi KK}$	0.26	0.10	0.26	0.10	0.085	0.02
$\epsilon_{\rho\pi\pi}$	−0.21	−0.09	−0.26	−0.12	−0.03	−0.01
$\epsilon_{\rho KK}$	−0.17	−0.04	−0.18	−0.04	−0.085	−0.03
$\epsilon_{M\pi K}$	0.15	0.05	0.18	0.055	0.07	0.04
$\epsilon_{M\eta K}$	−0.10	−0.02	−0.10	−0.01	−0.07	−0.04
m_M(MeV)	1180	1144	1228	1173	1066	1074
$(m_\rho/m_M)^2$	0.35	0.61	0.30	0.59	0.63	0.72
$(m_\varphi/m_M)^2$	1.10	1.03	1.02	0.98	1.11	1.01

(B) The δ_V and δ_Σ in Eq. (8) are averaged according to the contributions of the different V and PS mesons to the resonance in the degeneracy solution. In the degeneracy solution, $|D_r|$ may be written in the form $|D_r|=1-\alpha_0\sum_i F_{ii}$, so that the relative contributions of the V mesons may be computed from the relative contributions to $\sum_i F_{ii}$. The relative contributions of the PS mesons may be taken from the partial reduced widths of the resonance, i.e., the $\pi+\pi$ state contributes twice as much to the ρ as does the $K+\bar{K}$ state, since $\gamma_{\rho\pi\pi}^2/\gamma_{\rho KK}^2=2$. The results of such an average are

$$\begin{aligned}\Delta_\varphi&=-\tfrac{1}{2}(\delta_\varphi+\delta_\rho)+4\delta_K,\\ \Delta_\rho&=-\tfrac{1}{2}(\delta_\rho+\delta_\varphi)+(8/3)\delta_\pi+(4/3)\delta_K,\\ \Delta_M&=-\tfrac{1}{3}\delta_M-\tfrac{2}{3}\delta_\rho+2\delta_K+\delta_\pi+\delta_\eta.\end{aligned} \tag{9b}$$

(C) The same subtraction energy is used for all three sets of dispersion equations; this energy is computed by averaging the δ_V and δ_Σ of Eq. (8) over the eight PS and eight V mesons. The results of this average are

$$\Delta_\varphi=\Delta_\rho=\Delta_M=-\tfrac{3}{8}\delta_\rho-\tfrac{1}{8}\delta_\varphi-\tfrac{1}{2}\delta_M+\tfrac{3}{2}\delta_\pi+\tfrac{1}{2}\delta_\eta+2\delta_K. \tag{9c}$$

The prescription C is rather unphysical since the subtraction energy for a particular dispersion equation is determined partly by the masses of particles not involved in the equation. We include this prescription only for purposes of comparison.

III. RESULTS FOR QUASIDEGENERACY SOLUTION

We adopt the convention that the K mass is fixed, and set $m_K^2=\mu_0^2=1$. There is no loss of freedom in this assumption, since the dispersion relations do not involve absolute masses but only mass ratios. We denote the degeneracy-solution value of $\gamma_{\varphi KK}^2$ by γ_0^2. The quantity $\epsilon_i\gamma_0^2$ represents the deviation from the degeneracy-solution value of the interaction constant γ_i^2. Thus, $\gamma_{\varphi KK}^2=\gamma_0^2(1+\epsilon_{\varphi KK})$, $\gamma_{\rho\pi\pi}^2=\gamma_0^2(\frac{2}{3}+\epsilon_{\rho\pi\pi})$, etc., as may be seen from Eq. (7). The notation for mass deviations is the same as in the preceding section.

A value of $(-\frac{1}{4})$ for the input ratio δ_η/δ_π corresponds to the physical PS-meson masses, $m_\pi=138$ MeV,

$m_K=495$ MeV, and $m_\eta=550$ MeV. We may make any convenient choice for the magnitude of δ_π, since the quasidegeneracy equations are linear and homogeneous in the δ's and ϵ's. In order to facilitate comparison with experiment, we have listed in Table I the results corresponding to the physical *PS*-mass input values $\delta_\eta/\delta_\pi=-\frac{1}{4}$ and $\delta_\pi=(m_\pi^2/m_K^2)-1=-0.92$. We consider as physically meaningful only the results corresponding to the subtraction prescriptions A and B, and thus quote only results common to columns AI, AII, BI, and BII of the table. It is seen from these columns that the most striking effect of the *PS* mass differences is that the calculated ρ mass is lower than the M and φ masses, which is in agreement with experiment.

A. The Vector-Meson Masses

In order to make further comparisons with experiment, we shall assume that the linear (quasidegeneracy) approximation is valid even for the large physical value of δ_π. It is seen from Table I that the percentage deviations of all derived quantities are smaller than the input deviation of m_π^2, so this procedure is reasonable. However, it is expected that if the linear assumption were dropped, significant modifications would result. This point is discussed further in Sec. IV.

We use as experimental V-meson masses $m_\rho=750$ MeV, $m_M=885$ MeV, $m_\omega=785$ MeV, and $m_\varphi=1020$ MeV. The value of m_ρ^2/m_M^2 from the A and B columns of Table I is in the range 0.30–0.61, while the corresponding experimental value is 0.72. The computed value of m_φ^2/m_M^2 is in the range 0.98–1.10, while the corresponding experimental number is either $m_\omega^2/m_M^2=0.79$, $m_\varphi^2/m_M^2=1.33$, or somewhere between if the isoscalar member of the unitary-symmetry octet is a linear combination of the ω and φ. The present calculation is not accurate enough to be considered as evidence concerning whether the ω, φ, or a linear combination belongs with the octet.[14] However, it is clear that the predicted V-meson mass ratios are in rough agreement with experiment. It should be pointed out that when one replaces a wide resonance by a pole in convergent dispersion relations, an appropriate position for the pole is below that of the resonance.[15] Hence, a predicted value of m_ρ^2/m_M^2 somewhat lower than the experimental value is desirable.

The calculated M mass is in the range 1145–1230 MeV, while the experimental value is 885 MeV. We note that the value $m_M\sim885$ MeV obtained in Ref. 8 did not result from a complete bootstrap calculation for the V mesons, since m_ρ was taken from experiment. These two calculations are compared in Sec. IV.

The basic reason that the ρ mass is smaller than the M and φ masses in the present calculation is quite simple; the largest term in the ρ wave function results from the $\pi+\pi$ state, so that the small value of the π mass leads to a small value of the ρ mass. A large assumed deviation of the η mass leads to a corresponding effect. In order to illustrate this, we have calculated the effect of the unphysical input assumptions $\delta_\pi=0$ and $\delta_\eta=1$, using the proportionality and subtraction prescriptions A and I. The results are

$$\begin{aligned} \delta_\rho&=0.075\,, & \delta_M&=1.77\,,\\ \delta_\varphi&=0.23\,, & \epsilon_{\varphi KK}&=0.02\,,\\ \epsilon_{\rho\pi\pi}&=-0.16\,, & \epsilon_{\rho KK}&=-0.025\,,\\ \epsilon_{M\pi K}&=0.06\,, & \epsilon_{M\eta K}&=-0.05\,. \end{aligned} \tag{10}$$

In this case the M is the only V meson whose mass is changed greatly, because the $\eta+K$ state is important in the M wave function, while the ρ and φ are not coupled directly to the η. Because the quasidegeneracy equations are linear and homogeneous in δ_i and ϵ_i, Eq. (10) and column AI of Table I may be used to compute the results of any choice of δ_η and δ_π corresponding to the proportionality and subtraction prescriptions A and I.

The fact that the calculated relative deviations from degeneracy are smaller for the V-meson multiplet than for the *PS*-meson multiplet also has a simple explanation, namely, the wave function for a particular V meson is an average over the different *PS* mesons. Thus, in our model, the ρ is light because the $\pi+\pi$ is light, but m_ρ/m_φ is not as small as m_π/m_K because the ρ is a $K+\bar{K}$ part of the time.

The Gell-Mann–Okubo mass formula,[16] applied to the *PS* and V-meson octets, yields the relations

$$(m_\pi^2-m_K^2)=-3(m_\eta^2-m_K^2)$$

and

$$(m_\rho^2-m_M^2)=-3(m_\varphi^2-m_M^2)\,.$$

The assumptions on which this formula are based are quite different from those of the present paper, except for the one common assumption that only terms linear in the deviations from degeneracy are important. There is no relation between *PS* and V-meson masses implied by the Gell-Mann–Okubo assumptions. In the present model, one may choose the *PS*-meson masses to satisfy the Okubo formula (i.e., $\delta_\eta/\delta_\pi=-\frac{1}{3}$). However, it does not follow from such a choice that the V-meson masses satisfy the formula exactly. We have been unable to find a simple recipe for the proportionality and subtraction prescriptions that does lead to such an exact causal relation, and we do not know whether or not this causal relation would apply if a more exact procedure for writing and solving partial-wave dispersion relations in the quasidegeneracy approximation were found.

[14] In the author's opinion, it is also dangerous to decide this question on the basis of the Gell-Mann–Okubo mass formula, since this formula is also based on the nonphysical assumption that the deviations from degeneracy are small, (see Ref. 16).

[15] This is shown clearly in the work of Ball and Wong on form factors; J. S. Ball and D. Y. Wong, Phys. Rev. **130**, 2112 (1963).

[16] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Prog. Theor. Phys. (Kyoto) **27**, 949 (1962).

B. The Interaction Constants

It is seen from the A and B columns of Table I that the calculated deviations ϵ_i of the interaction constants from the values of the degeneracy solution (exact unitary symmetry) depend somewhat on the proportionality and subtraction prescriptions used. However, two important features of these calculated deviations are common to all the prescriptions. First, the ϵ_i are not huge; i.e., the calculated interaction constants are not extremely sensitive to the *PS*-meson mass splitting. Unitary symmetry remains a reasonable approximation to the interaction ratios despite the huge deviation from unity in the input value of m_π^2/m_K^2. This result is encouraging to the point of view that unitary symmetry may describe the physical *PS* mesons.

Secondly, the signs of the various deviations are a feature of the calculated ϵ_i that is common to the different proportionality and subtraction prescriptions. For example, the predicted ratio $\gamma_{\rho\pi\pi}^2/\gamma_{M\pi K}^2$ is lower than the unitary symmetry value of $\frac{4}{3}$. Future experimental measurements of the γ_i^2 will provide further tests of the bootstrap model. If these measurements indicate that unitary symmetry is approximately valid, and if the measurements are sufficiently accurate to detect deviations from the exact ratios of Eq. (7), they will distinguish between the bootstrap model and other models involving unitary symmetry. In those cases where the *V*-meson rest masses are lighter than those of the two *PS* mesons in the appropriate states, experimental determination of the coupling constants is difficult, of course, since it depends on some sort of extrapolation procedure.

Part of the reason for the insensitivity of the calculated ϵ_i to the *PS*-meson masses is that the two most important effects of deviations in these masses partially cancel. The most important such effect is the decrease of the momentum q_i of a state i (at fixed energy) that results from an increase of the mass of either *PS* meson in the state i. This causes the dispersion integral α_i to decrease. However, this effect is cancelled partially by the increase in the Born approximation to the amplitude T_{ii} that results from an increase in the mass of a *PS* meson in the state i.

The most striking features of the relative magnitudes of the calculated ϵ_i may be understood from the following considerations. For each resonance, the determinant of the denominator may be written, $|D|=1-\sum_i \alpha_i F_{ii}+x$, where the F_{ii} are quadratic in the coupling constants, and x is either zero (for the φ) or is quadratic in the α_i. This quantity x is small in all three cases, so that $\sum_i \alpha_{i,r}(m_V^2)F_{ii}\sim 1$, where m_V^2 is the resonance energy. In the resonance region, the $\alpha_i(s)$ are increasing functions of s. Therefore, if the rest mass of a vector meson is small compared to that of the constituent *PS* mesons, the $\alpha_i(m_V^2)$ are small, and large values of the coupling constants in the F_{ii} are required. For example, the ρ-meson rest mass is smaller than that of the M and φ, but is large compared to that of the most important constituent state, the $\pi+\pi$ state. Hence, the ρ coupling constants are smaller than the degeneracy-solution values. Similarly, the $\varphi/(K+\bar{K})$ mass ratio is small compared to either the $\rho/(\pi+\pi)$ or $M/(\pi+K)$ mass ratio, so $\epsilon_{\varphi KK}$ is positive.

Since the deviations of the γ_i^2 from the degeneracy-solution values are not large, the problem discussed in Ref. 8 is still present; the predicted $\gamma_{\rho\pi\pi}^2$ and $\gamma_{M\pi K}^2$ are about 2 or $2\frac{1}{2}$ times as large as the experimental values. This problem is discussed further in Sec. V.

IV. COMPARISON WITH NONQUASIDEGENERATE CALCULATION

The author has applied the bootstrap technique to the $M(K^*)$ meson, without using the approximation of small deviations from degeneracy.[8] The M system is not completely self-determining; there are two more input than output parameters in the calculation of Ref. 8. Barbour and Nishimura are investigating the possibility of determining all the *V*-meson masses and *V*-*PS*-*PS* interaction constants from the observed *PS*-meson masses without using the quasidegeneracy approximation.[17] These authors also have treated the φ meson separately.[18]

Some insight into the effect of removing the quasidegeneracy assumption may be obtained from a comparison of the present results with those of Ref. 8. The calculated M mass in Ref. 8 is approximately the experimental value of 885 MeV, which is lower than that of the present paper. This discrepancy does not result from the fact that the ρ mass is taken from experiment in Ref. 8 because the calculated M mass is not very sensitive to the ρ mass. Part of the discrepancy results from the different subtraction prescriptions used. The calculated M mass is sensitive to the subtraction energy used in the M-meson dispersion relations. In Ref. 8, only the cuts resulting from exchanged M mesons are used to determine the subtraction energy. We may use a corresponding prescription in the present paper, by writing $\Delta_M=-\delta_M+PS$-meson terms [see Eqs. (8) and (9)]. The calculated M mass then depends only on the M-meson dispersion relations, and is 1065 MeV or 1080 MeV, depending on which proportionality prescription is used. We believe that the discrepancy between $\sim$1070 MeV and $\sim$885 MeV results primarily from the quasidegeneracy assumption used here.

In Ref. 8, only two relations between the three interaction constants $\gamma_{M\pi K}^2$, $\gamma_{M\eta K}^2$, and $\sqrt{2}\gamma_{\rho\pi\pi}\gamma_{\rho KK}$ are obtained. However, if we assume that the ratio $\gamma_{M\eta K}^2/\gamma_{M\pi K}^2$ lies in the range 0.59–0.88 (obtained from the A and B columns of Table I of the present paper), then the equations of Ref. 8 yield the results,

$$1.85<(\gamma_{M\pi K}^2/4\pi)<2.12\,,$$
$$1.09<(\sqrt{2}\gamma_{\rho\pi\pi}\gamma_{\rho KK}/\gamma_{M\pi K}^2)<1.13\,. \qquad (11)$$

[17] I. Barbour and K. Nishimura (private communication).
[18] I. Barbour and K. Nishimura, Nuovo Cimento **29**, 288 (1963).

The corresponding quantities, determined from the A and B columns of Table I of the present paper, are in the ranges,

$$1.42 < (\gamma_{M\pi K}^2/4\pi) < 1.75\,,$$
$$0.52 < (\sqrt{2}\gamma_{\rho\pi\pi}\gamma_{\rho KK}/\gamma_{M\pi K}^2) < 1.05\,. \qquad (12)$$

It is seen that both models predict that the ratio $\sqrt{2}\gamma_{\rho\pi\pi}\gamma_{\rho KK}/\gamma_{M\pi K}^2$ is smaller than the value $\frac{4}{3}$ predicted by exact unitary symmetry. Much of the discrepancy between Eqs. (11) and (12) is associated with the smaller M mass occurring in Ref. 8 (see the discussion in Sec. III B of the present paper). However, we may conclude tentatively that if the quasidegeneracy assumption is removed, the comparison between calculation and experiment will be better for the average V-meson mass, and worse for the magnitudes of the interaction constants.

V. CONCLUSIONS

In the bootstrap model of the V-meson octet, the sensitivity of the physical V-PS-PS-meson interaction constants to the PS-meson mass ratios is sufficiently small so that the ratios of the interaction constants are described approximately by unitary symmetry, even if physical PS-meson masses are used. The calculated average V-meson mass, and the calculated V-meson mass splitting that result from the physically observed PS-meson mass splitting, are in rough agreement with experiment. On the other hand, the absolute magnitudes of the calculated interaction constants are about 2 or $2\frac{1}{2}$ times larger than those obtained from the observed $\rho \to \pi+\pi$ and $M \to \pi+K$ decay widths. It has been pointed out that if the coupling of other states to the $PS+PS$ states is important, inclusion of the other states probably would improve the agreement between the calculated and experimental values of the interaction constants.[8,11] However, the question arises as to whether or not such an inclusion would at the same time destroy the agreement with respect to the average V-meson mass and the V-meson mass splitting. We discuss this question here.

It is likely that the inclusion of other states in the model would affect the magnitudes of the interaction constants more than it would affect the average V-meson mass. Such an effect has been observed in Ref. 1, and also by Balazs.[19] In the bootstrap model of the V-meson octet in the degeneracy approximation (Ref. 1), the coupling of the isotopic spin 1, $\pi+\pi$ state to the $K+\bar{K}$ state reduces $\gamma_{\rho\pi\pi}^2$ by $\frac{1}{3}$, but does not change the (V-meson/PS-meson) mass ratio at all. Similarly, in the bootstrap model of $\pi+\pi$ resonances of Balazs, the effect of approximating other channels by including inelastic processes in the unitarity condition is to reduce resonance widths without altering the resonance positions significantly.[19] In order to make clear the basic reason for this kind of effect, we note that if the matrix N/D method used in Sec. II A is applied to the P wave, $\pi+\pi$ elastic amplitude in the presence of an arbitrary number of coupled channels, the amplitude may be written in the form, $T=(\mathfrak{N}+y)/|D|$, where $\mathfrak{N}$ is the Born approximation for the T, $|D|$ is the determinant of the denominator matrix, and y is a sum of terms of order $\geqq 1$ in the dispersion integrals [integrals analogous to the α_i of Eq. (2d)]. If y is zero and $|D|$ is linear in s, the calculated ρ mass depends only on the $\pi+\pi$ channel, although the $\rho\pi\pi$ reduced partial width may depend on many channels. Hence, the average V-meson mass will be insensitive to the inclusion of other channels if the added terms y in the numerators of the appropriate amplitudes are small and $|D|$ is approximately linear.

We now turn to the question of the V-meson mass splitting. Let us assume that unitary symmetry is a valid approximation for all strong interactions. The deviations from degeneracy of the V multiplet then depend on the deviations from degeneracy of other multiplets. It is observed experimentally, both for states of baryon number one (the baryons and the $P_{3/2}$, baryon-PS meson resonances) and for states of baryon number zero, that the relative mass splittings are greatest within the lightest multiplet. This effect is in agreement with the predictions of the present bootstrap model. It is reasonable to suppose that the greatest relative deviations from degeneracy occur within the PS-meson multiplet. These deviations are important in the V-meson dispersion relations because the left-hand cuts are close to the physical regions for the $PS+PS$ states. Therefore, one expects the PS-meson mass splitting to be the dominant cause of the V-meson mass splitting, *provided that the important forces can be described approximately by unitary symmetry.* In fact, one advantage of applying dispersion relations to particle multiplets in a theory in which a basic interaction symmetry is present, is that the approximation of neglecting distant singularities is expected to be particularly good when one compares the different states within a multiplet.

We conclude that the present bootstrap model is incomplete, but that the basic relations between PS and V-meson masses occurring in the model may be real. If the model is extended so that the PS mesons themselves develop as poles in the appropriate states, as was done in Ref. 11 for degenerate multiplets, additional relations among the various mass splittings will be obtained.

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[19] Louis A. P. Balazs, Phys. Rev. Letters **10**, 170 (1963).