Six-Fermion Weak Interactions*

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Six-fermion weak interactions, in addition to the usual four-fermion weak interactions, may exist. Limits on the strength of such interactions deduced from low-energy decay phenomena are not sufficient to exclude observable effects of six-fermion couplings at high energies. In particular, the process $\nu+p\rightarrow\nu+\mu+\bar{e}+p$ may be attributed either to six-fermion couplings, or to the existence of intermediary vector bosons.

IT is important to determine whether or not there are vector bosons mediating weak interactions.[1] Their existence would justify the view that all forces are mediated by mesons. Alternative to this Yukawa picture is Fermi's idea of direct multiparticle weak couplings.[2] Indeed, four-fermion couplings are sufficient to generate all known weak processes. Both models are subject to divergence difficulties more severe than those of quantum electrodynamics, but recent progress[3–5] may allow both kinds of interaction to be consistently handled.

We point out the possibility that six-fermion interactions may exist. Although they are relatively ineffective and difficult to detect at low energies, they may give important effects at higher energies which are difficult to distinguish from the effects of the conjectured intermediate boson.

Six-fermion couplings may arise in different ways:

(1) They will always occur at higher order in weak interactions. In this case, one would expect them to be very weak. An enhancement of such interactions along lines suggested by the work of Feinberg and Paris[4] *seems* ruled out by more recent considerations.[6,7] Six-fermion interactions also arise with electromagnetic participation, but again they are quite weak (and, in general, calculable).

(2) In a vector-meson theory of weak interactions the vector meson may enjoy *strong* interactions, so long as these are bilinear in the vector-meson field. For example, strong interactions of the form $f\bar{W}_\mu W_\mu\pi^2$ would induce six-fermion weak interactions of the form $(\bar{\mu}\nu')(\bar{\nu}e)(\bar{p}p)$, with coupling strength $\sim fGm_B{}^{-2}m_\pi{}^{-1}$. These six-fermion interactions could give observable effects well below the threshold for real W production. Moreover, the process $\nu'+p\rightarrow\mu+W+p$ no longer requires electromagnetic participation and is therefore enhanced by $\sim(137)^2f^2/4\pi$. The possibility of a *light* W with strong bilinear couplings is ruled out by present experiments.[8,9] Further, this model gives rise to copious pair production of W at sufficiently high energy. We also note that $f\bar{W}_\mu W_\mu\pi^2$ leads to small induced matrix elements for $\nu+p\rightarrow\nu+p$ and for $\nu+p\rightarrow\nu+p+\pi^0$.[10]

(3) Direct six-fermion interactions could exist in a Fermi theory of weak interactions without vector intermediaries. Given that there are direct four-fermion couplings, we know of no convincing argument that excludes the possible existence of six-fermion couplings of comparable strength. Efimov[5] has constructed covariant local field theories free of ultraviolet divergences by taking as an interaction Lagrangian a suitable function of the fields, not just a monomial. If a similar procedure could be applied to the four-fermion couplings to obtain a finite theory, it could provide a natural origin to direct six-fermion couplings.

Consider a six-fermion coupling of the form

$$\lambda^{-3}2^{-1/2}GJ_\mu J_\mu{}^\dagger(\bar{p}p)\,,$$

where $G=10^{-5}/m_N{}^2$ is the usual four-fermion coupling strength, $J_\mu=\bar{\mu}\gamma_\mu(1+\gamma_5)\nu'+\bar{e}\gamma_\mu(1+\gamma_5)\nu$ is the usual lepton current, and λ is a coupling parameter with dimensions of mass. This coupling leads to the decay $\pi^0\rightarrow\mu^++e^-+\bar{\nu}+\nu'$, but with a very unfavorable branching ratio. It also causes a stimulated decay of muons bound in hydrogen,[11]

$$\bar{\mu}+p\rightarrow e^-+\bar{\nu}+\nu'+p\,. \tag{A}$$

We calculate for this process the rato

$$\Gamma_{\rm H}=(4\pi^5 7!)^{-1}G^2(m_\mu/\lambda)^6m_\mu{}^2|\phi(0)|^2$$

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[3] C. N. Yang and T. D. Lee, Phys. Rev. **128**, 885 (1962).
[4] G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963).
[5] G. Efimov (to be published).
[6] C. Bouchiat, B. d'Espagnat, and J. Prentki (to be published).
[7] W. Frazer (private communication).
[8] G. Danby, J. M. Gaillard, K. Gouilanos, L. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962).
[9] The Danby experiment seems to require $m_W>750$ MeV in the absence of any strong W couplings. Since the cross section for W production varies with $\approx m_W{}^5$, we conclude that $f=1$ implies $m_W\geq 3$ GeV.
[10] The induced matrix element for $\nu+p\rightarrow\nu+p$ is of order $Gf^3m_W{}^{-6}$ and that for $\nu+p\rightarrow\nu+p+\pi^0$ is of order $Gf^2m_W{}^{-4}$. The absence of these processes in the Danby experiment (Ref. 8) tells us once again that large f is incompatible with small m_W.
[11] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).

in units with $\hbar=c=1$, where $\phi(0)$ is the muon wave function at the origin. Putting $|\phi(0)|^2 \sim \pi^{-1}(\alpha m_\mu)^3$, we obtain $\Gamma_{\rm H}=5\times10^{-5}(m_\mu/\lambda)^6$ sec^{-1}. Stimulated decay of muons in orbit is greatly enhanced for heavy nuclei in analogy with muon capture. Since the capture rate in lead is 4×10^4 greater than in hydrogen,[12] we estimate[13] the stimulated decay rate in lead to be $\Gamma_{\rm Pb}\sim 2(m_\mu/\lambda)^6$ sec^{-1}. Since the measured muon decay rates in heavy elements agree fairly well with that of the free muon[14] (corrected for binding and Coulomb effects[15]), and since decay in orbit is indistinguishable from stimulated decay, we obtain the upper bound $\Gamma_{\rm Pb}<10^{-2}\Gamma_\mu$, where Γ_μ is the decay rate of the free muon. This gives $\lambda>28$ MeV.

The six-fermion interaction becomes more important at high energies, e.g., in

$$\nu'+p\rightarrow\mu+\bar{\mu}+\nu'+p \text{ (or, } \mu+\bar{e}+\nu+p). \qquad \text{(B)}$$

With a point proton we calculate $\sigma(\eta)$, the total cross section for (B) with proton recoil (in the lab system) less than η:

$$\sigma(\eta)=(2\pi^4 7!)^{-1}(G^2/\pi)(E/\lambda)^6E^2 \quad \text{for} \quad E\leq\tfrac{1}{2}\eta;$$
$$=(96\pi^2)^{-2}(G^2/\pi)(\eta/\lambda)^6\left[1-\frac{8}{7}\beta+\frac{3}{8}\beta^2-\frac{1}{80}\beta^4\right]E^2 \quad \text{for} \quad E\geq\tfrac{1}{2}\eta;$$

where E is the lab energy of the incident neutrino and $\beta=\eta/E$. The square of the proton's vector form factor falls by a factor of two at a recoil of 400 MeV/c.[16] An estimate of the total cross section for (B) is obtained by making a sharp cutoff at 400 MeV/c, i.e., $\sigma\sim\sigma$(400 MeV/c). At an incident neutrino energy of 1 GeV we obtain $\sigma\sim(m/\lambda)^6 10^{-39}$ cm^2. Reaction (B) is not a dominant weak process caused by $\sim$1 GeV neutrinos, and furthermore conventional neutrino-induced processes have a cross section 10^{-38} cm^2 at such energies.[8] We obtain the upper bound $(m_\mu/\lambda)^6\leq1$ or $\lambda\geq100$ MeV. The neutrino experiment gives a far better upper limit on the six-fermion coupling strength than does muon capture.

The strong energy dependence of the cross section for process (B) suggests that experiments with cosmic-ray neutrinos may be very relevant. The estimated[17] flux of energetic cosmic-ray neutrinos is 0.4 $E^{-2.67}\times(E+30)^{-1}$ sr^{-1} cm^{-2} sec^{-1} GeV^{-1} (with E in GeV). Exposed to this flux, each nucleon undergoes process (B) at a rate $(m_\mu/\lambda)^6$ 3×10^{-39} sec^{-1}, whereas, we estimate that it undergoes the "elastic" processes ($\nu+n\rightarrow e^-+p$ or $\bar{\nu}+p\rightarrow e^++n$) at the rate 10^{-39} sec^{-1}. Thus, the limit on the strength of the six-fermion coupling from present machine neutrino experiments does not rule out the possibility that six-fermion interactions may be more important than four-fermion interactions at cosmic-ray energies.

The possible existence of such a six-fermion coupling as we consider has great significance in any search for the hypothetical weak interaction intermediary W. A typical W event in a high-energy neutrino experiment is $\nu'+p\rightarrow\mu+W+p$, with the subsequent decay $W\rightarrow\nu'+\bar{\mu}$ (or, $\nu+\bar{e}$). The final state has the same particles as process (B), and it would be difficult to distinguish between them. Without detailed analysis, the detection of such events tells us that there exist either vector bosons or six-fermion interactions (or both). In principle, true W events could be distinguished from direct six-fermion events by the sharp $(\nu'\bar{\mu})$ mass, or by the threshold behavior of W production. The same difficulty applies to cosmic-ray experiments at great depth[18] with the difference that one may possibly detect the resonant production of W in exceedingly energetic $(\bar{\nu}e)$ collisions.[19]

Note added in proof. Other authors, in other contexts, have suggested the possible existence of six-Fermion weak interactions: S. Nakamura and S. Sato, Progr. Theor. Phys. (Kyoto) **28**, 323 (1962); M. Taketani and T. Tati, *ibid.*, **28**, 757 (1962); Y. Katayamo and K. Matumoto, Progr. Theor. Phys. (Kyoto) **29**, 798 (1963); S. Trieman (preprint, 1963).

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[13] This is certainly an overestimate, because (i) stimulated decay depends upon a higher power of the available energy than muon capture, and is more suppressed in heavy nuclei where the binding energy is appreciable; and (ii) stimulated decay involves lower proton recoil momenta than capture, so that the effect of Pauli exclusion is more significant.

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[15] R. W. Huff, The Enrico Fermi Institute, The University of Chicago, Chicago, Illinois, Report EFINS-61-27, 1961 (unpublished).

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[18] For example, M. Menon, P. Ramana Murthy, B. Sreekantan, and S. Miyake, Phys. Letters **5**, 272 (1963); Nuovo Cimento (to be published).

[19] S. L. Glashow, Phys. Rev. **118**, 316 (1960). Only electron antineutrinos contribute to this process.