Production of a Strangeness −3 Baryon in Antiproton-Deuterium and K^--Proton Collisions*

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Theoretical estimates are given for the production cross sections for a strangeness −3 baryon with spin-parity $\frac{3}{2}^+$ and mass of about 1.7 BeV, denoted by Ω^-, in the following reactions:

$$(1)\quad \bar{p}+D\rightarrow\Omega^-+K^++K^0+K^0$$
$$(2)\quad K^-+p\rightarrow\Omega^-+K^++K^0.$$

For 2.8-BeV/c antiproton laboratory momentum, we obtain a *lower* limit on the cross section for the first reaction of about 0.01 μb. We also compute the ratio of Ω^- to Ξ^- production in antiproton-deuteron collisions and find this to be about 1% at the above momentum. For 3.5-BeV/c K^- laboratory momentum we obtain a cross section for reaction (2) of about 0.1 μb.

In this paper we present theoretical estimates of some production cross sections for a strangeness −3 baryon with spin-parity $\frac{3}{2}^+$, isotopic spin zero, and a mass of about 1.7 BeV. This particle, denoted by Ω^-, is, according to the unitary symmetry of strong interactions,[1–3] the last experimentally undetected member of a decuplet of which the other nine members consist of the N^*(1238 MeV), the Y_1^*(1385 MeV), and the Ξ^*(1530 MeV). The mass of 1.7 BeV is estimated from the Gell-Mann-Okubo mass formula[4,5] for any unitary multiplet split by suitable[4] symmetry-breaking interactions. Since the estimated mass of the Ω^- is too small to allow the strong decay into a Ξ and a $\bar{K}$, the particle will be metastable and could be observed to traverse macroscopic distances before decaying weakly. As such, it would appear as the first metastable baryon with spin greater than $\frac{1}{2}$. The experimental search for the Ω^- is a matter of great current interest. We outline here theoretical estimates, in the "pole" approximation, for production cross sections of the Ω^- in two, perhaps *a priori* favorable processes. A comparison of the estimated cross sections for these processes, and knowledge of their smallness (in the submicrobarn range) may be of use in planning the experimental effort. In one process we also compute the ratio of Ω^- production to Ξ^- production and find this to be about 1%. This ratio is less sensitive to calculational uncertainties than are the absolute cross sections.

Consider first the reaction

$$\bar{p}+\mathrm{D}\rightarrow\Omega^-+K^++K^0+K^0. \qquad (1)$$

Although there are four particles in the final state and although further, the process involves an annihilation within the deuteron with a strange baryon emerging, this process has an unusually low threshold (at a center-of-mass total energy of about 3.2 BeV). Presently available antiproton beams reach substantially above this threshold. Our calculation will be performed for 2.8 BeV/c-antiproton laboratory momentum, the momentum of an experiment currently in progress in deuterium.[6] At this momentum one is about 740 MeV above threshold in the center-of-mass. The "dispersion" (Feynman) graph to be calculated is illustrated in Fig. 1(a). The spin $\frac{3}{2}$ Ω^- will be described according to the formalism of Rarita and Schwinger,[7] as first utilized by Kusaka.[8] We have for the imaginary part of the

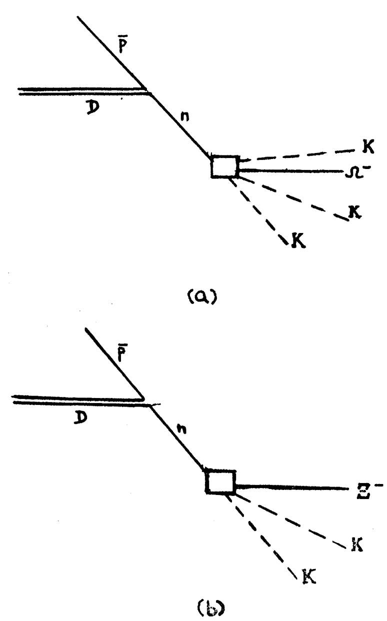

FIG. 1. (a) Feynman graph for the reaction in Eq. (1). (b) Feynman graph for the reaction in Eq. (18).

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matrix element $\mathfrak{M}$ describing the graph in Fig. 1(a).

$$\mathrm{Im}\mathfrak{M}(q^2)=\pi\sum_n \delta(n-q)\bar{v}(p)\bar{z}_\mu(l) \times\langle K^+K^0K^0|J_\mu|n\rangle\langle n|j|D\rangle, \quad (2)$$

with n, p, l, and q denoting, respectively, the four-momenta of the neutron, the antiproton, the Ω^-, and the total center-of-mass energy-momentum; $\bar{v}(p)$ denotes an antiparticle spinor, $\bar{z}_\mu(l)$ denotes the wave function of the Ω^-, and J_μ and j are, respectively, the currents coupled to Ω^- and nucleons. One matrix element in Eq. (2) is completely known. We have[9,10]

$$\bar{v}(p)\langle n|j|D\rangle=\bar{v}(p)\Gamma\bar{u}^T(n)(m/n_0)^{1/2},$$
$$\Gamma=(a/\sqrt{8})(\gamma\cdot D+m_D/m_D)(\gamma\cdot\zeta)C, \quad (3)$$
$$a^2/4\pi=8[B/m(1-\alpha r_t)]^{1/2}\cong 0.475.$$

In Eq. (3), D is the deuteron four-momentum, m and m_D are the nucleon and deuteron masses, respectively; B, α, and r_t are the deuteron-binding energy, inverse size, and the triplet-effective range. C is the charge-conjugation matrix and ζ is the deuteron-polarization vector, satisfying

$$\zeta\cdot D=0,\quad \zeta\cdot\zeta=3. \quad (4)$$

The matrix element $\langle 3K|J_\mu|n\rangle$ is obtained by considering a phenomenological interaction term of the following form:

$$\frac{G}{2}\int\bar{\Omega}_\mu(\mathbf{r}_3)K^\dagger(\mathbf{r}_4)\boldsymbol{\tau}\partial_\mu N(\mathbf{r}_4)\cdot K^\dagger(\mathbf{r}_1)\boldsymbol{\tau}\tau_2 K^\dagger(\mathbf{r}_2) \times\rho(\mathbf{r})\rho'(\mathbf{r}')\delta\left(\mathbf{r}_3-\frac{\mathbf{R}}{2}\right)d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4$$
$$=\frac{G}{2}\int\bar{\Omega}_\mu(\mathbf{r}_3)J_\mu(\mathbf{r}_3)d\mathbf{r}_3, \quad (5)$$

with

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$$
$$\mathbf{R}=\mathbf{r}_1+\mathbf{r}_2$$
$$\mathbf{r}'=\mathbf{r}_3-\mathbf{r}_4.$$

In Eq. (5) the corresponding field operators are denoted by the particle symbols, the τ_i are the Pauli isotopic spin matrices, and ρ and ρ' are density functions which allow for a nonlocality of this effective interaction or, equivalently, to a correlation between various momenta. A typical term in the matrix element of J_μ in momentum space contains, in addition to a delta function giving over-all momentum conservation, the form factors $\rho(\mathbf{k}_1-\mathbf{k}_2)\rho'(\mathbf{p}-\mathbf{k}_3)$, where $\mathbf{p}$ is the momentum of the nucleon and $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ are the momenta of the K mesons. We will subsequently choose the factor ρ so as to restrict the integration of the square of the complete matrix element, $\mathfrak{M}$, over the four-body phase space to an analytically performable rigorous *underestimate* of the complete integration, thus obtaining a lower limit on the production cross section from the graph in Fig. 1(a). The factor of $\frac{1}{2}$ takes into account the identity of two of the K mesons; *neglecting* interference, the above interaction generates six identical terms in the integration of the square of $\mathfrak{M}$ over the four-body phase space. For a typical term in $\mathfrak{M}$ we have

$$\langle 3K|J_\mu|n\rangle=Gn_\mu u(n)(m/n_0)^{1/2}. \quad (6)$$

The coupling G may be written in terms of a dimensionless constant g and a characteristic mass, m_x as

$$G=g/m_x^3. \quad (7)$$

Substituting from Eqs. (3) and (6) into (2), performing the sum, and using $C\bar{v}^T=-u$, we obtain (suppressing form factors),

$$\mathrm{Im}\mathfrak{M}(q^2)=\pi\delta(n^2-q^2)(aG/(8)^{1/2}m_D)\bar{z}_\mu(l)n_\mu Ou(p),$$

with (8)

$$O=(\gamma\cdot n+m)(\gamma\cdot\zeta)(-\gamma\cdot D+m_D).$$

The complete matrix element for the "dispersion" graph in Fig. 1(a) is then

$$\mathfrak{M}[(D+p)^2]=\pi^{-1}\int\frac{\mathrm{Im}\mathfrak{M}(q^2)dq^2}{q^2-(D+p)^2} =\frac{aG}{(8)^{1/2}m_D}\frac{z_\mu(l)n_\mu Ou(p)}{(D+p)^2-m^2}. \quad (9)$$

The total cross section is given by

$$\sigma(W)=(2\pi)^4\int V^{-1}\delta(D+p-l-k_1-k_2-k_3) \times\left(\frac{mM}{16p_0D_0l_0\omega_1\omega_2\omega_3}\right)\sum|\mathfrak{M}(W)|^2\frac{d\mathbf{l}d\mathbf{k}_3d\mathbf{y}d\mathbf{x}}{2(2\pi)^{12}}, \quad (10)$$

with

$$\mathbf{x}=\mathbf{k}_1+\mathbf{k}_2$$
$$\mathbf{y}=\mathbf{k}_1-\mathbf{k}_2. \quad (10)$$

Here $W=D_0+p_0$ is the total center-of-mass energy, V is the center-of-mass relative velocity of antiproton and deuteron, and the sum denotes a sum over the initial and the final spin substates of antiproton and deuteron, and of Ω^-, respectively. The performance of these spin sums is facilitated by noting that n_μ has only a fourth component in the center of mass and that, in a helicity representation, only two of the four wave functions, $\bar{z}_\mu$, have nonzero fourth components. These correspond to the helicity $\pm\frac{1}{2}$ wave functions, and are given explicitly by

$$z_0^{-1/2}(l)=(\tfrac{2}{3})^{1/2}\frac{|\mathbf{l}|}{M}u_{-1}(l)$$
$$z_0^{1/2}(l)=(\tfrac{2}{3})^{1/2}\frac{|\mathbf{l}|}{M}u_1(l). \quad (11)$$

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Here M is the Ω^- mass and $u_{-1}(l)$ and $u_1(l)$ are Dirac spinors for spin-down and spin-up, respectively. For $\mathbf{l}$ at an angle θ with the antiproton beam direction (taken as the quantization axis), we have

$$u_{\pm1}(l)\rightarrow N(l)e^{i\sigma_2(\theta/2)}\chi_{\pm1}\begin{bmatrix} l_0+M \\ \pm|\mathbf{l}| \end{bmatrix}, \qquad (12)$$

where $\chi_{\pm1}$ are ordinary Pauli spinors and $N(l)$ is the normalization factor $[2M(l_0+M)]^{-1/2}$. We give the result for the spin sum over the square of the matrix element in the center-of-mass system.

$$V^{-1}(2\pi)^{-5}\left(\frac{mM}{16p_0D_0l_0\omega_1\omega_2\omega_3}\right)\sum|\mathfrak{M}(W)|^2$$
$$=\left(\frac{1}{96\pi}\right)\left(\frac{a^2}{4\pi}\right)\left(\frac{g^2}{(4\pi)^3m_x^6}\right)\left(\frac{D}{m_D}\right)^2\left(\frac{l}{M}\right)^2$$
$$\times\frac{W^2(W+m)^2(p_0+m)(l_0+M)}{(W^2-m^2)^2WDl_0\omega_1\omega_2\omega_3}$$
$$\times\left(1+\frac{D_0+m_D}{p_0+m}\right)^2. \qquad (13)$$

This expression gives only the leading terms in an expansion of $|\mathfrak{M}(W)|^2$ in powers of (D/D_0+m_D) and the zeroth-order terms in an expansion in powers of $[l(W-m)/(l_0+M)(W+m)]$. For 2.8-BeV/c incident antiprotons the maximum values of these parameters are 0.32 and 0.175, respectively. In Eq. (13), l and D denote the magnitudes of the corresponding particle momenta, and ω_1, ω_2, and ω_3 denote the total energies of the K mesons. Inserting Eq. (13) into Eq. (10) leads to a rather complicated series of integrations over the four-body phase space. We have chosen to perform these integrations analytically by making an approximation which *rigorously* underestimates the complete integration and leads to a *lower* limit on the production cross section. The approximation consists in setting $\mathbf{x}\cdot\mathbf{y}=(\mathbf{l}+\mathbf{k}_3)\cdot\mathbf{y}=0$. This limits the angular variation of $\mathbf{y}$, the relative momentum of two of the K mesons, to be perpendicular to the total momentum of these two mesons. Then

$$\omega_1=\omega_2=\omega \qquad (14)$$
$$2\omega=(\mathbf{x}^2+\mathbf{y}^2+4m_K^2)^{1/2}.$$

The two correlated mesons behave as a particle with mass, $4m_K^2+\mathbf{y}^2$ and momentum, $(\mathbf{l}+\mathbf{k}_3)$. The correlation can be thought of as being achieved through the form factor $\rho(\mathbf{k}_1-\mathbf{k}_2)$ implicitly contained in $\mathfrak{M}$. The integration over the angle between $\mathbf{k}_3$ and $\mathbf{l}$ can be taken out by the delta function, the integration over ω_3 between limits which depend upon $|\mathbf{y}|$ and $|\mathbf{l}|$ can be performed and followed by an integration over $|\mathbf{y}|$ between zero and an upper limit dependent upon $|\mathbf{l}|$. Finally the integration over $\mathbf{l}$ can be performed approximately from zero up to the maximum $\mathbf{l}_m$. The resulting expression for the cross section is

$$\sigma(W)=\left(\frac{1}{96\pi}\right)\left(\frac{a^2}{4\pi}\right)\left(\frac{g^2}{(4\pi)^3m_x^6}\right)\left(\frac{DW}{m_DM}\right)^2$$
$$\times\frac{(p_0+m)(W+m)^2}{DW(W^2-m^2)^2}\left(1+\frac{D_0+m_D}{p_0+m}\right)^2Q(l_m), \qquad (15)$$

where

$$Q(l_m)\cong\frac{8}{W-m}\int_0^{l_m}x^4dx\left\{A(x)[a(x)-x^2]^{3/2}-\frac{1}{5}\frac{[a(x)-x^2]^{5/2}}{(W-x_0)^2-x^2}\right\},$$

with

$$A(x)=\frac{1}{3}\left[1-\frac{4m_K^2}{(W-x_0)^2-x^2}\right],$$
$$a(x)=(W-x_0-m_K)^2-4m_K^2,$$
$$x_0=(x^2+M^2)^{1/2}. \qquad (16)$$

In Eq. (16), l_m is the maximum value of the Ω^- center-of-mass momentum, which is about 1.05 BeV/c for 2.8-BeV/c incident antiprotons. Approximate evaluation then gives $Q(l_m)\sim0.36$. From Eq. (16) with the characteristic mass, m_x, taken to be the Ω^- mass we obtain

$$\sigma(W=3.94\text{ BeV})\cong0.84[g^2/(4\pi)^3]\mu\text{b}. \qquad (17)$$

With $g^2/(4\pi)^3=0.01$ [for the origin of the $(4\pi)^3$ recall that this constant might be a phenomenological representation of the square of the product of three Yukawa-type couplings] we obtain a *lower* limit on the cross section of 0.0084 μb. This should be considered a conservative estimate for $g^2/(4\pi)^3$.

It is a direct matter to compute the production cross section for the process

$$\bar{p}+D\rightarrow\Xi^-+K^++K^0. \qquad (18)$$

The relevant graph is shown in Fig. 1(b). We assume the cascade spin-parity to be $\frac{1}{2}^+$ and we take at the unknown vertex, a phenomenological interaction density of the form

$$iG_1\bar{\Xi}\boldsymbol{\tau}\tau_2K^\dagger\cdot K^\dagger\boldsymbol{\tau}N \quad\text{with}\quad G_1=g_1/m_c. \qquad (19)$$

Here m_c is the cascade mass. In the ratio of the cross sections for processes (1) and (18), unknown corrections to the deuteron vertex and the neutron propagator cancel out, as these are the same in the graphs of Figs. 1(a) and (b). We obtain, for 2.8-BeV/c incident antiprotons, as a *lower* limit

$$\frac{\sigma(\Omega^-)}{\sigma(\Xi^-)}\cong\frac{g^2/(4\pi)^3}{g_1^2/(4\pi)^2}\frac{3Q(l_m)W^2m_c^2}{2\pi(l_m)^3M^8}=0.84\% \qquad (20)$$

with

$$\frac{g^2/(4\pi)^3}{g_1^2/(4\pi)^2}=\frac{1}{2}.$$

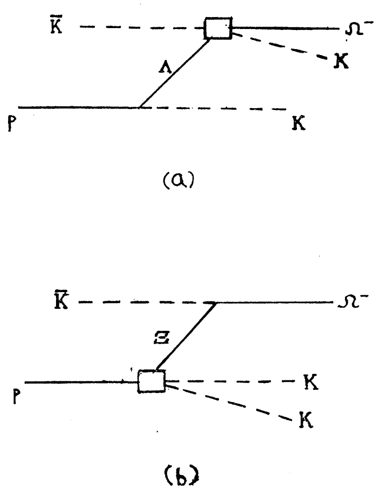

FIG. 2. Feynman graphs for the origin of the phenomenological coupling term in Eq. (5).

This would give a cross section of about 1.0 μb for Ξ^- production in the reaction (18). It is worth noting that the Ω^- production cross section grows extremely rapidly with $W(\sim W^8)$ and the ratio (20) goes like W^6. Consequently it is very sensitive to the characteristic mass chosen in the dimensional coupling constants in Eqs. (7) and (19). If, instead of choosing these masses as M and m_c, respectively, we had chosen them both as m_K, the ratio (20) would be $\sim$1.8! A power of W^2 in the ratio (20) is attributable directly to the spin $\frac{3}{2}$ nature of the Ω^-.

Finally, we have estimated the production cross section for Ω^- in the process

$$K^- + p \rightarrow \Omega^- + K^+ + K^0. \tag{21}$$

This is, of course, the first process in which one would look for the Ω^-. The difficulty is that, in the experiment underway at the present time,[11] the K^- laboratory momentum reaches to about 3.5 BeV/c, which is only about 100 MeV above the threshold for the reaction, (21), in the center-of-mass system. We can compute the matrix element directly from our phenomenological interaction introduced in Eq. (5). This interaction may be thought of as a representation of the graphs in Fig. 2, for example.[12] We obtain for the total cross section

$$\sigma(W) \cong (g^2/(4\pi)^3)\left(\frac{m}{M}\right)^2 \frac{4}{p} M^{-6} I,$$

with (22)

$$I \cong \left(\frac{1+\gamma}{\gamma}\right) M^3 (\gamma v)^4 \gamma l_m [(W^2-p^2)^{1/2} - M]\left\{\frac{2\gamma M v}{p}\right\}$$

and

$$v = p/W,$$
$$\gamma = (1-v^2)^{-1/2}.$$

In these equations, p is the *laboratory* momentum of the K^-, W is the total *laboratory* energy, and l_m is the maximum momentum of the Ω^- in the center-of-mass system. We find, for 3.5-BeV/c incident K^-

$$\sigma \cong 8.4(g^2/(4\pi)^3)\ \mu\text{b} = 0.084\ \mu\text{b with } g^2/(4\pi)^3 = 0.01. \tag{23}$$

This cross section is about ten times the *lower* limit for the cross section estimated for process (1) at 2.8-BeV/c incident antiproton momentum. Nevertheless, if even more energetic and intense antiproton beams become available prior to similarly improved K^- beams, a search for the Ω^- in reaction (1) may become feasible.

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[11] W. J. Willis (private communication).

[12] It is gratifying that a *direct* computation of the cross section from the second graph in Fig. 2, using at one vertex the interaction in Eq. (19), and at the other vertex a term, $(g_2/m_c)\bar{\Xi}_\mu K\tau_2\partial_\mu\Xi$, yields, with $g_1^2g_2^2/(4\pi)^3 = 0.01$, essentially the same value as the cross section given by Eqs. (22) and (23).