Energy Levels in Sm^{148} and Sm^{150}†

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Energy levels in Sm^{148} and Sm^{150} have been observed via the reactions Sm$^{147}(d,p)$Sm^{148}, Sm$^{149}(d,p)$Sm^{150}, Sm$^{148}(p,p')$Sm^{148}, and Sm$^{150}(p,p')$Sm^{150} at 12-MeV bombarding energy. Reaction products were analyzed in a 0.1% energy resolution magnetic spectrograph using emulsion techniques. Levels below 1.9-MeV excitation are found to have much smaller (d,p) cross sections than those above this energy. Many previously unknown levels above 1-MeV excitation have been found in both these nuclei. The observed levels are compared with previous studies of these nuclei. Several of the positive parity levels in Sm^{148} can be fitted to an asymmetric rotor-vibrator spectrum. The excited states in Sm^{150} can be approximated, but not fitted exactly, with this model. The observed low-lying states in these nuclei are discussed in terms of expected level structure for collective models. The ground-state Q values for the reactions Sm$^{147}(d,p)$Sm^{148} and Sm$^{149}(d,p)$Sm^{150} are determined to be 5920±10 keV and 5764±4 keV, respectively.

INTRODUCTION

RECENTLY, extensive experimental studies of the level schemes of nuclei in the transition region between $N=82$ (closed shell nuclei) and $N=90$ (beginning of a deformed region) have been published. Most of these studies were based on beta- and gamma-ray spectroscopy. The selection rules for these transitions usually cause a significant fraction of the states to go unobserved. This fact suggests the additional study of these nuclei by other methods. As part of a program of study of the level schemes of rare-earth nuclei, we have investigated the reactions Sm$^{147}(d,p)$Sm^{148}, Sm^{149}-(d,p)Sm^{150}, Sm$^{148}(p,p')$Sm^{148}, and Sm$^{150}(p,p')$Sm^{150}. The over-all aim of the program is the study of level trends through neighboring nuclei and the interpretation of these trends (and discontinuities in such trends) in terms of current ideas on nuclear structure. In addition, these new data provide a valuable check and verification of the previous studies of these nuclei.

EXPERIMENTAL METHOD

Protons from the (d,p) and (p,p') reactions were detected in a magnetic analyzer of the Browne-Buechner[1] type. This instrument can detect particles over an energy range $0.5E_0 \leq E \leq 1.2E_0$, where E_0 is determined by the field strength. The radius of curvature of particles with energy E_0 in the uniform magnetic field is 61.6 cm. The instrument has a characteristically small average solid angle which for these experiments was from 2.2×10^{-4} to 3.6×10^{-4} sr. The average energy resolution along the focal curve is 0.1%.

Outgoing protons from the reactions were detected in Eastman NTA 50 μ nuclear track plates, which are clamped along the focal curve. Aluminum foils 0.005-in. thick were used to prevent deuterons and α particles from reaching the emulsions during the experiments. Deuteron background is particularly troublesome at forward angles in (d,p) experiments unless such precautions are taken.

The developed track plates are scanned on microscopes equipped with calibrated stages. Usually scanning is done in $\frac{1}{2}$ mm strips and at $\frac{1}{2}$ mm intervals. The location of a peak is determined by the point on the high-energy side which is $\frac{1}{3}$ of the peak height.

The magnetic rigidity as a function of distance along the focal curve was determined by using Po^{210} α particles, by elastic scattering from a number of target nuclei with precisely known Q values for various (d,p) reactions, and by analysis of O^{16} ions of identical energy but different charge states. Calculations of Q values and other pertinent information have been programmed for IBM 650 and IBM 709 use. Relativistic corrections have been included in the Q equation and in the energy-$B\rho$ relationships. The energy of the incident beam is determined by a self-consistent method in the computer program from the position of groups of known origin. The groups used as incident energy calibrators were protons from C$^{12}(d,p)$C^{13} $(Q=2722.3\pm0.7$ keV), C$^{13}(d,p)$C^{14} $(Q=5951.3\pm0.8$ keV), and O$^{16}(d,p)$O^{17} $(Q=1917\pm0.8$ keV). Peaks due to light impurities are determined by their kinematic shift as a function of reaction angle. Therefore, at least two observations at different angles are required. A kinematical shift of light relative to heavy nucleus reactions occurs if the angle is fixed and the incident energy is changed. In these heavy nuclei, kinematic shifts of neighboring isotopes are very small and diffi-

TABLE I. Enrichments of samarium oxide targets.

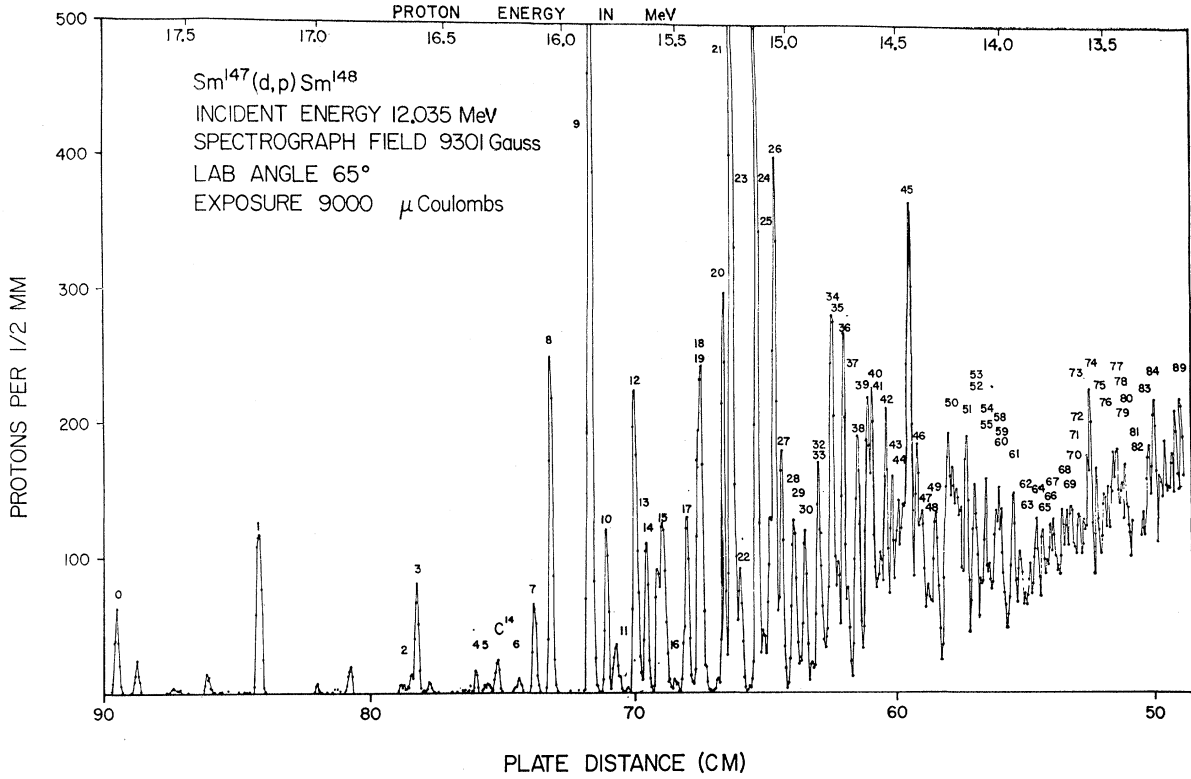| Enriched isotope	Percentages of samarium isotopes						
	Sm^{144}	Sm^{147}	Sm^{148}	Sm^{149}	Sm^{150}	Sm^{152}	Sm^{154}
Sm^{147}	0.08	97.80	0.91	0.51	0.17	0.34	0.21
Sm^{148}	0.36	0.81	96.26	1.41	0.34	0.55	0.28
Sm^{149}	0.08	0.33	0.55	97.46	0.65	0.70	0.30
Sm^{150}	0.10	1.00	1.40	8.40	81.00	6.50	1.60

FIG. 1. Proton spectrum at 65 degrees for the reaction $Sm^{147}(d,p)Sm^{148}$. Only groups from this reaction are numbered.
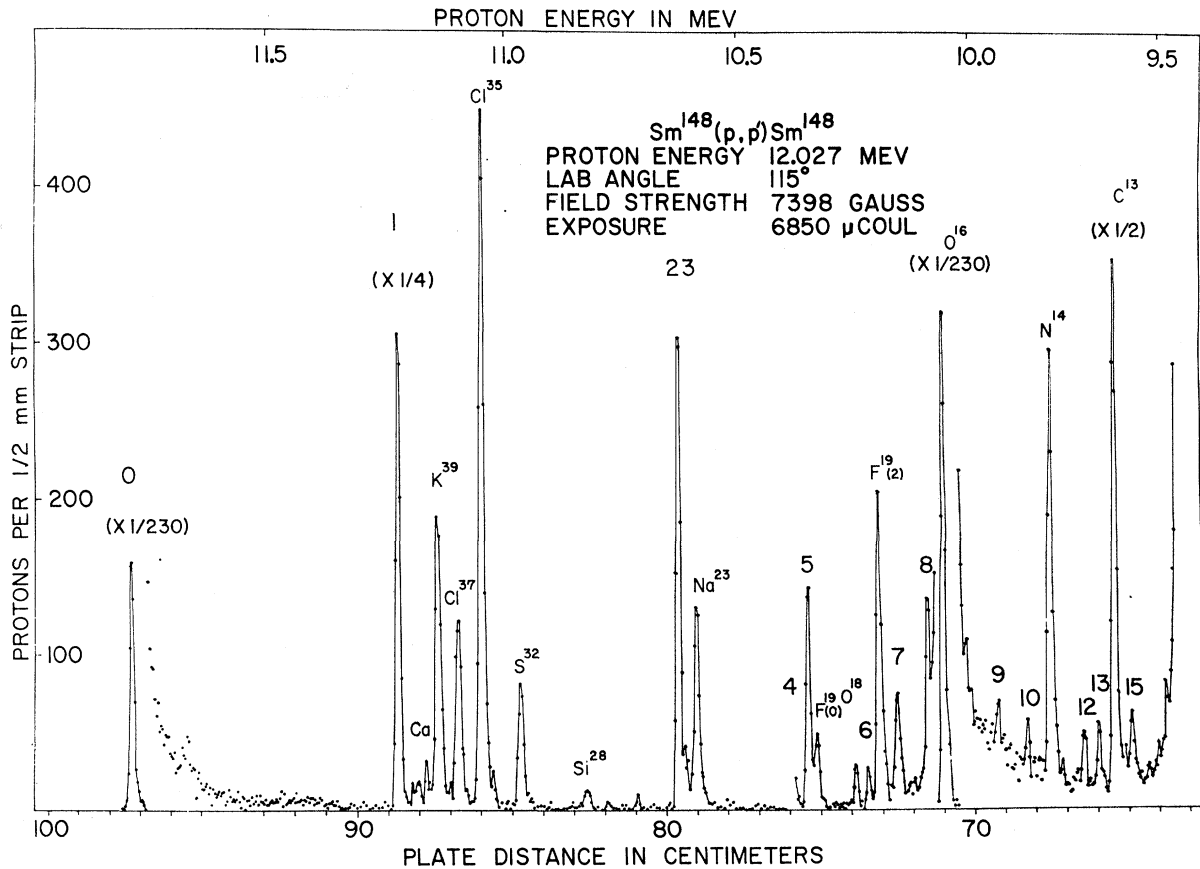

FIG. 2. Proton spectrum from the reaction $Sm^{148}(p,p')Sm^{148}$ at an angle of 115°. The Sm^{148} groups are numbered and various impurities are identified.

TABLE II. The observed intensities for $Sm^{147}(d,p)Sm^{148}$ at a reaction angle of 65 degrees. Intensities are not given for groups 70 through 93 because of an uncertainty in background and summing.

Peak No.	Q (keV)	Relative intensity	Peak No.	Q (keV)	Relative intensity	Peak No.	Q (keV)	Relative intensity
0	5920	1.0	24	3238	0.59	48	2470	2.00
1	5368	2.78	25	3210	2.25	49	2445	3.27
2	4755	0.22	26	3195	7.00	50	2394	4.60
3	4778	1.46	27	3157	3.25	51	2364	3.45
4	4488	0.19	28	3106 ⎱ 3.00		52	2334 ⎱ 3.80	
5	4456	0.16	29	3087 ⎰		53	2299 ⎰	
6	4325	0.09	30	3055	2.20	54	2262 ⎱ 4.10	
7	4251	1.21	31	3018	0.35	55	2252 ⎰	
8	4180	3.90	32	2989 ⎱ 3.55		56	2208 ⎱ 4.20	
9	4011	12.70	33	2970 ⎰		57	2176 ⎰	
10	3938	1.80	34	2929	6.75	58	2159 ⎱	
11	3879	0.27	35	2900	1.54	59	2140 ⎬ 5.55	
12	3816	4.53	36	2875	4.10	60	2126 ⎰	
13	3766		37	2852	1.10	61	2073	4.00
14	3719	1.94	38	2814	4.20	62	2030	2.37
15	3698	3.26	39	2765	4.60	63	1996	0.93
16	3640	0.13	40	2744	4.33	64	1967	3.50
17	3590	2.68	41	2702	2.60	65	1930	2.20
18	3533 ⎱ 6.40		42	2677	3.90	66	1892 ⎱ 6.00	
19	3516 ⎰		43	2647	3.95	67	1877 ⎰	
20	3420	4.55	44	2621	3.45	68	1835 ⎱ 6.10	
21	3384	11.30	45	2574	4.90	69	1812 ⎰	
22	3345	1.70	46	2531	3.50			
23	3268	8.65	47	2508	3.15			

cult to detect. This fact requires one to use isotopically enriched targets.

The ion beam of the F. S. U. Tandem Van de Graaff is bent through a 90° analyzing magnet of 34 in. radius. A difference amplifier system regulates the terminal voltage by keeping the beam current balanced on two slits located outside the exit end of this magnet. Long-term drift in the 90° magnetic field is corrected by the experimentalist by using an NMR fluxmeter as a monitor. The experimentalist also corrects drift in the magnetic analyzer by monitoring an NMR fluxmeter. Following the analyzing magnet the beam is deflected by a switching magnet and focused by a magnetic quadrupole lens on the target. Beam current at the Faraday cup (after passing through $\frac{1}{4}$-×3-mm slits and the target) was as high as 1 μA.

In (p,p') experiments at 12 MeV exposures of 4000 μC or more were usually taken. This minimum exposure length was set by the low inelastic cross sections and target thickness effects on resolution. When the target is even-odd in a (d,p) experiment, an exposure of 6000 μC or more was found necessary. In the case of $Sm^{147}(d,p)Sm^{148}$ and $Sm^{149}(d,p)Sm^{150}$, states up to 2-MeV excitation are weakly excited and longer exposures were necessary. In addition to the main exposure a short exposure of 30–100 μC was made in order to allow the counting of very intense groups.

Targets were fabricated from enriched samarium oxides obtained from the Separated Isotopes Division of Oak Ridge National Laboratory. The enrichments for the targets used here are given in Table I. These oxides were evaporated from small carbon crucibles by an electron gun technique, which has been previously described in the literature.[2] Targets prepared by this method had thicknesses ranging up to 300 μg/cm² with carbon backing thicknesses of 10–50 μg/cm². Target orientation in these experiments was usually chosen so as to give minimum energy spread to groups from the ground state and low-lying excited states.

RESULTS

Sm¹⁴⁸

Figure 1 shows one of the (d,p) spectra from which the level scheme of Sm¹⁴⁸ was determined. The C^{13}-$(d,p)C^{14}$ ground state which appears between groups 5 and 6 was used to obtain the incident energy. The numbered groups (beginning with 0 for the ground state) are levels in Sm¹⁴⁸ while the other groups are due to various impurities present in the target. The target slits were set at $\frac{1}{4}$×3 mm during these experiments. The solid angle subtended by the spectrograph in this exposure was $2.2×10^{-4}$ sr. Figure 2 shows one of the (p,p') spectra taken in this study of the Sm¹⁴⁸ level scheme. Additional (d,p) spectra were taken at 45° and 65° (11.264 MeV). A proton spectrum at 115° was also taken.

The interpretation of the spectra was fairly straight forward since few impurity groups were present and the target was highly isotopically enriched. No groups from other samarium isotopes were strong enough to be observed except for very weak groups which proved to be the first excited 2+ states coulomb excited in the (p,p') experiments. Agreement in excitation energy be-

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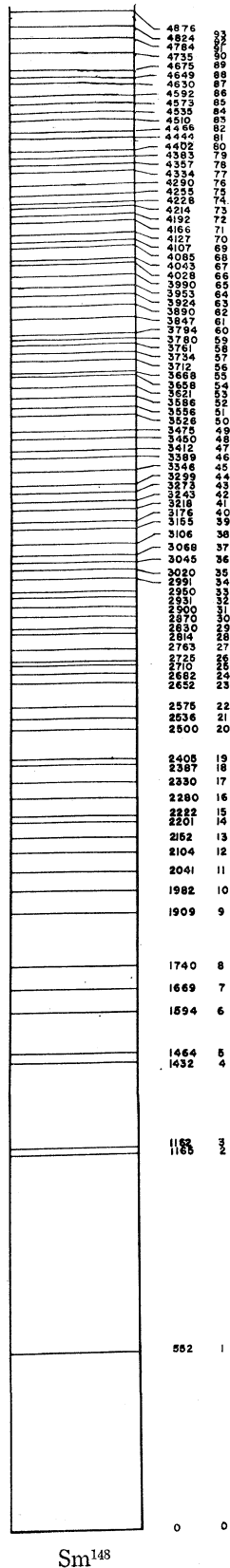[2] M. C. Oleson and B. Elbek, Nucl. Phys. 15, 26 (1960).

FIG. 3. Level scheme for Sm^{148} from these experiments. Excitation energies are in keV and the numbering of the levels is in correspondence with Figs. 1 and 2 and Table II.

Sm^{148}

tween the various angles was within 3 keV up to 3 MeV and within 6 keV up to 4.5 MeV excitation. The level at 1165 keV appears as a bump on the front edge of the 1182 keV level (Group 3) in Fig. 1. This level is strongly excited in (p,p') reaction (Fig. 2). In the (d,p) spectrum which is shown here, the weak 2041 keV level happens to be hidden by an impurity group.

A number of close doublets are observed in this work. However, doublets of less than approximately 7 keV separation would probably be missed. The broadening of the base of the 2387 keV peak suggests a weak level at 2405 keV. Similar doublet structure is observed for the 2710 and 2950 keV groups. The observed relative intensities of reaction product groups are given in Table II. The ground-state Q value for the reaction $Sm^{147}(d,p)Sm^{148}$ was measured to be 5920 ± 10 keV. The level scheme for Sm^{148} deduced from these experiments is given in Fig. 3.

Sm^{150}

Only one target of Sm^{150} was successfully fabricated. This was on a particularly thick backing. All other attempts at fabrication were failures due to inferior strength of the backing material. Inelastic (p,p') spectra were taken at 90°, 95°, and 133° and are shown in Fig. 4. These spectra show heavy contaminant groups. The fluorine originates in the carbon backing. Levels at 335, 743, 777, 1048, 1074, 1172, 1196, and 1697 keV were observed. Of these levels only the 1172- and 1196-keV levels were not seen in the (d,p) spectra. A weak group at 668 keV is seen at 95° and 133°. However, a spurious group of similar intensity and position has on occasion been observed in other (p,p') spectra. The origin of this peak is not known. These (p,p') spectra were taken with a mean spectrograph solid angle of 3.5×10^{-4} sr and a beam spot size of $\frac{1}{2}\times3$ mm. The incident energy was determined by the position of the elastic group. Some peaks from other isotopes (Sm^{152} and Sm^{154}) were observed near the ground state. Their weak intensity rules out the possibility of any of the higher states originating in isotopes other than Sm^{150} since the intensity of such higher energy groups is down an order of magnitude or more from the low-lying coulomb-excited 2+ and 4+ states.

The (d,p) spectra on Sm^{149} were taken at 12 MeV with a solid angle of 3.0×10^{-4} sr and target slits at $\frac{1}{2}\times4$ mm. A representative spectrum appears in Fig. 5. In the important 700–800 keV excitation region two groups are seen at 743 and 777 keV. The levels above 1.5 MeV are very certain, agreeing in excitation energy and approximate intensity between angles. The ground-state Q value for the reaction $Sm^{149}(d,p)Sm^{150}$ was determined as 5764 ± 4 keV relative to the $C^{13}(d,p)C^{14}$ ground-state group. The levels in Sm^{150} deduced from these experiments are shown in Fig. 6. The relative intensities in (d,p) excitation are given in Table III.

A few comments on certain of the levels are neces-

TABLE III. The observed intensities for $Sm^{149}(d,p)Sm^{150}$ at a reaction angle of 45 degrees.

| Peak No.	Q (keV)	Relative intensity	Peak No.	Q (keV)	Relative intensity	Peak No.	Q (keV)	Relative intensity
0	5764	1.0	24	3514	Obsc.	48	2516	3.84
1	5429	1.20	25	3488⎫		49	2490	6.10
2	5021	0.71	26	3474⎭	3.43	50	2430	1.57
3	4987	0.19	27	3430	0.30	51	2417	2.25
4	4716	1.29	28	3392	1.60	52	2398	2.50
5	4690	0.15	29	3364	4.47	53	2360	4.80
6	4592	0.0	30	3296	5.22	54	2333	3.50
7	4568	0.0	31	3240	8.37	55	2299	3.90
8	4395	0.05	32	3189	6.90	56	2276	4.10
9	4339	0.25	33	3140	3.58	57	2236	1.95
10	4304	0.89	34	3109	4.83	58	2208	4.10
11	4249	0.55	35	3045	1.43	59	2178	5.30
12	4112	1.20	36	3017	5.60	60	2116	3.30
13	4078	0.0	37	2943	9.10	61	2088	0.90
14	4067	0.08	38	2899	8.35	62	2076	3.60
15	4004	0.08	39	2857	12.40	63	2024	4.33
16	3974	0.58	40	2830	2.84	64	1984	6.00
17	3938	1.61	41	2759	6.20	65	1929	6.25
18	3790	1.92	42	2718	4.50	66	1897	7.00
19	3738	2.13	43	2676⎫		67	1868	
20	3695	1.63	44	2660⎭	3.66	68	1839	
21	3646	11.10	45	2629	5.90	69	1816	5.75
22	3615	1.19	46	2583	2.91	70	1788	
23	3559	3.91	47	2552	3.07	71	1776	3.90
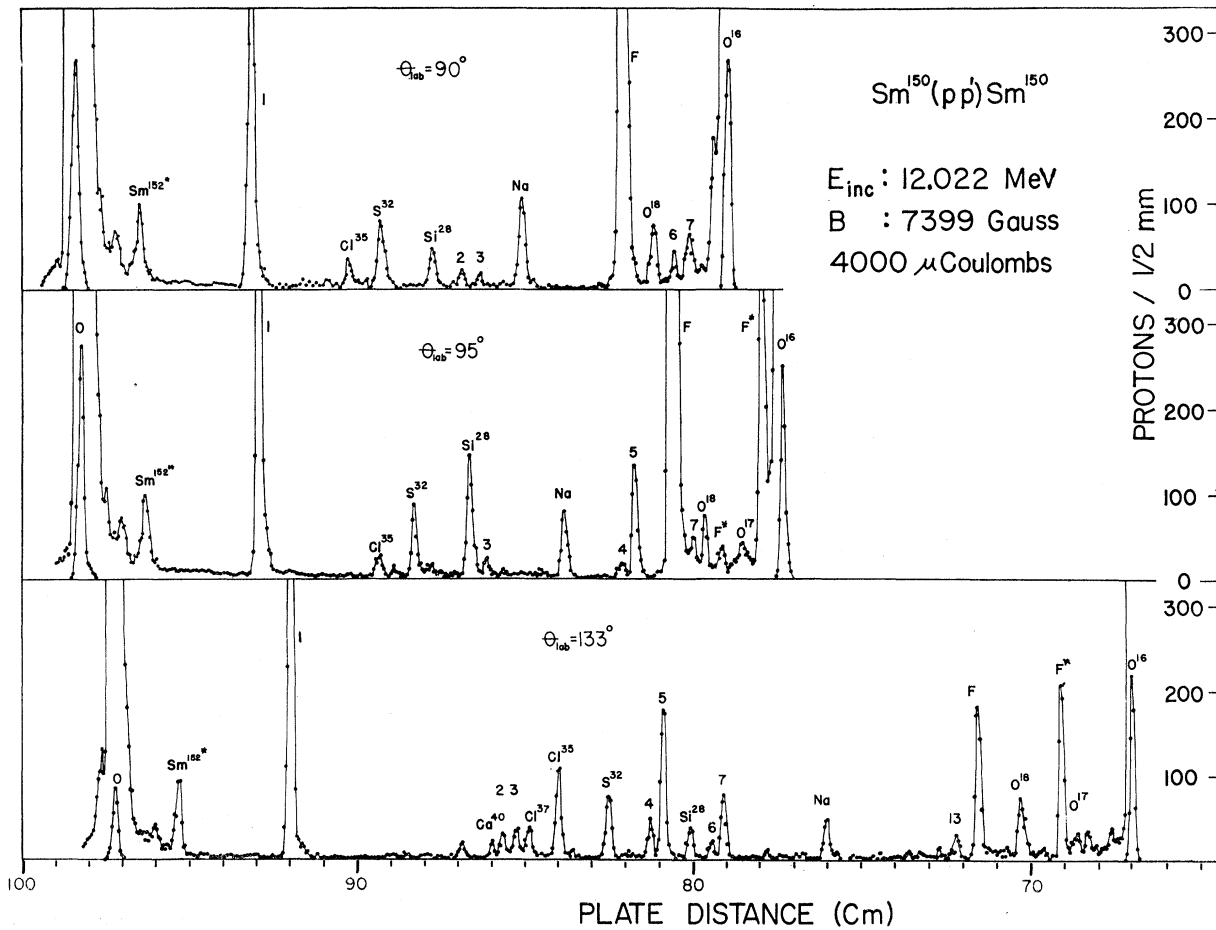
FIG. 4. Proton spectra at various angles for the reaction $Sm^{150}(p,p')Sm^{150}$. The Sm^{150} groups are numbered and various impurities are identified.
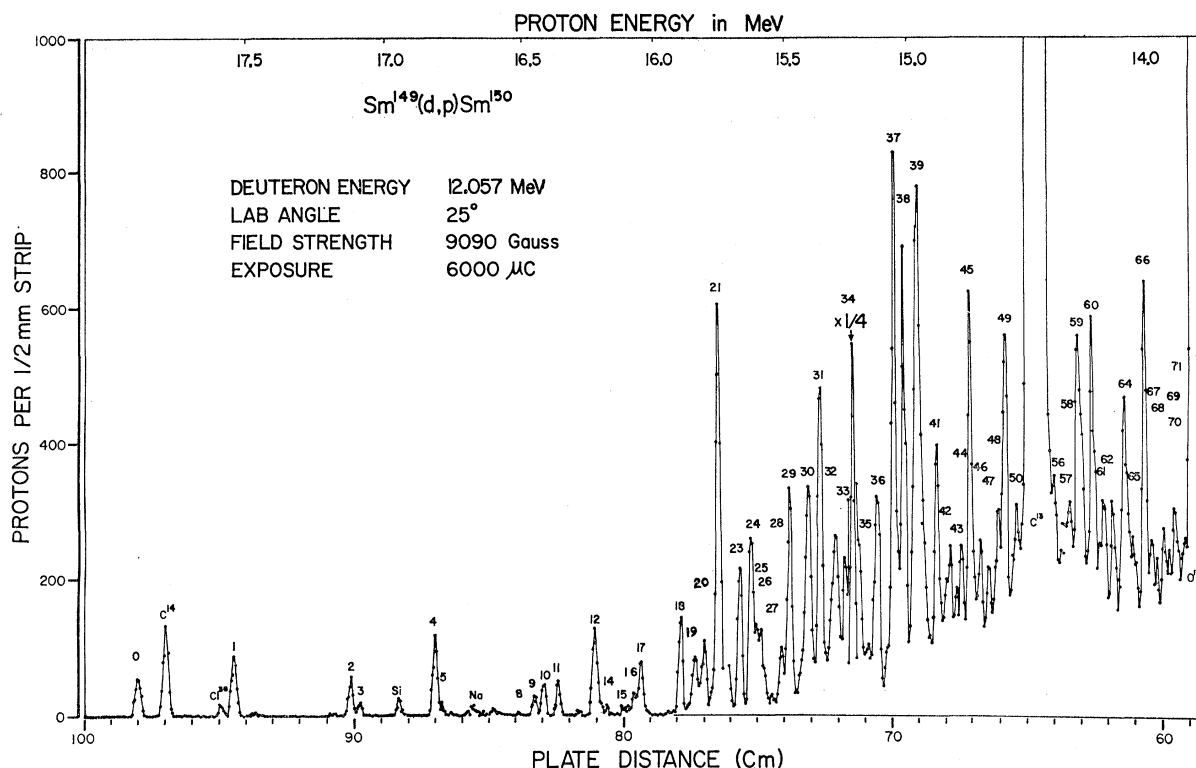
FIG. 5. Proton spectrum at 25° for the reaction Sm149(d,p)Sm150. Only groups from this reaction are numbered.

sary. Levels 8, 14, 15, and 27 are extremely weak and should be considered somewhat tentative. However, the excitation energies determined from the 25° and 45° spectra are in excellent agreement, and no other samarium isotopes were present in significant amounts in the Sm149 target. Group 24 happened to fall on a joint in the emulsion surface at this angle and is missing from the spectrum. Group 25 was also partly obscured at this angle.

ERRORS

The main sources of error in the determination of Q values and excited state energies are; (1) reproducibility of emulsion position during the experiment and subsequent scanning, (2) finite width of the scanning swaths, (3) differential hysteresis which distorts the magnet calibration (4) statistical variation in peak shape. Effects (1) and (2) have been analyzed and contribute a maximum error of ±1.5 keV. Effect (3) is usually the main contribution to error and has been estimated from comparisons of a large number of very precisely known excitation energies and Q values. Errors in excitation energies from this effect are estimated at ±2 keV up to 500 keV, then ±3 keV up to 800 keV, ±5 keV up to 1200 keV, ±8 keV up to 2.5 MeV and ±10 keV above 2.5 MeV excitation. The error in absolute Q value will depend on the distance along the focal curve between the group in question and the peak used to determine the incident energy.

Effect (4) contributes significantly only when peak height is less than about 25 counts. In these cases the additional uncertainty in energy is estimated to be ±5 keV.

COMPARISON WITH PREVIOUS STUDIES

Sm148

Gamma transition measurements were made by Schwerdtfeger, et al.[3] on the Eu148 decay. These have been used by Jha et al.[4] in conjunction with their gamma-gamma coincidence and summing techniques, to determine levels at excitation energies of 551(2+), 1181(4+), 1595, 1906, 2096, 2196(?), 2400, 2520, 2610, 2820, 2920, and 3020(?) keV in Sm148. An identical level scheme, but without the 2920 and 3020 keV levels, was proposed by Aleksandrov and Shelike.[5] The work of Schwerdtfeger et al.[6] gives levels at 551(2+), 1182(4+), 1595(5±, 3−), 1887(3+), 1908(6+), 2030, 2049, 2097, 2198(5, 3±), 2697(?), and 2782(6+) keV. Sugiyama,[7] in a study of the Eu148 activity, reported

[3] C. F. Schwerdtfeger, E. G. Funk, and J. W. Mihelich, Bull. Am. Phys. Soc. 5, 425 (1960).

[4] S. Jha, R. K. Gupta, H. G. Devare, and G. C. Pramila, Nuovo Cimento (10) 25, 28–40 (1962).

[5] Y. A. Aleksandrov and P. Shelike, Izv. Akad. Nauk USSR, Ser. Fiz. 26, 1162 (1962).

[6] C. F. Schwerdtfeger, E. G. Funk, and J. W. Mihelich, Phys. Rev. 125, 1641 (1962).

[7] K. Sugiyama, J. Phys. Soc. Japan 17, 264 (1962).

levels at 555(2+), 1183(4+), 1470(1), 1598(3,5), 1745(4+), 1911(3,4), 2148, 2220, 2460, 2500, 2740, 2830, 3100, and 3370 keV. In a recent study of the decay of Pm^{148}, Reich et al.[8] used beta-gamma coincidence, gamma-gamma coincidence, and angular correlations to propose levels at 0.55(2+), 1.18(4+), 1.46(1), 1.59, 1.90(6+)(?), 2.02, 2.09(5,6+), and 2.19(6+) MeV in Sm^{148}. Their decay scheme was in good agreement with previously published studies of the Pm^{148} decay, although some disagreement about beta intensities exists.[6,9,10] Coulomb excitation of the 551(2+) keV and 1165(3−) states has been previously observed.[11–14]

The reaction studies of Sm^{148} described here confirm levels at 551, 1182, 1460, 1595, 1910, 2039, 2098, 2202, and 2697 keV which were found in the previous experiments. Evidently the 1887(3+) keV level reported by Schwerdtfeger et al. is not excited by these reactions. All states observed by Jha et al. (excepting the 2520 keV and 2610 keV levels) have been observed in this study. The excitation energies derived from conversion electrons should be considered somewhat more accurate than the reaction values in the case of the 2820 and 2920 keV levels which are members of incompletely resolved doublets in our work. It is not clear from the excitation energy whether the 2039 keV level weakly excited in this (d,p) experiment is the 2030 keV level or the 2049 keV level found in decay studies. Although its weak intensity prevents a definitive statement, this level does not appear to be unusually broad.

Sm^{150}

From a recent study of the decay of Pm^{150}, Gove and O'Kelley[15] have deduced levels at 0.335(2+), 0.740, (1.05), 1.18, 1.66, 1.96, and 2.10 MeV in Sm^{150}. Smither[16] has found evidence for levels at 712(2?), 737(2?), 743(0+), and 774(2+) keV by looking at the gamma spectrum of the reaction Sm^{149}(n,γ)Sm^{150} with a bent crystal spectrometer. Harmatz et al.[17] studied the electron capture and positon spectrum of Eu^{150m} and Eu^{150}, finding levels at 335(2+)-, 743(0+)-, 773(4+)-, 1153-, and 1260(0−)-keV excitation in Sm^{150}. These spins have been subsequently verified by the directional correlation experiments of Guttman et al.[18]

[8] C. W. Reich, R. P. Schuman, J. R. Berreth, M. K. Brice, and R. L. Heath, Phys. Rev. 127, 192 (1962).
[9] J. S. Eldridge and W. S. Lyon, Nucl. Phys. 23, 131 (1961).
[10] S. K. Bhattacherjee, B. Sahai, and C. V. K. Baba, Nucl. Phys. 12, 356 (1959).
[11] N. P. Heydenburg and G. M. Temmer, Phys. Rev. 100, 150 (1955).
[12] H. Mark and G. T. Paulissen, Phys. Rev. 100, 813 (1955).
[13] B. Simmons, K. Famularo, and G. Freier, Phys. Rev. 100, 1265(A) (1955).
[14] O. Hansen and O. Nathan, Nucl. Phys. 42, 197 (1963).
[15] N. B. Gove and G. D. O'Kelley, Bull. Am. Phys. Soc. 7, 352 (1962).
[16] R. K. Smither, Bull. Am. Phys. Soc. 7, 316 (1962).
[17] B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. 123, 1758 (1961).
[18] M. Guttman, H. J. Prask, J. J. Reidy, E. G. Funk, and J. W. Mihelich, Bull. Am. Phys. Soc. 6, 429 (1961).
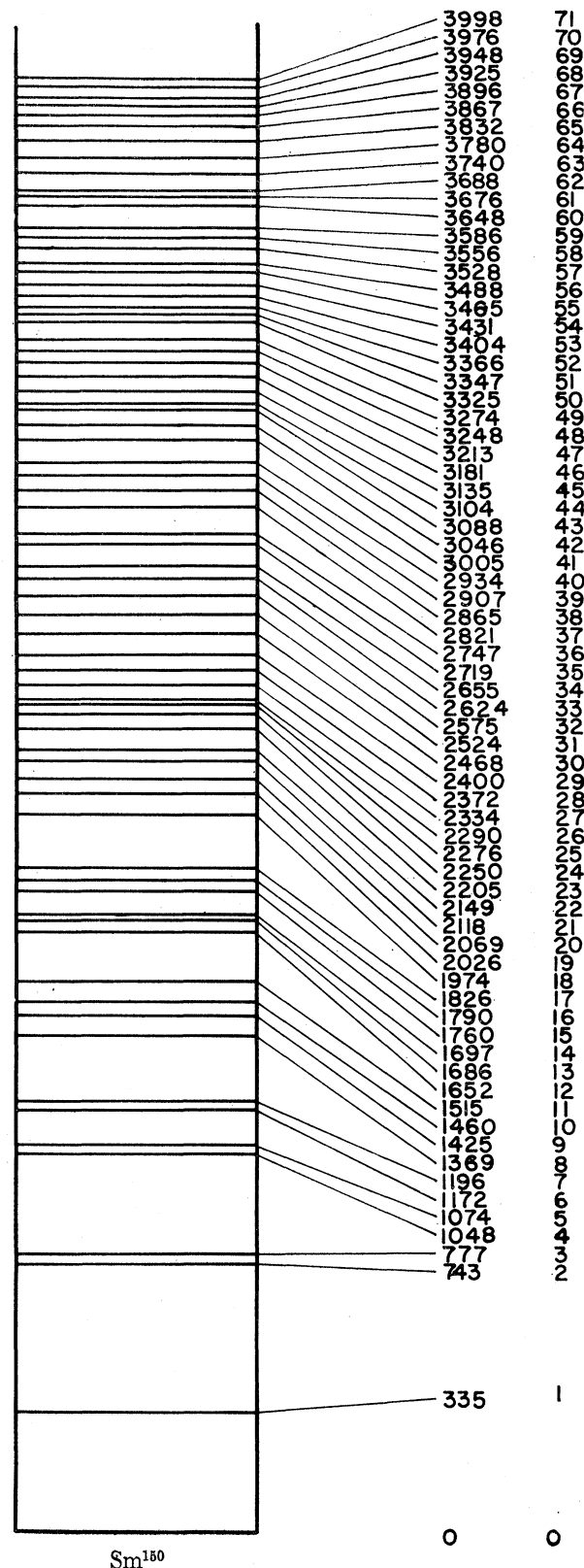
FIG. 6. Level scheme for Sm^{150} from these experiments. Excitation energies are in keV and the numbering of the levels is in correspondence with Figs. 4 and 5 and with Table III.

who also found evidence for additional levels at 1.07-, 1.36-, 1.81-, and 1.97(1−(?))-MeV excitation. Ricci *et al.*[19] found levels at 334, 739(0+), 1050(2+), 1170, 1250, 1920, and 1980(?) keV in another study of the Eu^{150} decay. Coulomb excitation of the 335(2+)- and the 1074(3−)-keV levels has been recently reported.[14] Previously Coulomb excitation of a level at 770±20 keV had been observed.[20] A most extensive study of Sm^{150} which used results of β-spectrometer studies of conversion electrons and bent crystal spectrometer observations on $Sm^{149}(n,\gamma)Sm^{150}$ has been reported by Bieber *et al.*[21] On the basis of energy differences they propose levels at 333.95, 736.96, 740.34, 773.34, 988.0, 1046.0, 1082.7, 1192.8, 1343.9, 1449.1, 1503.9, 1642.0, 1708.4, 1809.0, and 2033.3 keV.

The levels at 335, 743, 777, 1048, 1074, 1176, and 1974 keV which were reported in previous studies have been confirmed. The level reported at 712 keV by Smither is not observed. It is extremely doubtful that we could resolve the 737 and 741 keV levels even if they were of equal intensity. The levels previously reported at 988, 1153, 1256, and 1920 keV are apparently not excited by (p,p') and (d,p) reactions at 12 MeV. Here again, as in Sm^{148}, the excitation energies from this work appear systematically higher than those from the electron data and the latter should be considered more accurate. The differences are in all cases within the estimated experimental error.

DISCUSSION

The variation in the cross sections of (p,p') and (d,p) reactions with the nature of the states in both Sm^{148} and Sm^{150} is striking. In particular the collective nature of the low-lying states is clearly indicated by very small (d,p) cross sections relative to higher excited states (see Fig. 1 and Fig. 5) and relatively large (p,p') cross sections (see Fig. 2 and Fig. 4). The energy gap corresponding to the breaking of the first neutron pair is discernible as a clear increase in (d,p) cross section in both Sm^{148} and Sm^{150} at slightly under 2-MeV excitation. The systematics of these cross sections is explained by the essentially single particle nature of the (d,p) reaction, which inserts a neutron into the target configuration with a certain angular momentum.

Both Sm^{148} and Sm^{150} show additional features in their excitation spectra which are described in a qualitative way by the quadrupole surface phonon model.[22−26]

Most prominent are the nearly equal spacing between first and second excited states and the large reduced transition probability of the first excited 2+ state under Coulomb excitation. The latter are many times larger than the single particle estimate[27] $B(E2)=0.024e^2 \cdot 10^{-48}$ cm². The relatively close spacing of the two phonon triplet in Sm^{150} might indicate that small perturbing terms in the collective Hamiltonian could account for the observed spectrum. Such an analysis, including cubic terms, has recently been put forth.[28] However, this approach does not account for the spectra of Sm^{148} or Sm^{150} since it requires at least two of the members of the two phonon triplet to have an excitation energy less than twice the energy of the first excited state. It remains to be seen whether some other perturbation approach can account for the low-lying states of these nuclei.

A somewhat different collective approach is found in the asymmetric rotor model of Davydov and Fillipov[29] as extended by Davydov and Chaban[30] to include a vibration-rotation interaction. Level spacings in this model depend on γ, the asymmetry parameter, and μ, the nonadiabaticity parameter (μ approaches zero in the limit of no vibrations). Using tables of energy level ratios based on this model,[31] fits to the experimentally observed spectra have been made. Energies of the first excited state and two higher excited states determine γ and μ, which then determine a large number of other positive parity levels. Ratios given in the tables were graphed and smooth curves were drawn through the points for interpolation purposes. The observed energies of the first 2+ and 4+ levels restricts the parameters to $22° \leq \gamma \leq 30°$ and $0.6 \leq \mu \leq 1.0$. There exists no pair of values for γ and μ which will simultaneously fit the 1182(4+) and 1909(6+) keV levels. This may not be a serious difficulty since the 1909 keV level is strongly excited in the (d,p) reaction. As mentioned earlier this is an indication of a noncollective nature. In order to avoid the appearance of low-lying states which are not experimentally observed, a low value for γ (and a correspondingly high value for μ) must be chosen. A value of 22.4° was chosen for γ, which fixed μ equal to 0.94. The resulting spectrum below 2.1 MeV is compared with experiment in Fig. 7. There is weak evidence in some of the spectra for a level at ~1120 keV as predicted. The small cross section of this possible level makes it too uncertain to be definitely put in the level scheme. A 2+ level is predicted at the energy of the known 3− level and would be obscured. The predicted levels at 1434 and 2050 keV are consistent with the experimental observations.

[19] R. A. Ricci, R. van Lieshout, G. B. Vingiand, S. Monaro, and B. van Nooijen, Nucl. Phys. **32**, 490 (1962).

[20] O. Nathan and V. Popov, Nucl. Phys. **21**, 631 (1961).

[21] E. Bieber, T. v. Egidy and O. W. B. Schult, Z. Physik, **170**, 465 (1962).

[22] N. Bohr and F. Kalckar, Kgl. Danske Videnskab. Selskab Mat. Fys. Medd. **14**, No. 10 (1937).

[23] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **26**, No. 14 (1952).

[24] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).

[25] L. Wilets and M. Jean, Phys. Rev. **102**, 788 (1956).

[26] T. Miyazima and T. Tamura, Progr. Theoret. Phys. (Kyoto) **15**, 255 (1956).

[27] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 595.

[28] A. K. Kerman and C. M. Shakin, Phys. Letters **1**, 151 (1962).

[29] A. S. Davydov and G. F. Filippov, Nucl. Phys. **8**, 237 (1958).

[30] A. S. Davydov and A. A. Chaban, Nucl. Phys. **20**, 499 (1960).

[31] P. P. Day, E. D. Klema, and C. A. Mallman, ANL-6220, 1960 (unpublished).

In view of the partial success of the Davydov-Chaban model in fitting the level spectrum of Sm¹⁴⁸, a fit to Sm¹⁵⁰ was also attempted. There is no combination of values for γ and μ which will yield the observed energy ratios of the 737(2+), 740(0+), and 773(4+) keV levels to the 334(2+) keV level. In fact, the 0+ and 4+ levels cannot be fitted exactly even if the second 2+ excitation energy is considered as a free variable. The additional state at 712 keV previously reported was not observed here and is evidently due to an accidental summing of two precisely known gamma-ray energies. The two levels at 1172 and 1196 keV are believed to be phonon states since they are relatively strong in the (p,p') spectra and completely missing in the (d,p) spectrum. For this reason values for γ and μ were chosen to yield states near those energies and to closely approximate the positions of the well-established 0+ and 4+ two phonon levels. The resulting level scheme, obtained using $\gamma=20.8°$ and $\mu=0.8$ is compared with experiment in Fig. 8. It is interesting to

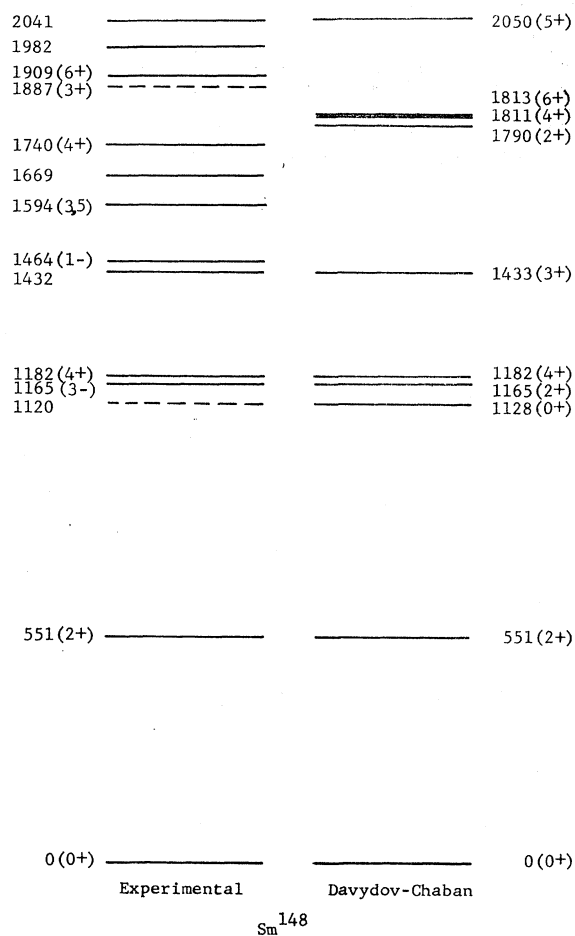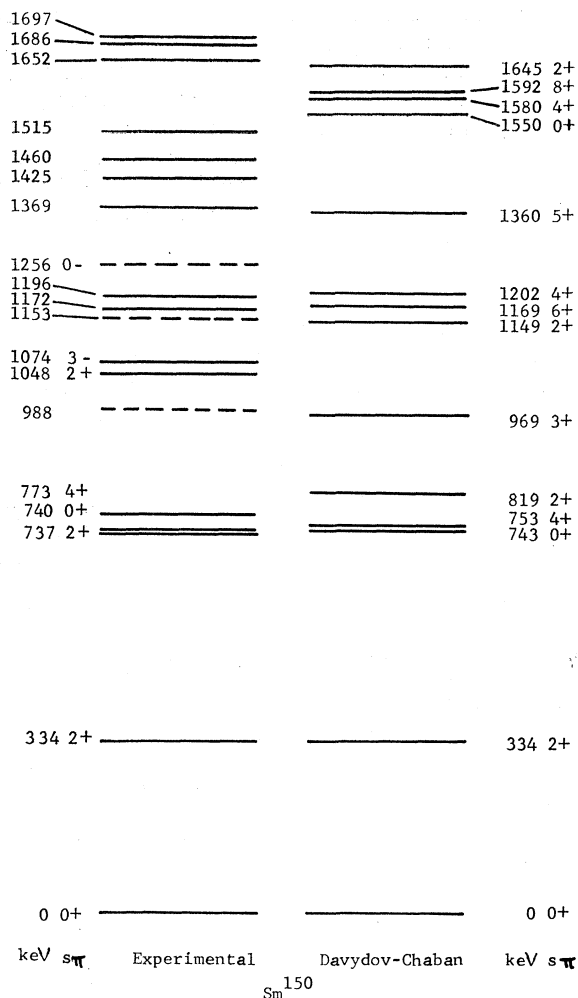FIG. 8. Comparison of positive parity levels predicted by Davydov and Chaban for $\gamma=20.8$ and $\mu=0.8$ with experimental Sm¹⁵⁰ levels. Dashed levels have been seen from other work, but were not observed in these experiments.

FIG. 7. Comparison of the positive parity levels of Sm¹⁴⁸ predicted by the Davydov-Chaban model with experiment. Spins and parities are taken from other experiments. For this spectrum $\gamma=22.4°$ and $\mu=0.94$. Only weak evidence for a state at 1120 keV was obtained. The 1887-keV level seen in the decay of Eu¹⁴⁸ was not observed in either (p,p') or (d,p) experiments.

note that the Davydov-Chaban model predicts that the 0+ and 3+ levels in the three phonon multiplet are unusually high and low, respectively. Qualitative considerations of (p,p') and (d,p) relative cross sections in the spectra of Sm¹⁴⁸ and Sm¹⁵⁰ indicate that the 1074 keV level is the 3− octupole state.[31a] The neighboring $N=88$ nucleus Gd¹⁵² has a known 3− level at 1124 keV. The smooth variation of the energy of 3− states with neutron number has previously been pointed out.[14]

The cross section of the 1434-keV level in Sm¹⁴⁸ is relatively high in the (p,p') spectra. This tends to

[31a] *Note added in proof.* A recent (n,γ) study of Sm¹⁵⁰ by L. V. Groshev *et al.* [Nucl. Phys. 43, 264 (1963)] places a 3− state at 1071 keV. This recent work also supports the possibility of 5+ assignment to the state at 1369 keV. A discrepancy of 8 keV for excitation energies between 1200 and 1700 keV exists between the (n,γ) studies and our reaction studies. There is substantial agreement on the excitation energies of a large number of additional levels up to 3 MeV.

support the hypothesis that this and other low-lying 1— states in the region $N>82$ are collective in nature.[32,33] These and other negative parity states can be accounted for by an extension of the asymmetric rotor model to include such octupole deformations.[34] It is somewhat anomalous that a state with appreciable (p,p') cross section, analogous to the 1464(1—) state in Sm^{148}, is not observed in the Sm^{150} spectrum at 1300–1500 keV excitation.

When considered in terms of the spherical surface oscillator model, the (d,p) stripping reaction is forbidden for phonon states.[35] This is due to the orthogonality of states of the harmonic oscillator, the target being in the oscillator ground state. In actuality the surface oscillator model is not an exact description and one must expect transitions to the higher phonon states. These are estimated to be reduced relative to the ground state by a fraction to a power equal to the number of phonons (e.g., if one phonon is $\frac{1}{10}$ of the ground state intensity the two phonon intensity would be 1/100). It is evident that these predictions are not

in agreement with the experimental results for Sm^{147}-$(d,p)Sm^{148}$ and $Sm^{149}(d,p)Sm^{150}$.

Collective spectra of even-even spherical nuclei have been recently considered in terms of the shell model plus a pairing and long-range force.[36–39] In this description the phonon states are made up of a superposition of many quasiparticle states and the stripping amplitudes depend upon this composition. It is possible, under certain circumstances, for the higher phonon states to be stronger than the ground state transition.[39] This is more in accord with these data than the previous theory.

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[32] C. J. Gallagher and V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 2, No. 2 (1962).
[33] M. Sakai, Phys. Letters 3, 338 (1963).
[34] P. O. Lipas and J. P. Davidson, Nucl. Phys. 26, 80 (1961).
[35] G. R. Satchler, Ann. Phys. 3, 275 (1958).

[36] L. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 32, No. 9 (1960).
[37] M. Baranger, Phys. Rev. 120, 957 (1960).
[38] S. Yoshida, Phys. Rev. 123, 2122 (1961).
[39] S. Yoshida, Nucl. Phys. 38, 380 (1962).