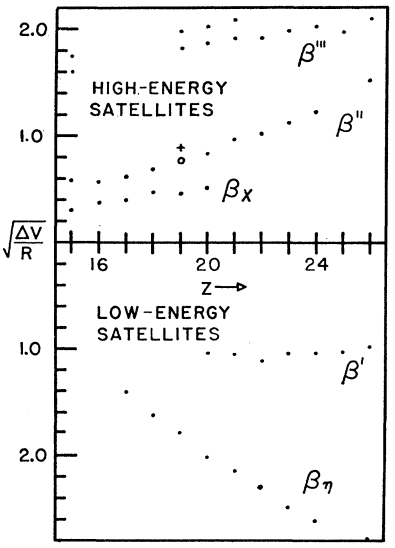

FIG. 8. A semi-Moseley diagram for the more prominent satellites appearing near $K\beta_{1,3}$ in the lighter elements.

fact that the absorption effects appear as quite small changes in the total cross section need not be inconsistent with the large double-vacancy yield apparently required by the present work.

C. $K\beta$ Satellite in Neighboring Elements

It is of some interest to consider the possible extension of the model of Sec. II to the neighboring elements and to see what suggestions empirical regularities in these spectra might make regarding K^+ $K\beta_5$.

In Fig. 8 are collected some data on the high-energy satellites in neighboring elements mostly from the summary of Sandstrom[29] except that some of the results for atomic numbers 17, 18, and 19 are taken from the present work. The energy separation ΔE of each satellite from its presumed parent ($K\beta_{1,3}$) was expressed in rydberg units. As is conventional, the square root of this separation as a function of Z is shown.

For this discussion the first and second high-energy structures are arbitrarily labeled β_x and β'', respectively, even though this conflicts with historical usage. An open circle has been placed for potassium where Jossem[26] has located a diffuse region of surplus intensity in the spectrum of metallic potassium. The point labeled with the cross is the position of K^+ $K\beta_5$ in KCl. The KCl data do not preclude a similar diffuse region to that for metallic potassium at the location of the open circle. Identification of K^+ $K\beta_5$ from KCl as the cross transition is favored by the apparent absence of a likely counterpart in neighboring spectra.

ACKNOWLEDGMENTS

The writer is indebted to Professor J. S. Levinger for several helpful discussions and to Takeshi Watanabe for numerous discussions and for checking several of the calculations. He also wishes to thank Dr. R. LaVilla, H. Steffen Peiser, and John W. Cooper for comments and suggestions.

[29] A. E. Sandstrom, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XXX.

PHYSICAL REVIEW VOLUME 133, NUMBER 2A 20 JANUARY 1964

Hall Effects and Magnetoresistance in Some Nickel-Copper-Iron Alloys*†

A. C. EHRLICH, J. A. DREESEN, AND E. M. PUGH
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received 29 July 1963)

The ordinary and extraordinary Hall coefficients and transverse magnetoresistance of six Ni-Cu-Fe alloys have been measured in the temperature range of 20°K to room temperature. All six alloys contain nominally 70 at.% Ni, with Cu and Fe additions to provide electrons concentrations from 27.7 to 28.2 electrons per atom. The ordinary Hall coefficient is found: (i) to be insensitive to electron concentration; (ii) to vary with temperature in the way predicted by the 4-band model; (iii) to have a magnitude consistent with the 4-band model if anisotropic electron scattering is assumed. The extraordinary Hall coefficient is positive for the alloys with 27.7, 27.8, 27.9, and 28.0 electrons per atom, and negative for the alloys with 28.1 and 28.2 electrons per atom. The transverse magnetoresistance shows the typical behavior of ferromagnetic alloys, except that the resistance does not vary linearly with the magnetic induction above saturation. The reduced ideal resistivity is found to be a universal function of a reduced temperature for these six alloys and Ni.

INTRODUCTION

THE Hall electric field per unit current density ϵ_H is given by

$$\epsilon_H = R_0 B + R_s M,$$

where B is the magnetic induction and M the magnetization in the material, and where R_0 and R_s are the ordinary and extraordinary Hall coefficients, respectively.

* This work was sponsored by the U. S. Army Research Office, Durham, North Carolina.

† Submitted by one of the authors (ACE) in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Carnegie Institute of Technology.

The ordinary Hall coefficients of high impurity Ni alloys can be understood on the basis of a 4-band model introduced by Pugh.[1] The central idea of this model is that the spin-up and spin-down d electrons in ferromagnetic metals should not be considered as being in the same band, since they have different occupation levels. Further, since s electrons with a given spin can be expected to interact preferentially with the d states of the same spin (e.g., s-d electron transitions involving a spin flip are probably less likely than those that do not involve a spin flip), the s electrons should also be divided into spin up and spin down subbands.

For a single spherical (not necessarily parabolic) band with isotropic scattering, R_0 is related to the number of conduction electrons per atom n by

$$n=-1/R_0Ne, \tag{1}$$

where N is the number of atoms per m^3 and e is the magnitude of the electronic charge. In a multiband model, if a conductivity and Hall coefficient can be defined for each band j, Pugh[1] has shown that the measured Hall coefficient R_0 can be related to the individual band conductivities and Hall coefficients R_{0j} by the equation

$$R_0=\sum_{j=1}^{\mu}\left(\frac{\sigma_j}{\sigma}\right)^2 R_{0j}, \tag{2}$$

where σ, the total conductivity, is

$$\sigma=\sum_{j=1}^{\mu}\sigma_j$$

and μ is the number of bands taking part in electrical conductivity. In a multiband metal it is generally convenient to define an effective number of conduction electrons n in analogy with Eq. (1) by

$$n^*=-1/R_0Ne. \tag{3}$$

If it is assumed that the bands of Ni and its alloys consist of two spherical bands of electrons (the two halves of the s band) and two spherical bands of holes (the two halves of the d band) and that these four subbands have isotropic relaxation times τ_j then Eqs. (1), (2), and (3) can be combined to give

$$\frac{1}{n^*}=\frac{1}{n_{sp}}\left(\frac{\sigma_{sp}}{\sigma}\right)^2+\frac{1}{n_{sa}}\left(\frac{\sigma_{sa}}{\sigma}\right)^2-\frac{1}{n_{dp}}\left(\frac{\sigma_{dp}}{\sigma}\right)^2-\frac{1}{n_{da}}\left(\frac{\sigma_{da}}{\sigma}\right)^2, \tag{4}$$

where n_{sp} is the number of s electrons whose spin magnetic moments are parallel to the magnetization, n_{da} the number of d holes in the antiparallel half of the d band, etc. Similarly, σ_{sp} is the conductivity of the electrons in the parallel half of the s band, etc.

In Ni and Ni alloys of the first transition series, the magnetization data[2] indicate that there are about 0.6 s electrons per atom. The gyromagnetic ratio data[3] suggest a (roughly) 10% orbital contribution to the magnetization. Therefore the number of s electrons n_s should be very nearly 0.54, if the number of holes n_{dp} in the lower energy half of the d band is assumed to be negligibly small. Equation (4) can explain values of n^* as small as 0.27, provided that the mobilities of the d holes and the antiparallel s electrons are assumed small compared to the mobility of the parallel s electrons. At temperatures far below the Curie temperature this assumption is reasonable.[4] As the temperature is raised toward the Curie temperature the distinction between the two halves of the d band vanishes, and therefore also the distinction between the s subbands, so that n^* should rise with temperature and extrapolate to 0.54 at the Curie point. This is exactly the behavior found in a considerable number of Ni alloys provided the alloy has about 25 or more atomic percent impurities.[5]

For these alloys, as the percentage of impurities in Ni drops below about 25%, n^* rises rapidly, and peaks at a value of about 1.4 at the pure metal. The dependence of n^* on impurity content has been found to be almost independent of the particular impurity[6] and the electron concentration.[5] Since the total impurity content appears to be significant, it would be interesting to investigate a series of ternary alloys, uncomplicated by change of total impurity content or phase, and with electron concentrations ranging over the anomalous peak in n^*. We have measured such a series of six alloys at six temperatures between 20°K and room temperature, each alloy consisting of about 70 at.% Ni, with the necessary amounts of Fe and Cu needed to obtain electron densities of 27.7, 27.8, $\cdots$, 28.2 electrons per atom. We have also determined the extraordinary Hall constant for these same alloys at the same temperatures.

In addition, the transverse magnetoresistance and resistivity of these six alloys have been measured at the same temperatures as the Hall coefficients.

EXPERIMENTAL METHODS

The samples used in this investigation were prepared by melting under vacuum in a pure Al_2O_3, crucible Ni and Fe with a reported purity of 99.998% and Cu with a reported purity of 99.999%. The Ni and Fe, originally in the form of sponge, were pressed into pellets prior to melting. The resulting ingots were cold rolled and machined into wire, probes, and samples, which were

[1] E. M. Pugh, Phys. Rev. **97**, 647 (1955).

[2] Richard M. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1951), p. 156.

[3] G. G. Scott, Rev. Mod. Phys. **34**, 102 (1962).

[4] N. F. Mott, Proc. Roy. Soc. (London) **A153**, 699 (1936).

[5] E. R. Sanford, A. C. Ehrlich, and E. M. Pugh, Phys. Rev. **123**, 1947 (1961).

[6] P. Cohen, Office of Naval Research Technical Report, June 1955 (unpublished); Thesis, Carnegie Institute of Technology (unpublished); F. E. Allison and E. M. Pugh, Phys. Rev. **102**, 1281 (1956); S. Foner and E. M. Pugh, Phys. Rev. **91**, 20 (1953).

then annealed in vacuum for approximately 12 h at 925°C and cooled to room temperature over a period of 12 h at a rate not exceeding 2 deg/min. The physical dimensions of the samples were nominally 1 cm wide, 10 cm long, and 1 mm thick. The methods for measuring sample dimensions and the sample mounting and Hall voltage measurement procedures have been described elsewhere.[5]

The transverse magnetoresistance and resistivity data were taken using a Wenner potentiometer and a Leeds and Northrup dc null detector. A method analogous to the incremental method[7] was used.

The Hall, magnetoresistance, and resistivity data were taken at six temperatures. The samples were in direct contact with the baths of liquid hydrogen (20°K), liquid nitrogen (77°K), liquid methane (112°K), liquid ethylene (169°K), liquid propane (231°K), and a high thermal conductivity silicon oil (room temperature).

DATA ANALYSIS

The Hall coefficients were obtained by a least-square fitting of the data for the magnetically saturated points to a straight line of the form

$$\epsilon_H = R_0 B + R_s M_0,$$

where M_0 is the saturation magnetization. The magnitude of B required for saturating the Hall effect could be obtained by visual inspection of graphs of ϵ_H plotted against B. It is much more difficult to saturate the magnetoresistance than the Hall effect since the greatest change in the resistivity as a function of magnetic induction occurs beyond the knee of the magnetization curve where changes in magnetization are due primarily to domain rotation.[8] As a consequence, in fitting the magnetoresistance data for saturated points to a quadratic in B (see below) it was sometimes necessary to discard points for a given B as being unsaturated even though the same value of B was sufficient to saturate the Hall effect in the same sample at the same temperature.

EXPERIMENTAL RESULTS

Measurements were made on six ternary Ni-Cu-Fe alloys all of which contained approximately 30 at.% impurities, where both Cu and Fe are counted as impurities in the Ni. The exact composition of the samples together with the Hall coefficients, resistivity, and the saturation magnetizations are given in Table I. The saturation magnetizations given were used to determine R_s and are based on measurements of these alloys made by one of us (JAD) at 77°K. At other temperatures, the magnetizations were estimated by assuming that their temperature dependence is the same as that of Ni on a reduced magnetization versus reduced temperature graph. The estimated errors in R_0 (and n^*) are about 4% for 20°K, 77°K, and room-temperature determinations and 6% for the other three temperatures at which data were obtained. Allowing for errors in the values of M_0, the values of R_s we have obtained should be accurate to within 6% at all temperatures. The estimated error in the resistivity is 2% and is due primarily to uncertainties in the measurement of sample dimensions and the spacing of the resistivity probes.

The transverse magnetoresistance data are presented and discussed in terms of $\delta\rho(B)/\rho(0)$, defined as

$$\delta\rho(B)/\rho(0) = [\rho(B) - \rho(0)]/\rho(0),$$

where $\rho(B)$ is the resistivity in the field B. In Fig. 1

TABLE I. Summary of Hall coefficients, resistivity, and magnetic saturation data. Compositions are given in atomic percent.

T (°K)	$-R_0\times10^{11}$ (m³/C)	$-R_s\times10^{11}$ (m³/C)	$(\rho\times10^8)$[a] (ohm-m)	n^* (el./at.)	M_0 (Wb/m²)
69.1% Ni-10.7% Cu-20.2% Fe (Sample I—27.71 el./at.)					
20	30.54	−16.59	9.295	0.227	1.00
77	28.18	−20.20	10.19	0.246	1.00
112	26.36	−24.41	11.32	0.264	0.996
169	24.04	−32.15	13.78	0.290	0.989
231	22.16	−44.57	17.22	0.315	0.981
300	21.07	−68.12	22.19(303)	0.332	0.960
69.4% Ni-14.1% Cu-16.5% Fe (Sample II—27.81 el./at.)					
20	28.24	−25.63	12.17	0.249	0.864
77	26.38	−29.32	13.15	0.267	0.860
112	25.03	−33.90	14.32	0.282	0.857
169	22.91	−42.15	16.93	0.308	0.850
231	21.42	−55.56	20.55	0.331	0.841
305	20.13	−78.66	25.97(308)	0.352	0.821
69.1% Ni-17.7% Cu-13.2% Fe (Sample III—27.91 el./at.)					
20	29.59	−22.26	14.05	0.237	0.744
77	27.24	−25.96	15.07	0.257	0.740
112	25.71	−29.84	16.30	0.273	0.738
169	23.92	−37.79	18.87	0.294	0.731
231	21.52	−47.25	22.46	0.327	0.719
303	19.82	−62.68	27.92(307)	0.356	0.705
70.2% Ni-19.8% Cu-10.0% Fe (Sample IV—28.00 el./at.)					
20	28.19	−25.94	15.40	0.249	0.629
77	26.08	−28.93	16.49	0.269	0.625
112	24.85	−32.63	17.80	0.283	0.623
169	22.71	−39.61	20.57	0.310	0.617
231	20.63	−47.38	24.25	0.342	0.607
305	18.83	−63.98	29.90	0.375	0.594
72.0% Ni-21.1% Cu-6.9% Fe (Sample V—28.07 el./at.)					
20	28.52	40.18	17.15	0.247	0.539
77	25.99	45.70	18.35	0.271	0.535
112	24.23	51.21	19.83	0.291	0.533
169	21.83	63.98	22.42	0.323	0.527
231	19.68	81.65	26.14	0.359	0.517
301	17.40	101.3	31.69(308)	0.407	0.507
71.1% Ni-25.6% Cu-3.3% Fe (Sample VI—28.19 el./at.)					
20	24.65	143.8	20.64	0.285	0.393
77	22.50	162.5	22.06	0.313	0.390
112	20.86	180.2	23.74	0.338	0.387
169	18.69	215.8	26.74	0.377	0.380
231	15.97	260.4	30.80	0.447	0.371
305	15.07	287.0	36.45(307)	0.470	0.354

[a] The numbers in parentheses to the right of the room-temperature resistivities are the temperatures at which those resistivities were measured.

[7] S. Foner and E. M. Pugh, Phys. Rev. **91**, 20 (1953).

[8] Richard M. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1951), p. 745.

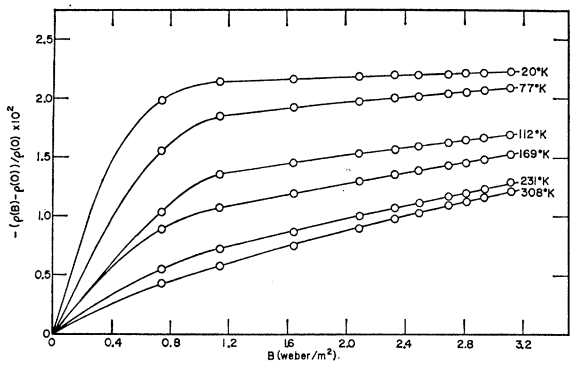

FIG. 1. Transverse magnetoresistance as a function of magnetic induction for an alloy of 69.4% Ni-14.1% Cu-16.2% Fe at the indicated temperatures. (Sample II—27.81 el./at.)

curves of $-\delta\rho(B)/\rho(0)$ as a function of B at the six temperatures studied are shown for sample II. While the general shape of the curves and the magnitudes of $\delta\rho(B)/\rho(0)$ are typical of the six alloys studied, the temperature dependence is not always so regular. Further, in contrast to most of the work reported[9] these Ni-Cu-Fe alloys do not show a linear variation of $\delta\rho(B)/\rho(0)$ with B above magnetic saturation. For these alloys there is usually a small but definite curvature. In particular, the experimental results for $\delta\rho(B)/\rho(0)$ above magnetic saturation can be fit very well by an expression of the form

$$-\delta\rho(B)/\rho(0)=\alpha+\beta B+\gamma B^2, \qquad (5)$$

where α, β, and γ are constants. The constants were determined by a least-square fit of the data, and the results are shown in Table II. Equation (5) will usually reproduce the experimentally determined values of $\delta\rho(B)/\rho(0)$ to within 0.1% and always to within 0.4%. The two positive values of γ should be considered as zero, since their magnitudes are within one standard deviation of zero.

Because a true zero applied field cannot be conveniently obtained with the magnet control system presently available in this laboratory, there may be an error as great as 10% in $\delta\rho(B)/\rho(0)$. The slope of the curve is not affected by this, however, and is known quite accurately.

DISCUSSION

Ordinary Effect

The data we have obtained can be interpreted quite well by Pugh's 4-band model. In its simplest form [Eq. (4)] this model can give values of n^* as small as, but no smaller than $n_s/2$, where n_s is the number of s electrons per atom. This requires the rather stringent condition that there be no contribution to the Hall effect from any electrons other than those s electrons whose spins are parallel to the magnetic field. In fact,

[9] H. C. Van Elst, Physica **25**, 708 (1959).

TABLE II. Results of a least-squares fit of the magnetoresistance to the quadratic in B $-\delta\rho(B)/\rho(0)=\alpha+\beta B+\gamma B^2$. Compositions of the samples are given in Table I.

Sample	T (°K)	$\alpha\times10^4$	$\beta\times10^5$ (m²/Wb)	$-\gamma\times10^6$ (m⁴/Wb²)
I	20	183	70	102
	77	199	132	64
	112	209	196	93
	169	185	243	57
	231	145	296	80
	303	65	322	55
II	20	210	43	2
	77	168	156	87
	112	114	206	86
	169	78	286	92
	231	38	308	63
	308	18	364	100
III	20	361	78	65
	77	276	176	94
	112	188	252	142
	169	121	299	105
	231	60	362	130
	307	40	399	98
IV	20	146	78	34
	77	113	174	62
	112	74	221	55
	169	48	310	111
	231	33	381	146
	305	78	431	140
V	20	226	65	−8
	77	185	190	94
	112	164	231	62
	169	81	331	145
	231	59	391	142
	308	43	487	200
VI	20	130	81	−16
	77	88	192	23
	112	76	276	90
	169	32	429	288
	231	34	455	176
	307	11	594	271

we have for samples I through V values of n^* at 0°K (extrapolated) that are roughly 20% smaller than $n_s/2$ (0.27). We consider, therefore, the assumptions upon which Eq. (4) is based.

The assumption that the d band is spherical is certainly not accurate.[10,11] This plays no part in the discussion, however, since our experimental results indicate no contribution to the Hall effect from the d band. To the contrary, the s band, hardly more than 25% full, is likely to be spherical, or very nearly spherical. The relaxation time was taken to be isotropic, but there are no compelling reasons to think it is. Cooper and Raimes[12,13] have shown that a spherical Fermi surface and an anisotropic relaxation time will make n^* smaller than the number of conduction electrons. It appears then that the current is carried primarily by one-half of

[10] B. Segall, Phys. Rev. **125**, 109 (1962).
[11] J. H. Wood, Phys. Rev. **126**, 517 (1962).
[12] J. Cooper and S. Raimes, Phil. Mag. **4**, 145 (1959).
[13] J. Cooper and S. Raimes, Phil. Mag. **4**, 1149 (1959).

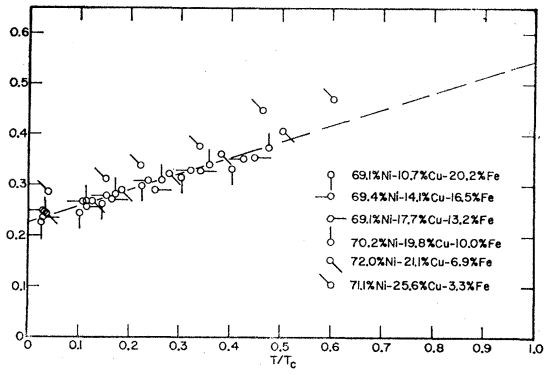

FIG. 2. The effective number of conduction electrons per atom as obtained from Hall data versus the reduced temperature.

the s electrons in a spherical band with anisotropic scattering time.

The temperature dependence of n^* for each of the samples investigated follows very well the predictions of the 4-band model. In Fig. 2 we have plotted n^* against T/T_c. n^* is seen to rise with temperature, and to extrapolate to values very nearly those of n_s at T/T_c.

Extraordinary Effect

A number of theoretical discussions of the extraordinary Hall coefficient have been given.[14–20] The theories usually suggest a relationship between R_s and the resistivity ρ. Logarithmic plots of R_s versus ρ for various pure metals and alloys have usually given straight lines, particularly for the higher resistivities. For the rather highly resistive alloy system studied here, plots of R_s versus ρ, where ρ is varied by changing the temperature or the alloy composition, might be expected to give straight lines. If such plots are made for the alloys investigated in this study by changing ρ for a given sample by varying the temperature, only samples I and II yield a good straight line (slopes of 1.56 and 1.45). The other four samples deviate from linear behavior at the higher temperatures.

From Table I it is clear that similar plots of R_s versus ρ as the alloy composition changes at a given temperature would not be illuminating. R_s is positive and hardly changes for samples I–IV, although the resistivity increases monotonically. Further, R_s changes sign abruptly at sample V and becomes quite large for sample VI even though the resistivity continues to increase at the same rate.

[14] R. Karplus and J. M. Luttinger, Phys. Rev. **95**, 1154 (1954).
[15] J. Smit, Physica **21**, 877 (1955).
[16] J. Smit, Physica **24**, 39 (1958).
[17] J. M. Luttinger, Phys. Rev. **112**, 739 (1958).
[18] C. Strachan and A. M. Murray, Proc. Phys. Soc. (London) **73**, 433 (1959).
[19] J. Kondo, Progr. Theoret. Phys. (Kyoto) **27**, 772 (1962).
[20] L. Berger, U. S. Office of Naval Research, Contract Nonr—760 (05) NR 018–301, Technical Report No. 15, 1962 (unpublished); see also L. Berger, J. Appl. Phys. **34**, 1360 (1963).

It has been pointed out by Berger[20] that R_sM_0 may be of greater physical significance than R_s alone. The argument is that R_sM_0 is the experimentally measured quantity and depends on the electrons at the Fermi surface. Division by M_0 is undesirable since it depends on all the electrons of the d band, and is therefore foreign to the transport properties. Further, assuming the Karplus and Luttinger[14] theory of R_s is correct, the dependence of R_sM_0 on the resistivity can be eliminated by considering the quantity γ_s, where

$$\gamma_s = R_sM_0/\rho^2.$$

In Fig. 3, γ_s has been plotted against electron concentration for the liquid hydrogen and room temperature data. The 169°K results are also plotted as representative of the four other temperatures, all of which fall between the hydrogen and room-temperature data. It can be seen that γ_s does vary with composition in a somewhat regular way.

Resistivity

The resistivities of the Ni-rich members of the Ni-Cu-Fe alloy system display a number of interesting features. For the samples investigated in this work, Table I shows that the resistivity increases monotonically with the Cu content of the samples at a given temperature, even though the total impurity content (Cu plus Fe) remains constant. This is not surprising

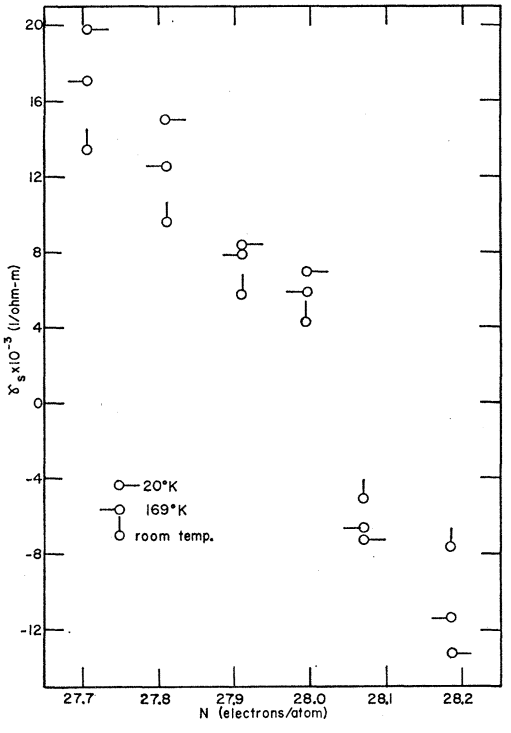

FIG. 3. Extraordinary Hall conductivity versus electron concentration per atom at the indicated temperatures.

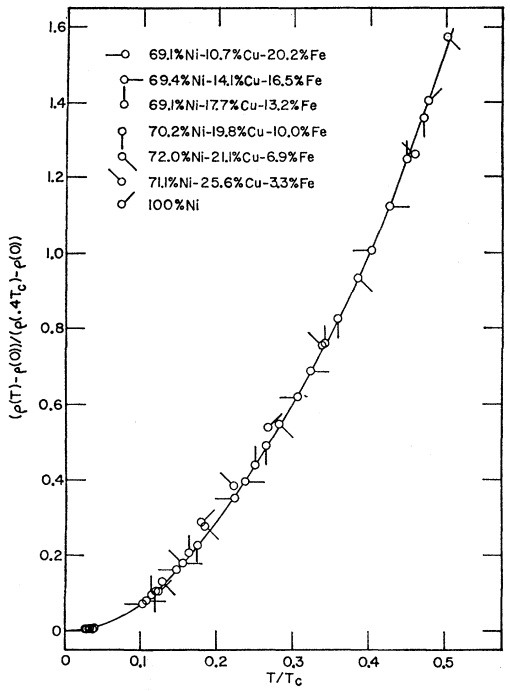

FIG. 4. Reduced resistivity versus reduced temperature for Ni and 6 Ni-Cu-Fe alloys.

in view of the fact that in the binary Ni-Cu and Ni-Fe alloys a given atomic fraction of Cu in Ni will cause a much greater resistivity than the same fraction of Fe in Ni.

Friedel[21] has given an explanation for this based on the idea that an impurity with atomic number greater than the host lattice (e.g., Cu in Ni) cannot be shielded as well as an impurity with atomic number smaller than the host lattice (e.g., Fe or Co in Ni) if the electrons that do the shielding belong to a band that is almost full. This is the case for Ni alloys where the shielding is done primarily by the d electrons.

Somewhat surprising is the fact that replacing Ni with Fe in a Ni-rich Ni-Cu alloy hardly affects the resistivity, or if anything, actually reduces it. For example: at 300°K a 20% Cu-0% Fe-80% Ni alloy has the same resistivity as a 21.2% Cu-9.6% Fe-69.2% Ni alloy. Similarly, 14% Cu-0% Fe, 14% Cu-6% Fe, and 15.2% Cu-13.4% Fe alloys (the remaining percentage of each being Ni) all have essentially the same resistivity at 300°K. It may be that increasing the number of d-band holes (by adding Fe), which improves the shielding of the Cu ions, is more than enough to compensate for the additional Fe scattering centers.

Mannevy-Tassy[22] has shown experimentally that on a reduced resistivity plot, i.e., $\rho(T)/\rho(T_c)$ versus T/T_c, Fe and Ni fall on the same curve from low temperatures to above the Curie point. That is, $\rho(T)/\rho(T_c)=F(T/T_c)$ only. It is easy to show that such a relation implies that $\rho(T)/\rho(\alpha T_c)=F'(T/T_c)$, where α is a number (which we will take as 0.4 in order to be able to use the available data), and F' is related to F by a numerical multiplying factor.

We have tried to extend this empirical relation, which has no present theoretical justification, to the alloys measured here. The residual resistivity ρ_0 should first be substracted from the measured values. We have therefore plotted in Fig. 4 $(\rho(T)-\rho_0)/(\rho(0.4T_c)-\rho_0)$ versus T/T_c, where ρ_0 is the residual resistivity. We include in Fig. 4 the Ni data of Dreesen and Pugh.[23] It is remarkable that all the data, including the Ni data, fall on the same curve. The same points on a logarithmic scale are given in Fig. 5. The points fit quite well a straight line (slope of 1.8) except at the lowest resistivities where a small error in the estimate of ρ_0 could have an appreciable effect on the values of the ordinate.

Magnetoresistance

The interpretation of our magnetoresistance data is somewhat complicated by the fact that $\delta\rho(B)/\rho(0)$ does not show a linear variation with B above magnetic saturation. The constant negative slope usually found in ferromagnetic alloys is associated with electron-magnon scatterings.[24,25] As the magnetic induction is

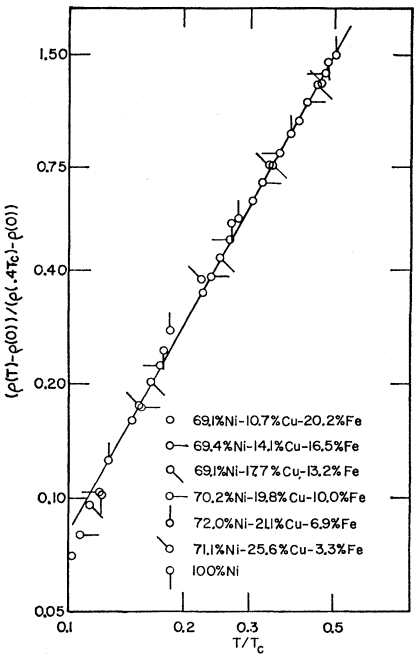

FIG. 5. Reduced resistivity versus reduced temperature for Ni and 6 Ni-Cu-Fe alloys.

[21] J. Friedel, Advan. Phys. **3**, 446 (1954).
[22] G. Mannevy-Tassy, Compt. Rend. **230**, 1150 (1950).
[23] J. A. Dreesen and E. M. Pugh, Phys. Rev. **120**, 1218 (1960).
[24] E. I. Kondorskii, O. S. Galkina, and L. A. Chernikova, Zh. Eksperim. i Teor. Fiz. **35**, 1070 (1958) [translation: Soviet Phys.—JETP **7**, 741 (1958)].
[25] O. S. Galkina and L. A. Chernikova, Zh. Eksperim. i Teor. Fiz. **38**, 3 (1960) [translation: Soviet Phys.—JETP **11**, 1 (1960)].

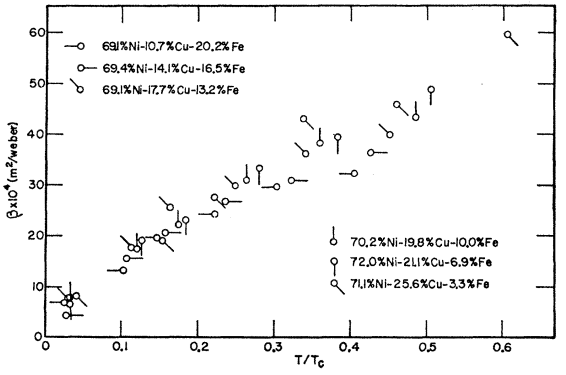

FIG. 6. The linear part of the saturated magnetoresistance versus the reduced temperature.

raised at a given temperature, the number of magnons is decreased and therefore the number of electron-magnon scatterings is decreased. Consequently the electrical resistivity is lowered. Furthermore, since there are no magnons at 0°K there should be no effect on the resistivity of increasing B above magnetic saturation, i.e., the slope of $\delta\rho(B)/\rho(0)$ should extrapolate to zero at 0°K.

It might be assumed for example, that the small positive curvature of $\delta\rho(B)/\rho(0)$ that we have observed is a superposition on the ferromagnetic effects of the usual increase in resistivity with magnetic induction found in nonferromagnetic metals. For such an assumption, the coefficient β in Eq. (5) should be associated with the constant slope of $\delta\rho(B)/\rho(0)$ usually found. While the temperature dependence of γ in Eq. (3) does not support such an assumption, since $\beta \gg \gamma$ it should be a good approximation to do this. Further, the values of β listed in Table II are consistent with the slopes found by other investigators for other alloys.[9]

According to theory, then, β should approach zero as the temperature goes to 0°K. In Fig. 6, β versus T/T_c has been plotted for the six alloys investigated. Since the variation of β with temperature is generally found to be strongest below 20°K, it appears that β does go to zero at 0°K for the six alloys investigated in this work.

Generally, in a ferromagnetic alloy, as the magnetization rises from zero to its saturation value, measurements show that the resistivity rises rapidly when the magnetic field is parallel to the current, but falls rapidly when the field is perpendicular to the current. The difference in the resistivity for these two cases divided by the resistivity in zero magnetic field is called the ferromagnetic anisotropy of resistivity and is defined by

$$\Delta\rho/\rho(0)=(\rho_{11}-\rho_{1})/\rho(0),$$

where ρ_{11} and ρ_{1} are, respectively, the resistivities measured parallel and perpendicular to the saturation magnetization.

A theory of this effect has been given by Smit.[26] It depends on the spin-orbit interaction operator which mixes the spin up $3d$ states with the spin down $3d$ states in a way not symmetrical in real space. This leads to an increase or decrease (depending on the direction of magnetization relative to the current direction) in the number of s-d electron scatterings and thus decreases or increases the electrical resistivity.

Although we have not measured the longitudinal magnetoresistance, we can estimate $\Delta\rho/\rho(0)$ by making use of the approximation[27] that $(\rho(B)-\rho(0))/\rho(0)$ for B parallel to the current is twice $-(\rho(B)-\rho(0))/\rho(0)$ for B perpendicular to the current when both these quotients are extrapolated to $B=0$ from the magnetically saturated points. For the 20°K data, where both β and γ of Eq. (5) are very small, this gives

$$\Delta\rho/\rho(0)\cong 3\alpha,$$

where α is the constant in Eq. (5). From Table II it can be seen that the values of 3α at 20°K vary from about 4×10^{-2} to 10×10^{-2}. These are typical of the values for $\Delta\rho/\rho(0)$ found in other ferromagnetic Ni alloys.[24]

Berger[21] has presented a theory which connects $\Delta\rho/\rho(0)$ with R_s when there is an intersection (or near intersection) of branches of the $3d$ band at the Fermi surface. If the Fermi level is moved in an alloy system by changing the electron concentration, Berger's theory predicts the occurrence of a strong maximum in $\Delta\rho/\rho(0)$ and a change of sign in R_s as the Fermi level passes through an intersection of branches. Our data, however, show no correlation between $\Delta\rho/\rho(0)$ and any of the other effects measured in this alloy system.

ACKNOWLEDGMENTS

We wish to thank the various members of the Department of Physics at the Carnegie Institute of Technology who have contributed helpful advice and discussion, particularly Dr. L. Berger. We are grateful also to the Department of Metallurgy and the Computation Center for the use of their facilities.

[26] J. Smit, Physica **17**, 612 (1951).
[27] J. P. Jan, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1957), Vol. 5, p. 1.