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Empirical Rules for Predicting Ground-State Spins of Light Nuclei

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Empirical rules are given which predict the ground-state spin of light nuclei. Possible theoretical justifications for the rules are discussed.

I. INTRODUCTION

THE ground-state spins of nuclei are well described by the individual particle shell model (IPSM) of the nucleus.[1] It has been shown by Racah[2] and by Edmonds and Flowers[3] that for a wide range of reasonable attractive residual two-body interactions among like particles the extra core nucleons should couple in pairs with total angular momentum zero. This yields the prediction of zero ground-state spin for even-even nuclei and predicts that the ground-state spins of even-odd nuclei are equal to the spin of the last unpaired nucleon. Indeed, these rules agree well with the experimental measurements except for some light even-odd nuclei where the ground-state spin J is given by $J=J_1-1$, where J_1 is the spin of the unpaired nucleon. Rules for the ground-state spins of odd-odd nuclei were given by Nordheim[4] in 1950. These rules were later modified by Brennan and Bernstein.[5] It was shown by de-Shalit and Walecka that in the odd group model[6] the rules of Brennan and Bernstein were valid for a broad field of attractive residual internucleon interactions. The predictions of Brennan and Bernstein agree well with measured odd-odd nuclei spins for nuclei with atomic numbers A in the range $20<A<120$. Moreover, their model gives a prediction for the magnetic moment which agrees well with measured values. Gallagher and Moszkowski[7] have found excellent agreement between ground-state spin predictions for odd-odd nuclei and experimental spins by use of coupling rules derived from the collective model.

In this paper, we give rules for predicting ground-state spins of nuclei with $A<26$ and point out certain regularities in the ground-state spins of light nuclei.

II. PROPOSED COUPLING RULES

The rules which we are proposing consider the coupling of groups of nucleons, namely, all protons outside of the closed shell for odd-Z nuclei, or all neutrons outside of the closed shell for odd-N nuclei, instead of only the last nucleon[1]; and all particles or holes outside of the closed shells for odd-odd nuclei, instead of the last two particles.[4,5,7] The angular momenta of all the considered particles are coupled in the L-S scheme as

TABLE I. Ground-state spins and magnetic moments of light nuclei.

Isotope	Observed[a] ground-state spin	Predicted ground-state spin	Magnetic Moments Experimental[b]	Magnetic Moments Schmidt value	Present calculations
H^2	1	1	0.857		0.88
H^3	$\frac{1}{2}$	$\frac{1}{2}$	2.98	2.79	2.79
He^3	$\frac{1}{2}$	$\frac{1}{2}$	−2.13	−1.91	−1.91
He^5	$\frac{3}{2}$	$\frac{3}{2}$			
Li^5	$\frac{3}{2}$	$\frac{3}{2}$			
Li^6	1	1	.82		0.88
Li^7	$\frac{3}{2}$	$\frac{3}{2}$	3.26	3.79	2.79
Li^8	2	2	±1.65		−2.94
Be^7	$\frac{3}{2}$	$\frac{3}{2}$			
Be^9	$\frac{3}{2}$	$\frac{3}{2}$	−1.18	−1.91	−5.73
B^8	2	2			
B^9	$>\frac{1}{2}$	$\frac{3}{2}$			
B^{10}	3	3	1.80		2.64
B^{11}	$\frac{3}{2}$	$\frac{3}{2}$	2.69	3.79	8.37
B^{12}	1	2			
C^{11}	$\frac{3}{2}$	$\frac{3}{2}$			
C^{13}	$\frac{1}{2}$	$\frac{1}{2}$	0.702	0.637	−1.91
C^{15}	$\frac{1}{2}$	$\frac{1}{2}$			
N^{12}	1	2			
N^{13}	$\frac{1}{2}$	$\frac{1}{2}$			
N^{14}	1	1	0.403		0.88
N^{15}	$\frac{1}{2}$	$\frac{1}{2}$	−0.283	−0.263	−0.263
N^{16}	2	2			
O^{15}	$\frac{1}{2}$	$\frac{1}{2}$			
O^{17}	$\frac{5}{2}$	$\frac{5}{2}$	−1.89	−1.91	−1.91
O^{19}	$(\frac{5}{2},\frac{3}{2})$	$\frac{3}{2}$			
F^{17}	$\frac{5}{2}$	$\frac{5}{2}$			
F^{18}	1	1			
F^{19}	$\frac{1}{2}$	$\frac{1}{2}$	2.63	2.79	2.79
F^{20}	2	2			
Ne^{19}	$\frac{1}{2}$	$\frac{1}{2}$			
Ne^{21}	$\frac{3}{2}$	$\frac{3}{2}$			
Ne^{23}	$\frac{5}{2}$	$\frac{5}{2}$			
Na^{21}	$\frac{3}{2}$	$\frac{3}{2}$			
Na^{22}	3	3	1.75		2.64
Na^{23}	$\frac{3}{2}$	$\frac{3}{2}$	2.22	4.79	8.37
Na^{24}	4	4			
Na^{25}	$\frac{5}{2}$	$\frac{3}{2}$			
Mg^{23}	$\frac{3}{2}$	$\frac{3}{2}$			
Mg^{25}	$\frac{5}{2}$	$\frac{5}{2}$	0.854	1.91	9.55
Al^{24}	4	4			
Al^{25}	$\frac{5}{2}$	$\frac{5}{2}$			
Al^{26}	5	5			
Al^{27}	$\frac{5}{2}$	$\frac{5}{2}$	3.64	4.79	16.74

[a] *Landolt-Bornstein Tables*, New Series Volume (Springer-Verlag, Berlin, 1961).
[b] U. S. Atomic Energy Commission 1960 Nuclear Data Tables, Part 4, p. 74; G. Laukien *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), vol. 38, p. 338.

[1] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons Inc., New York, 1955).
[2] G. Racah, *L. Farkas Memorial Volume* (Interscience Publishers, Inc., New York, 1952), p. 294.
[3] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A214**, 515 (1952).
[4] L. W. Nordheim, Phys. Rev. **78**, 294 (1950); Rev. Mod. Phys. **23**, 322 (1951).
[5] M. H. Brennan and A. M. Bernstein, Phys. Rev. **120**, 927 (1960).
[6] A. de-Shalit and J. D. Walecka, Nucl. Phys. **22**, 184 (1960).
[7] C. J. Gallagher and S. A. Moszkowski, Phys. Rev. **111**, 1182 (1958).

follows: the orbital angular momenta are coupled to the lowest possible value, $L_{\min}$; the spins are coupled to the maximum possible value, $S_{\max}$; naturally, the restrictions imposed by the Pauli principle must be taken into account.

If the shell is more than half full, the ground-state spin J is given by $J=|S_{\max}-L_{\min}|$; if the shell is less than half full, $J=|S_{\max}+L_{\min}|$.

In general, for odd-odd nuclei $L_{\min}=0$, so $J=S_{\max}$. The only known odd-odd nucleus with $L_{\min}\neq 0$ in the region considered is N^{16}.

For light nuclei there is little possibility of confusion in the calculation of $L_{\min}$ due to uncertainties in the single particle level order. For the nuclei that we are dealing with, $A<26$, the only ambiguity occurs when one has a single particle in the *s-d* shell. When this occurs, we look at experiments that tell us the orbital spin of the extra core particle.

III. DISCUSSION AND THEORETICAL CONSIDERATIONS

The coupling rules correctly predict the known ground-state spins of all nuclei with $A<26$, with the exceptions of B^{12}, N^{12}, and Na^{25}. Table I gives the known ground-state spins of odd A and odd-odd nuclei with $A<26$, along with our predictions.

For one particle outside of a closed shell these rules predict the same ground-state spins as the single particle shell model. As can be seen, the predictions of the ground-state spins are the same as those of the IPSM for odd A nuclei throughout the p shell. In the *s-d* shell, however, we predict that three particles outside the core would yield a ground-state spin of $\frac{3}{2}$ as is observed in Na^{21}, Na^{23}, O^{19}, and Ne^{21}. In the IPSM, these three particles would be in the $d_{5/2}$ orbit and the ground-state spin prediction would be $\frac{5}{2}$.

For $A<26$, the only known cases where $L_{\min}\neq 0$ are when one has either a particle or a hole outside of a closed shell and as mentioned, N^{16}. For odd-A nuclei and $L_{\min}\neq 0$, our rules reduce to the predictions of the IPSM. In other cases our rules reduce to the single rule $J=|S_{\max}|$.

Of the possible ground states of an even number of identical particles outside of a closed shell, the short-range pairing interaction lowers the energy of the states with seniority zero; thus the total angular momentum of an even number of identical particles would be zero in the ground state. The coupling rules that we have proposed would be expected to hold if the residual interparticle interaction had a spin dependent and spin-independent short-range force of the type suggested by Schwartz,[8] $V(\mathbf{r}_i,\mathbf{r}_j)=V_0(|\mathbf{r}_i-\mathbf{r}_j|)[1-\alpha+\alpha(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)]$, where V_0 is an attractive potential and α is positive. It is this interaction taken between the last two odd nucleons that gives rise to the Brennan-Bernstein amendment to Nordheim's rule which is successful in predicting the ground state spins of heavier odd-odd nuclei. The short-range part of the interaction tends to couple the orbital angular momenta to a minimum and the exchange part tends to align the intrinsic angular momenta.

Our coupling rules fail badly if one tries to apply them to nuclei with $A>26$. This is probably due to an increase in the effect of the spin orbit force which removes one from *L-S* coupling.

If one assumes that the ground state consist entirely of the *L-S* configuration that the coupling rules predict, then one gets a prediction of the ground-state magnetic moment. No claim is made to the effect that the actual ground state mostly consist of the considered configuration. The amount of admixture can be gauged by comparing the magnetic moments given by considered configuration with the experimentally observed values. These are given in Table I. From the discrepancies in the middle of the shells it is obvious that the ground state actually contains contributions from many configurations.

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[8] C. Schwartz, Phys. Rev. 94, 95 (1954).