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Decay Scheme of Fm^{255}

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The alpha decay scheme of Fm^{255} has been investigated by alpha particle, gamma ray, and electron spectroscopy. Nine alpha groups were observed with alpha-particle energies and abundances of: 7.122, $(0.09\pm0.01)\%$; 7.098, $(0.10\pm0.01)\%$; 7.076, $(0.43\pm0.05)\%$; 7.019, (93)%; 6.977, $(0.11\pm0.02)\%$; 6.960, $(5.3\pm0.1)\%$; 6.887, $(0.60\pm0.03)\%$; 6.803, $(0.12\pm0.02)\%$; and 6.58 MeV, $(4.5\pm0.9)\times10^{-2}\%$. The ground state of Cf^{251} was assigned a spin of $\frac{1}{2}$ and is probably the $\frac{1}{2}+[620\uparrow]$ Nilsson state. Five members of the rotational band based on the ground state were observed. The value of the decoupling parameter a, is 0.24 ± 0.03, and $\hbar^2/2\mathfrak{J}$ has a value of 6.4 ± 0.3. The ground state of Fm^{255} decays by an unhindered alpha transition to a 106-keV state in Cf^{251}. Both of these states are assigned a spin of $\frac{7}{2}$ and are presumably the $\frac{7}{2}+[613\uparrow]$ Nilsson state. Four members of the rotational band based on the 106-keV state in Cf^{251} were observed, and their energies correspond to a value of $\hbar^2/2\mathfrak{J}$ of 6.69 ± 0.03. The 106-keV state with the $\frac{7}{2}+[613\uparrow]$ assignment has a half-life of $(3.7\pm0.2)\times10^{-8}$ sec. The gamma rays which de-excite this state to the ground-state band are thought to originate through a multiple Coriolis interaction between two states 0.56 keV apart. Two other intrinsic levels in Cf^{251} were detected. One at 546 keV was given the assignment of $11/2-[725\uparrow]$ and the other, tentatively at 425 keV, was given the assignment $\frac{9}{2}+[615\downarrow]$. The appropriate alpha transition probabilities for the decay to the various states agree well with the expectations for unhindered and hindered alpha decay.

I. INTRODUCTION

The energy levels of many nuclides below the subshell[1] of 152 neutrons, have been thoroughly studied and classified.[2,3] There is, however, little experimental information on the energy-level characteristics for nuclides with more than 152 neutrons. Fm^{255} is one of the few nuclides which can be used to probe this region and which can be prepared in substantial amounts from conventional irradiations.

Fm^{255} was first identified by Ghiorso *et al.*[4] following an intense neutron irradiation of uranium in a thermonuclear explosion. Alpha particles were observed with an energy of 7.1 MeV and a half-life of ~1 day, sustained through β decay of a longer lived einsteinium parent. Later work[5,6] with reactor-produced heavy elements confirmed the initial results.

Jones *et al.*,[7] in work of higher precision, reported an alpha-particle energy of 7.08 ± 0.01 MeV and a half-life of 21.5 ± 0.1 h for Fm^{255}. These authors, as well as others,[8] had noted that the 7.08-MeV group, as measured in an ionization chamber, was significantly wider than expected for a single alpha group.

Asaro, Stephens, Thompson, and Perlman[9] measured the alpha spectrum with a magnetic spectrograph and found only one prominent Fm^{255} peak (94%) at 7.03 MeV (relative to 7.20 MeV for Fm^{254}) and weaker groups at 7.09 (0.4%), 6.97 (5%) and 6.90 MeV (0.8%). The observed broadening of the alpha peak in ionization chamber measurements was ascribed to intense conversion electrons in coincidence with the main alpha group. Indeed, α-L x-ray coincidence measurements showed 2.3 ± 0.2 conversion electron transitions (providing the transitions were converted in all of the L subshells) for each Fm^{255} alpha particle. Alpha-particle gamma-ray coincidence measurements showed, in addition to copious L x rays, gamma rays of ~58 and 80 keV, each in about 1.1% abundance.

There is still not enough information in the previous studies to formulate a decay scheme which could reasonably explain the available data. Even without evidence of several alpha groups, the large number of conversion electrons observed must mean that the decay scheme is quite complex. In the present study more detailed measurements of the alpha, gamma, and electron spectra of Fm^{255} have led to a consistent decay scheme which can be interpreted in terms of current theories and systematics of alpha decay and energy levels.

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[1] The term subshell has reference to an accidentally large energy gap in the spacing between adjacent neutron levels in deformed nuclei which occurs at neutron number 152. Its existence was first noted experimentally by an interruption in the smooth variation of α-decay energies in a series of californium isotopes.

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[4] A. Ghiorso, S. G. Thompson, G. H. Higgins, G. T. Seaborg, M. H. Studier, P. R. Fields, S. M. Fried, H. Diamond, J. F. Mech, G. L. Pyle, J. R. Huizenga, A. Hirsch, W. M. Manning, C. I. Browne, H. L. Smith, and R. W. Spence, Phys. Rev. **99**, 1048 (1955).

[5] G. R. Choppin, S. G. Thompson, A. Ghiorso, and B. G. Harvey, Phys. Rev. **94**, 1080 (1954).

[6] M. H. Studier, P. R. Fields, H. Diamond, J. F. Mech, A. M. Friedman, P. A. Sellers, G. Pyle, C. M. Stevens, L. B. Magnusson, and J. R. Huizenga, Phys. Rev. **93**, 1428 (1954).

[7] M. Jones, R. P. Schuman, J. P. Butler, G. Cowper, T. A. Eastwood, and H. G. Jackson, Phys. Rev. **102**, 203 (1956).

[8] A. Ghiorso (unpublished data).

[9] F. Asaro, F. S. Stephens, S. G. Thompson, and I. Perlman, in *Proceedings of the International Conference on Nuclear Structure at Kingston, Canada*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960) p. 925; University of California, Lawrence Radiation Laboratory Annual Report No. UCRL-9566, February 1961 (unpublished).

II. APPARATUS

A. Alpha Spectra

Most of the alpha spectra were measured with a double focusing magnetic spectrograph which has been described elsewhere.[10,11] The counting efficiency of the instrument was about 5×10^{-4} of 4π, and the alpha particles were detected with nuclear emulsions in which individual tracks were recorded. The full width at half-maximum (FWHM) of the alpha groups was about 10 keV at the acceptance angle employed.

A Frisch grid chamber was also used to measure alpha-particle energies. The counting efficiency was about 50% and the resolution (FWHM) was about 30 keV.

B. Coincidence Spectra

An alpha-particle gamma-ray coincidence system was available with which we could measure the gamma-ray spectrum in coincidence with specific alpha-particle energies or the alpha spectrum in coincidence with specific gamma-ray energies. This apparatus has been discussed in detail elsewhere.[12] It employed a silicon detector to measure alpha particles, a NaI detector for gamma rays, and an anthracene detector for conversion electrons. The resolving time of the apparatus was 4×10^{-8} sec for fast coincidences or 6×10^{-6} sec for slow coincidences. By inserting calibrated delay lines on the alpha-particle output, the half-life of delayed gamma-ray transitions could be measured. The same electronics system was also adopted for gamma-ray gamma-ray coincidence measurements with two 3-in. $\times$3-in. NaI detectors.

A time-to-height converter coupled to an alpha-particle gamma-ray coincidence apparatus also served to measure the lifetimes of delayed transitions.

C. Electron Spectra

The conversion electron spectra were measured with a 100-G permanent magnet spectrograph of a type which has been described elsewhere.[13] A wire coated with fermium or einsteinium activity served as the source for the electron spectrograph. The width of a 60-keV line was about 0.1 keV.

III. SOURCE PREPARATION

The Fm^{255} and associated activities were obtained by prolonged neutron irradiation of americium and curium by the Berkeley and Livermore divisions of the Lawrence Radiation Laboratory as part of a general program for the production of heavy elements. For reasons which can be visualized from the following decay properties, it is difficult to work with completely pure Fm^{255}.

$$\mathrm{Cf}^{253} \xrightarrow[17\ \text{day}]{\beta^-} \mathrm{E}^{253} \xrightarrow[20\ \text{day}]{\alpha}$$

$$\mathrm{Cf}^{254}\ (\beta\ \text{stable}) \quad \mathrm{E}^{254} \xrightarrow[37\ \text{h}]{\beta^-} \mathrm{Fm}^{254} \xrightarrow[3.2\ \text{h}]{\alpha}$$

$$\mathrm{E}^{255} \xrightarrow[\sim1\ \text{month}]{\beta^-} \mathrm{Fm}^{255} \xrightarrow[20\ \text{h}]{\alpha}.$$

The principal path to arrive at elements beyond californium is through the β^- decay of Cf^{253}, and E^{253} is by far the predominant activity in the trans-californium fraction. The isotope E^{253} captures neutrons successively to give E^{254} and E^{255}.

In the experiments, a fraction consisting of californium and higher elements was first separated from fission products and lower transuranium elements by a procedure described elsewhere.[14] The einsteinium and fermium were then separated from the californium by elution on a cation ion-exchange column with α-hydroxy isobutyric acid.[15] This fraction, which contained principally $E^{253,254,255}$ and $Fm^{254,255}$, could serve for some time as a reservoir for 20-h Fm^{255} because of the relatively long half-life of the E^{255} parent. In this fraction, however, the activity of the Fm^{255} was only about 0.01% that of the E^{253}. In order to delineate which features of the spectra belong to E^{253}, a pure sample of E^{253} was obtained by removing it after a growth period of a month from a previously purified californium fraction. In some experiments the 20-h Fm^{255} was separated from the einsteinium fraction by additional elutions on cation ion-exchange columns with α-hydroxy isobutyric acid.

The description given above for the chemical separations is incomplete in the sense that steps were taken to insure that the sources prepared for spectroscopic measurements would be virtually weightless. A standard procedure[16] was employed in which foreign ions are removed with ion-exchange columns both before and after a particular separation is made.

Sources for the alpha-particle spectrograph and ionization chamber were prepared by vacuum volatilization of the active material from a white hot tungsten filament onto a cold platinum plate. The length of the sources was about $\frac{3}{4}$ in. and the width varied from 0.020 to 0.12 in.

The electron spectrograph sources were prepared by volatilization of the activity onto one side of a platiunm wire, 0.010 or 0.020 in. in diameter.

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[10] D. L. Judd and S. A. Bludman, Nucl. Instr. **1**, 46 (1957).

[11] R. C. Pilger, Ph.D. thesis, University of California UCRL-3877, 1957 (unpublished).

[12] S. Bjørnholm, M. Lederer, F. Asaro, and I. Perlman, Phys. Rev. **130**, 2000 (1963).

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[15] G. R. Choppin, B. G. Harvey, and S. G. Thompson, J. Inorg. Nucl. Chem. **2**, 66 (1956).

[16] L. Phillips and R. Gatti, unpublished data referred to in National Academy of Sciences National Research Council, Nuclear Science Series NAS-NS 3031, 1960 (unpublished).

The source for the alpha-particle gamma-ray coincidence measurement was absorbed onto a sulfonated polystyrene disk.[17]

IV. RESULTS

A. Alpha-Particle Spectrograph Measurements

An alpha spectrum of chemically separated Fm^{255} is shown in Fig. 1. The source contained $\sim 1.2 \times 10^5$ dis/min of Fm^{255} and $\sim 6 \times 10^4$ dis/min of E^{253}. The Fm^{254} content was negligible. Table I shows a summary of the results obtained from a number of sources of equilibrated or enriched Fm^{255}. Unless otherwise noted, all alpha groups were observed with more than one source. The main group of E^{253} at 6.633 MeV (not shown) served as an energy standard. Incidental to the measurements, a better value for the α energy for Fm^{254} can be given. Previous measurements[9] had shown that there is 173-keV difference between Fm^{254} α_0 and Fm^{255} α_{106}, hence the energy for Fm^{254} α_0 is 7.192 ± 0.005 MeV.

The half-life of Fm^{255} was measured by following the alpha decay with a Au-Si surface barrier counter. The measured half-life was 20.07 ± 0.07 h.

TABLE I. Summary of alpha spectra data.

Alpha-particle energy (MeV)	Intensity (%)	Excited-state energy (keV)	Alpha decay hindrance factor[a]
7.122	0.09 ± 0.01	0	2.8×10^3
7.098	0.10 ± 0.01	25	2.1×10^3
7.076	0.43 ± 0.05	48	3.9×10^2
7.019 ± 0.004	93.4	106	1.03
6.977	0.11 ± 0.02	149	5.8×10^2
6.960	5.3 ± 0.1	166	10.3
6.887	0.60 ± 0.03	240	44
6.803	0.12 ± 0.02	326	97

[a] These were calculated in the manner indicated in Ref. 22.

An alpha spectrum of Fm^{255} was measured in an ionization chamber with $\sim 50\%$ geometry. The spectrum is shown in Fig. 2 along with the peak shape of E^{253}, which serves as the criterion for a single line. It is seen that the Fm^{255} main group, (α_{106}), is much spread out; it has its greatest intensity at about 75 keV higher than the true energy and extends to about 100 keV. This is caused by intense conversion electrons in coincidence with the main group and indicates that the main alpha group populates a state about 100 keV above the ground state and that the de-excitation of this state is complex.

B. Electron Spectra

An exposure of an E-Fm source was made for 1 month on the permanent magnet electron spectrograph, and a similar exposure was made with a source containing enriched E^{253}. The E-Fm source contained 6×10^7 dis/min of E^{253} and 6×10^3 dis/min of Fm^{255}. Many electron lines belonging to E^{253} decay were observed but only two lines (L_{II} and L_{III} lines of an 81.1-keV transition) could be attributed to Fm^{255}. A further series of exposures were made with separated Fm^{255} and additional transitions of 58.3 and 80.5 keV were observed. All of these transitions were of sufficient intensity that they could only follow the main alpha group of Fm^{255}. Table II shows a summary of the observed electron lines with separated Fm^{255}.

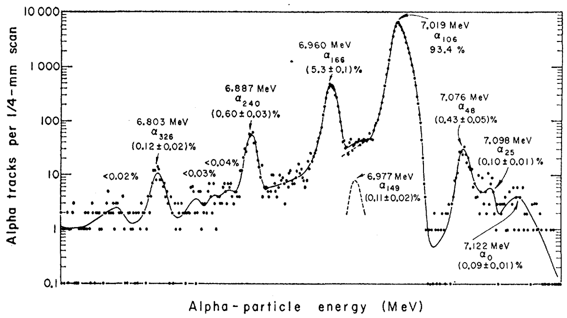

FIG. 1. Fm^{255} alpha-particle spectrum taken with a magnetic spectrograph. 0 events are plotted at 0.1.

It was possible to obtain rough relative subshell conversion coefficients for these transitions, so even without the gamma-ray measurements, it is possible to say something about multipolarities.

$M1$ transitions in this energy region convert principally in the L_I subshell, $E2$ transitions convert principally in the L_{II} and L_{III} subshells in nearly equal amounts and $E1$ transitions convert in all three L subshells in nearly equal amounts.

Thus, the 81.1- and 80.5-keV transitions would be predominantly $E2$ whereas the 58.3-keV transition would be predominantly $M1$.

Higher-order transitions are ruled out by absolute conversion coefficient and lifetime considerations which are discussed subsequently.

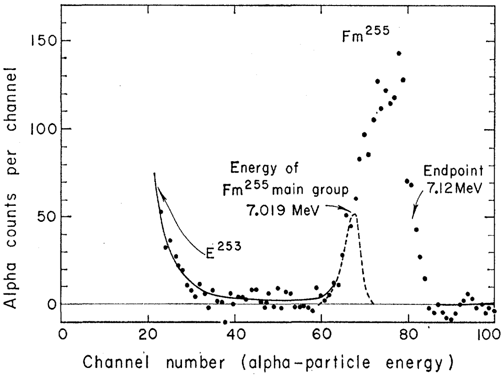

FIG. 2. Fm^{255} alpha-particle spectrum measured with a 2π solid-angle ion chamber. The background has been subtracted out. - - - peak shape of E^{253} in the sample shown at the energy of the Fm^{255} main group.

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[17] S. Bjørnholm and M. Lederer, Nucl. Instr. Methods **15**, 2331 (1962).

TABLE II. Fm^{255} electron lines.

Electron energy (keV)	Subshell	Binding energy (keV)	Gamma-ray energy (keV)	Electron relative intensities	Transition intensities (%)
32.31	L_{I}	25.98	58.29	$\sim$0.5	
51.49	M_{I}	6.78	58.17	$\sim$0.1	
		Best value	58.3 $M1$		$\sim$19
55.46	L_{II}	25.07	80.53	$\sim$0.2	
60.63	L_{III}	19.95	80.58	$\sim$0.2	
74.12	M_{II}	6.37	80.49	$\sim$0.1	
75.32	M_{III}	5.13	80.45	$\sim$0.1	
		Best value	80.5[a] $E2$		$\sim$16
56.01	L_{II}	25.07	81.08	1	
61.18	L_{III}	19.95	81.13	$\sim$0.8	
74.69	M_{II}	6.37	81.06	$\sim$0.3	
75.89	M_{III}	5.13	81.02	$\sim$0.3	
		Best value	81.1[a] $E2$		$\sim$65

[a] The best value for the energy separation between the 81.1- and 80.5-keV gamma rays is 0.56±0.02 keV.

C. α-γ and γ-γ Coincidence Measurements

1. *Low-Lying States*

The gamma-ray spectrum in coincidence (resolving time $\sim$6 μsec) with the main alpha group of Fm^{255} is shown in Fig. 3. Two gamma rays were observed with energies and intensities of 60 keV (0.9±0.2)% and 82 keV (1.1±0.2)%. These results are in good agreement with the previous measurement.[9] From the electron intensities shown in Table II, we can deduce that 20% of the observed photons at $\sim$82 keV belong to the 80.5-keV transition and 80% to the 81.1-keV transition.

We have shown earlier that the main alpha group populated a state $\sim$100 keV above the ground state. Since the 58- and 81-keV transitions follow the main group, they cannot be in coincidence with each other as this would give a decay energy which is about 40–60 keV too high. The relative intensities of the conversion lines of the transitions are known and we can now calculate the conversion coefficients assuming there are no other parallel transitions. These conversion coefficients, as given in Table III, indicate the 81-keV gamma rays have predominantly $E2$ multipolarity, and the 58-keV gamma ray has predominantly $M1$ multipolarity in conformity with the assignments made from the subshell conversion ratios. These assignments are incorporated in the decay scheme shown in Fig. 4.

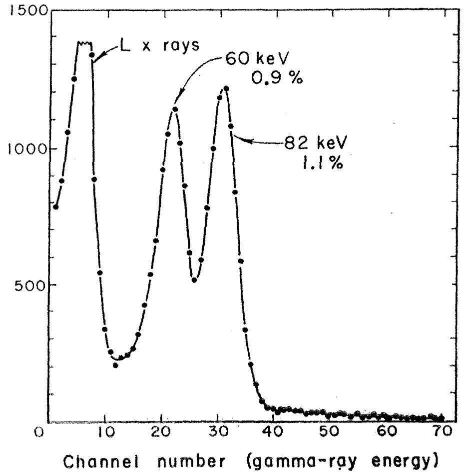

FIG. 3. Fm^{255} gamma-ray spectrum in coincidence with alpha particles.

The good agreement between the gamma-ray energies and the level spacings probably indicates the 58-keV transition takes place between the 106-keV state and the 48-keV state. If there were only one 81-keV transition its obvious place would be between the 106- and 25-keV states. It will be shown in the discussion section that there are two excited states at $\sim$106 keV and that the two 81-keV transitions can arise from the de-excitation of these states to the 25-keV state.

The gamma-ray spectrum in coincidence with L x rays was measured with a resolving time of $\sim$6 μsec, and is shown in Fig. 5. Beside K x rays, both the 58- and 81-keV gamma rays were seen. With the previous measured value[9] of 2.3 electron vacancies per Fm^{255} alpha particle (provided the transitions have energies above the L binding energies), the intensity of the 58-keV gamma ray in coincidence with transitions heavily converted in the L subshells was $\sim$1.3% per Fm^{255} alpha particle. The corresponding intensity for the 81-keV radiation was $\sim$1.2% per Fm^{255} alpha particle. The significance of this measurement is that there is about 1 highly converted transition (whose energy is over the L subshell binding energies) in coincidence with each 81-keV gamma ray, and between one and two such transitions in coincidence with each 58-keV gamma ray. This fits with the interpretation of the 48- and 25-keV levels as members of the ground-state rotational band (see Fig. 4).

TABLE III. Multipolarity assignment of 58- and 81-keV gamma rays.

Transition energy (keV)	Intensity of gamma rays (%)	Experimental conversion coefficient	Theoretical[a] conversion coefficients $E1$	$E2$	$M1$	Assignment
58.3	0.9±0.2	$\sim$26	0.7	290	40	$M1$
81.1 80.5	1.1±0.2	$\sim$70	0.3	60	12	$E2$

[a] The theoretical L conversion coefficient was taken from L. A. Sliv and I. M. Band, Coefficients of Internal Conversion of Gamma Radiation, Academy of Sciences of the USSR, Publication of the Leningrad Physico-Technical Institute: Part I K-Shell (1956), Part II L-Shell (1958), Issued in USA as Reports 57 *ICC K*1 and 58 *ICC L*1 Physics Department, University of Illinois, Urbana, Illinois. The ratio of $L:L+M+\cdots$ was assumed to be 0.7.

The gamma-ray spectrum in coincidence with alpha particles was measured with a resolving time of 0.04 μsec. The intensity of the two gamma rays dropped substantially from that obtained with a 6 μsec resolving time and thus indicated a delay between the alpha-

TABLE IV. Retardation of the 58- and 81-keV gamma rays.

Energy of gamma ray (keV)	Multi-polarity	Partial photon half-life (sec)	Single proton half-life[a] (sec)	Retardation
81.1	$E2$	4.2×10^{-6}	3.4×10^{-7}	12
80.5	$E2$	?	3.4×10^{-7}	?
58	$M1$	4.1×10^{-6}	3.7×10^{-11}	1.1×10^{5}

[a] The single proton half-lives were calculated from the equations given in Ref. 27.

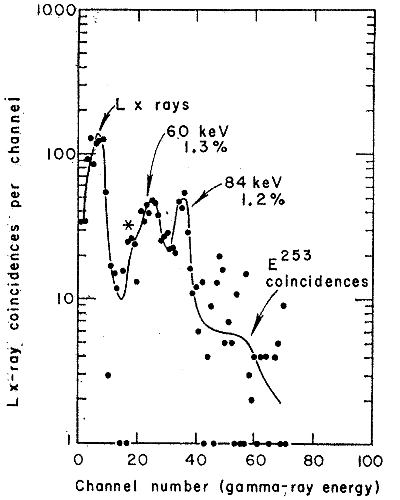

FIG. 5. Fm^{255} gamma-ray spectrum in coincidence with L x rays. The gamma-ray spectrum in coincidence with radiations above 20 keV was subtracted from the spectrum in coincidence with all detected radiations. "Negative" or 0 net events are plotted as 1 event. ★ ~50-keV escape peak of ~84-keV gamma ray in coincidence with residual I K x rays in the gate.

particle and gamma-ray emission. A measurement of alpha-particle gamma-ray delayed coincidences with a time-to-height converter showed that the 106-keV level has a half-life of $(3.7\pm0.2)\times10^{-8}$ sec (Fig. 6). The partial half-lives for the individual gamma rays are then 4.2×10^{-6} sec for the 81.1-keV gamma ray and 4.1×10^{-6} sec for the 58-keV gamma ray. These values and the respective single-particle values are given in Table IV. It will be shown later that the intensity of the 80.5-keV transition is governed by the half-life of the preceding 0.56-keV transition. It is seen that the 81.1-keV $E2$ gamma ray is about a factor of 12 slower than the single-particle value, while the 58-keV $M1$ gamma ray is retarded by a factor of about 10^5. The significance of these retardation factors in view of K forbiddenness ($K=\frac{7}{2}\rightarrow K=\frac{1}{2}$) will be discussed in a later section.

2. Higher Lying States

Alpha transitions of very low intensity, generally leading to relatively high-lying states, have been measured successfully by alpha-particle gamma-ray and alpha-particle electron coincidence counting employing silicon detectors for the alpha particles.[12] In the present study, the alpha spectrum was measured in coincidence with all conversion electrons of energy greater than 100 keV. A single alpha group attributable to Fm^{255} was seen at 6.58 MeV, which means that a state exists at about 550 keV above the ground state. The intensity of the alpha group in coincidence with electrons was $(2.6\pm0.8)\times10^{-4}$ relative to total Fm^{255} alpha particles. Similar measurements in which the gating pulses were gamma rays of energy >275 keV showed the same alpha group in an intensity of $(1.9\pm0.5)\times10^{-4}$. The total intensity of alpha population of the state at 550 keV is, therefore, $(4.5\pm0.9)\times10^{-4}$.

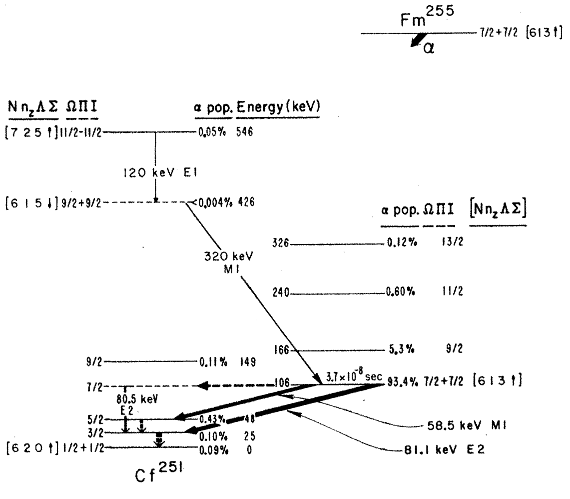

FIG. 4. Fm^{255} decay scheme.

In coincidence with the alpha group at 6.58 MeV were found gamma rays of 120 and 320 keV with relative intensities of 1 to 0.4. The photon peak at 120 keV would include K x rays of californium. The most likely arrangement for these two gamma rays is shown in Fig. 4 where it will be noted that their sum, 440 keV, agrees with the spacing between the 6.58-MeV group and the main alpha group (7.019 MeV) leading to the 106-keV state. The conversion coefficient of 1.3 ± 0.5, inferred from the above mentioned data, must apply to

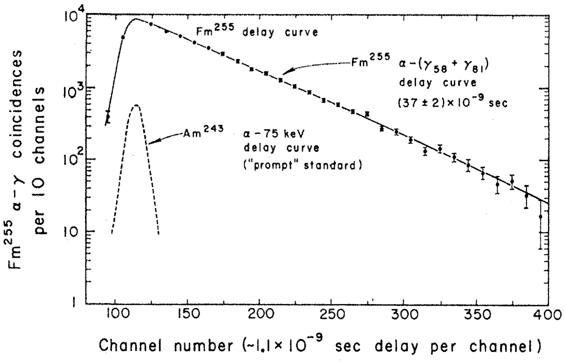

FIG. 6. Time delay between gamma rays of 50–100 keV and Fm^{255} alpha particles.

TABLE V. Fm^{255} alpha transitions to higher-energy excited states.

Alpha particle energy (MeV)	Intensity (%)	Excited state energy (MeV)	Alpha decay hindrance factors[a]
6.58	$(4.5\pm0.9)\times10^{-2}$	546	25
6.70	$<4\times10^{-3}$	426	>1000[b]

[a] The hindrance factors were calculated in the manner indicated in Ref. 22.

[b] This hindrance factor is valid *if* a state at ~426 keV de-excites by the 320-keV gamma ray.

the 320-keV transition and is in agreement with the total theoretical $M1$ value of 1.5. Since the measurements show that the direct population of the 426-keV level is negligible we can use the $M1$ assignment of the 320-keV gamma ray to correct the 120-keV radiation intensity for K x rays and then determine the conversion coefficient for the 120-keV transition. This turns out to be 0.6 ± 0.3 which is higher than the $E1$ value (0.12) but much lower than those for $M1$ (6.0) and $E2$ (12). The assumptions made in arriving at the experimental estimate are such that the value 0.6 is more likely too high than too low. On this basis the transition has been given an $E1$ assignment.

From the experimental data alone it cannot be said which transition, the 120 or 320 keV, precedes the other. As will be discussed later, it is most likely the 120-keV gamma ray precedes the 320 keV. In this event, the maximum population to the intermediate state at 426 keV is 4×10^{-5} of the Fm^{255} alpha decay. Direct decay to the intermediate state would lower the observed conversion coefficient of the 120-keV gamma ray and improve the agreement with the theoretical value. A summary of the alpha energy data for these transitions and the hindrance factors are given in Table V.

V. SPIN ASSIGNMENTS

As seen in Table I, the alpha population of the 106-keV state has a very small hindrance factor.[18] An unhindered decay of this type occurs in the region of strong nuclear deformation when the initial and final states have the same configuration.[19] Alpha decay to the entire rotational band is said to be favored and the relative intensities of the respective alpha groups are predictable. The spins of the rotational states generally increase by 1 unit for each state, and the energies of these states follow the relationship[20]

$$E=W+\hbar^2/2\mathfrak{F}[I(I+1)+\delta_{K,1/2}a(I+\tfrac{1}{2})(-1)^{I+1/2}], \quad (1)$$

where E is the excited state energy, $\mathfrak{F}$ is the moment of inertia, W is a constant, and I is the nuclear spin. K, the projection of the spin on the nuclear symmetry axis, is a constant for a given rotational band and is usually equal to the lowest spin in the band. The last term in the equation only applies to rotational bands with $K=\frac{1}{2}$. From the energy spacings between the 106-, 166-, and 240-keV states (60.2 ± 0.3 and 73.8 ± 0.4 keV), we calculate that I for the 106-keV level is 3.43 ± 0.14, i.e., $\frac{7}{2}$; hence, K for the entire band is also $\frac{7}{2}$. The states of 166 and 240 keV will then have spins of $\frac{9}{2}$ and 11/2. The value of $\hbar^2/2\mathfrak{F}$, 6.69 ± 0.03, is somewhat larger than the largest value previously found in this region, 6.38 keV, determined from the 287- and 330-keV levels in Pu^{239}.

The identification of three members of rotational band fixes the parameters of Eq. (1), and the positions of higher members can then be calculated. The energy of the spin 13/2 member of the $K=\frac{7}{2}$ rotational band, as calculated from Eq. (1), is 220.8 ± 1.1 keV above the spin $\frac{7}{2}$ state, in good agreement with an observed value, 219.7 ± 1.5 keV.

If the spin assignments are correct, it should be possible to calculate the relative alpha intensities to the various states in the band. The alpha decay of an odd-mass nucleus to a rotational band which has the same nuclear intrinsic configuration as the parent (favored decay) is very similar to the alpha decay of an even-even nuclide ($K=0$) to the ground-state rotational band of the daughter ($K=0$). It has been shown[19] that the hindrance factors for the alpha waves of the same angular momenta are nearly the same in both cases. The alpha transition probabilities to the various rotational states in favored alpha decay are given by the equation

$$P=P_E/N\sum_{L=0,2,4}\frac{\{C_{K_i}{}^{I_i}{}_{K_f-K_i}{}^{L}{}_{K_f}{}^{I_f}\}^2}{\mathrm{HF}_{L(e-e)}}, \quad (2)$$

where P_E is the alpha transition probability expected from simple barrier penetration theory, $HF_{L(e-e)}$ is the hindrance factor from adjacent even-even nuclides for alpha emission with angular momentum L, C is a Clebsch-Gordan coefficient, and the indices i and f refer to the initial and final states, respectively. N usually has a value between 1 and 2 for favored alpha decay. It is normally treated as a parameter and deter-

TABLE VI. Calculated alpha populations to the $K=\frac{7}{2}$ band.

Energy of state (keV)	I	Predicted relative intensities (%) $L=0$	$L=2$	$L=4$	Σ	Experimental intensities (%)
106	7/2	83.4 (norm)	9.8	0.20	93.4 (norm)	93.4
166	9/2	···	5.01	0.43	5.44	5.3 ± 0.1
240	11/2	···	0.64	0.30	0.94	0.60 ± 0.03
326	13/2	···	···	0.075	0.075	0.12 ± 0.02
427 (calc)	15/2	···	···	0.0057	0.0057	<0.05

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[18] The alpha decay hindrance factor for a particular alpha group is the ratio of the observed partial half-life for the group to the theoretical half-life deduced from simple barrier penetration considerations. See Ref. 22 for a more detailed explanation.

[19] A. Bohr, P. O. Froman, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 10 (1955); P. O. Froman, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **1**, No. 3 (1957).

[20] A. Bohr and B. R. Mottelson, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Chapt. XVII.

mined empirically, although it can be calculated from a detailed knowledge of the nuclear wave functions.[21]

As alpha decay data on the even-even nucleus Fm^{256} are not available, only hindrance factors deduced from Fm^{254} alpha decay were used for Eq. (2). The relative hindrance factors[22] for $L=0$, 2, 4, alpha emission from Fm^{254} are 1:4:30. The hindrance factor for the $L=6$ wave[23] is ~1300 and is of negligible concern here. Table VI shows the experimental and calculated intensities of the Fm^{255} alpha groups populating the $K=\frac{7}{2}$ band. The agreement is reasonably good. The ratio of the population to the 3rd member of the band to that to the 2nd member has been found in other odd-mass alpha emitters to be usually about 20–30% lower than the calculated value. Thus, our Fm^{255} result which is 36% lower than the calculated value, is consistent with empirical expectations. Deviations from the simple Bohr-Froman and Mottelson calculations described above have been interpreted for U^{233} alpha decay in terms of the interaction of the nuclear quadrupole moment with the outgoing alpha wave.[24] As yet the quantitative agreement between theory and experiment is not satisfactory.

The simplest explanation for the remaining alpha groups populating states near the ground state is in terms of a single rotational band. Examination of the spacings of the first two excited states will show that only a $K=\frac{1}{2}$ assignment is possible. With $K=\frac{1}{2}$ the order of the spins of the various levels depends on the value of a, the decoupling parameter. Considerations of the levels populated by the gamma-ray de-excitation of the 106-keV state ($K=\frac{7}{2}$, $I=\frac{7}{2}$) the gamma-ray multipolarities, and the number of converted transitions in coincidence with these gamma rays, lead to the spin sequence $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ for the ground, 1st, and 2nd excited states, respectively. With energy spacings shown in Fig. 4, the values of the decoupling parameter and $\hbar^2/2\mathfrak{J}$, as calculated from Eq. (1), are 0.24±0.03 and 6.4±0.3 keV, respectively. The value of $\hbar^2/2\mathfrak{J}$ is quite consistent with values for other nuclides near this region. The value of the decoupling parameter will be considered in the section on Nilsson level assignments. From the parameters given above, the positions of the higher members of the $K=\frac{1}{2}$ rotational band can be calculated.

TABLE VII. Energy spacings and alpha populations of the $K=\frac{1}{2}$ band.

	Energy of state (keV)		$\frac{P_E}{P_{E=0}}$	Alpha abundance (%)	
I	Calc.[a]	Observed		Calc.[b]	Exp
1/2	0	0	1.00	0.11	0.09±0.01
3/2	25.5 (norm)	25.5±2	0.80	0.10 (norm)	0.10±0.01
5/2	48.3 (norm)	48.3±2	0.65	0.43 (norm)	0.43
7/2	105±5	Masked	0.37	0.13	Masked
9/2	146±5	149±3	0.25	0.086	0.11±0.02
11/2	236±11	Masked	0.105	0.009	Masked
13/2	294±11	···	0.060	0.0015[c]	···
15/2	417±20	···	0.017	0.00003[c]	···

[a] The energies were calculated from Eq. (1) with $\hbar^2/2\mathfrak{J}=6.4$ and $a=0.24$.

[b] The abundances were calculated from Eq. (3) with $HF_L=160$ and $b_L=0.2$ for the $L=4$ alpha wave and with the relative values for the alpha-particle barrier penetrability given in column 4.

[c] These groups would have been too weak to see.

The values are shown in Table VII. It is seen that $K=\frac{1}{2}$, $I=\frac{7}{2}$ member should be very close in energy to the $K=\frac{7}{2}$, $I=\frac{7}{2}$ level (the most heavily populated state). The next member of the band, $K=\frac{1}{2}$, $I=\frac{9}{2}$, should be at 146±5 keV which is within the experimental uncertainty of the observed state of 149±3 keV. As shown in Fig. 4, the spin values are quite consistent with the assigned multipolarities of the 81- and 58-keV gamma rays. The 81.1- and 80.5-keV $E2$ transitions drop from the spin $\frac{7}{2}$ states with $K=\frac{7}{2}$ and $\frac{1}{2}$, respectively, to a state with a spin $\frac{3}{2}$ which de-excites to the ground state. The 58-keV $M1$ leads from the spin $\frac{7}{2}$ state with $K=\frac{7}{2}$ to one of spin $\frac{5}{2}$ which, in turn, probably de-excites to the ground state through the spin $\frac{3}{2}$ state. It will be seen that this $K=\frac{1}{2}$ band is best assigned to an even-parity Nilsson orbital as is the $K=\frac{7}{2}$ state.

Alpha decay between states of the same parity can proceed only by even parity alpha waves: $L=0$, 2, 4, 6, ···. In addition, the selection rule involving K restricts the values of L to $L\geq|K_f-K_i|$, which for the present case means that $K=4$, 6, ···. If $L\geq K_f+K_i$, alpha decay can take place not only between states of K_i and K_f but also between states of K_i and $-K_f$. The theory[19] indicates the following relationship should hold:

$$P=P_E\sum_L \frac{\{C_{K_i}{}^{I_i}{}_{K_f-K_i}{}^{L}{}_{K_f}{}^{I_f}+b_L(-1)^{I_f+K_f}C_{K_i}{}^{I_i}{}_{-K_f-K_i}{}^{L}{}_{-K_f}{}^{I_f}\}^2}{\mathrm{HF}_L}, \qquad (3)$$

where HF_L are the intrinsic hindrance factors for alpha waves of different angular momenta between states of K_i and K_f, HF_L/b_L^2 are the intrinsic hindrance factors between states of K_i and $-K_f$, and the other values have the same meanings as given earlier. The values of HF_L and b_L are treated as adjustable parameters, but they could be calculated from the Nilsson wave functions.[21] This type of analysis of appropriate transitions in Cm^{243} decay has given satisfactory agreement with the theory.[21,25] For the α transition $\frac{7}{2}+\rightarrow\frac{1}{2}+$ in Fm^{255}

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[21] H. J. Mang and J. O. Rasmussen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **2**, No. 3 (1962). H. J. Mang, abstract of paper to be presented at the Washington Meeting of the American Physical Society, 1963, University of California, Lawrence Radiation Laboratory Report UCRL-10648 (unpublished); H. D. Zeh and H. J. Mang, Nucl. Phys. **29**, 529 (1962).

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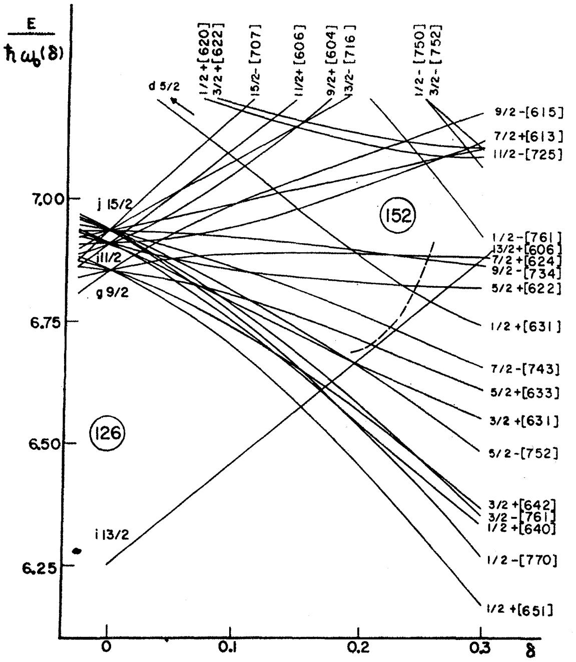

FIG. 7. Nilsson diagram for odd-neutron nuclei. (Reference 2.)

decay, only the $L=4$ wave is possible as is the case for the transition $\frac{7}{2}+\to\frac{3}{2}+$. The assumption will be made that only the $L=4$ wave is of significance in the other transitions as well.

Table VII shows experimental and calculated intensities to the various members of the $K=\frac{1}{2}$ band for an $L=4$ alpha wave with $b=+0.2$. The agreement is moderately good.

The spin assignments to higher energy states are made by comparison with the predicted Nilsson levels, which are discussed in the next section.

VI. NILSSON LEVEL ASSIGNMENTS

The energy levels in the region of strong deformation have been explained with considerable success in terms of Nilsson single-particle orbitals.[2,3] These are the calculated levels[26] in a spheroidal potential well. A diagram of the neutron levels for the region of interest is shown in Fig. 7. These states are described by the quantum numbers $\Omega\pi[Nn_z\Lambda\Sigma]$. Ω is the projection of the particle angular momentum, j, on the nuclear symmetry axis and has parity, π. For the relatively low-energy odd-mass states, $\Omega=K=I_0$, where I_0 is the lowest spin in the rotational band based on Ω. N is the total number of nodes in the wave function. The number of nodal planes perpendicular to the nuclear symmetry axis is denoted by n_z while Λ and Σ are the orbital and intrinsic spin components of Ω, respectively. We shall indicate the relative orientation of Λ and Σ by means of an arrow which has the usual *spin-up*, $\Omega=\Lambda+\frac{1}{2}$, or *spin-down*, $\Omega=\Lambda-\frac{1}{2}$, meaning. For Fm^{255} with 155 neutrons and $K=\frac{7}{2}$, the most reasonable Nilsson state assignment is $\frac{7}{2}+\ [613\uparrow]$. This would also be the assignment for the $K=\frac{7}{2}$ band in Cf^{251}. The ground state of Cf^{251}, with $K=\frac{1}{2}$ and the same parity as the $\frac{7}{2}+$ band, is very likely the state $\frac{1}{2}+\ [620\uparrow]$. The decoupling parameter, a, can be calculated for this level from the given wave functions.[26] For a deformation (η) of 6, the calculated value of a is $+0.29$ in good agreement with the experimental value (see Table VII and corresponding text).

As was discussed earlier, an $E2$ transition between the $K=\frac{7}{2}+$ and $K=\frac{1}{2}+$ rotational bands occurs with a retardation of only 12 over the single proton transition rate. This transition is K forbidden in that $2<|K_f-K_i|$, and it has been found that K forbidden transitions are usually retarded by $\sim 10^2$ for each unit of K forbiddeness. The relatively small retardation which we observed can be explained if we assume an interaction between the two near lying states at 106 keV, each with spin $\frac{7}{2}$.

The 106-keV level with a K of $\frac{7}{2}$ would then have a small admixture of $K=\frac{1}{2}$ and the close lying level with $K=\frac{1}{2}$ would have the same admixture of $K=\frac{7}{2}$. This admixture can be calculated if we assume that the 81.1-keV $E2$ is due to the $K=\frac{1}{2}$ admixture in the $K=\frac{7}{2}$, $I=\frac{7}{2}$ state. This transition will then be collective in nature and have a transition probability roughly equal to the product of the single proton value, a factor of 200 due to the collective nature,[27] the square of the appropriate Clebsch-Gordan coefficient (0.26), and the unknown admixture. The unknown admixture, "a^2", is then easily calculated to be 0.2%. If we had assumed instead that the 80.5-keV transition was the interband transition, the calculated admixture would have been even smaller. Thus, for the 106-keV states the calculated admixture of $K=\frac{1}{2}$ state is $\leq 0.2\%$. Both of the two 81-keV transitions have intensities of at least 16% as can be seen from Table II. From the alpha population expected from Eq. (3) or the amount expected from a 0.2% admixture of $K=\frac{7}{2}$, it is difficult to see how the direct alpha population to this state with $K=\frac{1}{2}$, $I=\frac{7}{2}$ could be greater than 0.5%. This can be confirmed by another line of reasoning. If the *interband* 81-keV $E2$ transition is "slow" because of K forbiddeness, the other *intraband* 81-keV transition should not decay with the same half-life unless it is from a state not directly populated in alpha decay. Therefore, the $K=\frac{1}{2}$, $I=\frac{7}{2}$ state would then lie lower in energy than the $K=\frac{7}{2}$, $I=\frac{7}{2}$ state and would be populated by a gamma-ray transition from the $K=\frac{7}{2}$ state. The 81.1-keV gamma ray would drop from the $K=\frac{7}{2}$, $I=\frac{7}{2}$ state and the 80.5-keV gamma ray would drop from the $K=\frac{1}{2}$, $I=\frac{7}{2}$ state.

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[26] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

[27] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 439 (1956).

As these gamma rays both presumably populate the spin $\frac{3}{2}$ state, the energy difference between the two spin $\frac{7}{2}$ states and the energy of the transition between them is 0.56 keV (Table II).

If the foregoing analysis is correct it should be possible to calculate roughly from the appropriate Nilsson wave functions,[26] the $M1$ transition probability of the 58.3-keV transition between the $K=\frac{7}{2}$, $I=\frac{7}{2}$ state and the $K=\frac{1}{2}$, $I=\frac{5}{2}$ state. With the previously calculated admixture, a nuclear deformation (η) of 6 and values of $g_r=Z/2A$ and $g_s=-2.6$ as given by Rasmussen and Chiao,[28] the calculated $M1$ transition probability for the 58.3-keV transition is $\frac{1}{6}$ of the observed value. We do not believe that this disagreement seriously affects the interpretation as the highly retarded $M1$ transition would be very sensitive to details of the nuclear wave functions and admixtures of other states. The 58.3-keV transition should have an $E2$ component, but the intensity of its conversion lines, as determined[29] from the appropriate Clebsch-Gordan coefficients and the intensity of the 81.1-keV transition is about 10% of the 81.1-keV transition. This would have been below our limits of detection.

It is also possible to calculate the intensity of the 0.56-keV transition between the two spin $\frac{7}{2}$ states. The $M1$ transition probability for this transition as calculated from Nilsson wave functions is over an order of magnitude higher than that of the 58.3-keV transition for conversion in comparable shells. The restrictions on the conversion opportunities of the 0.56-keV transition (0 shell and higher shells) bring its calculated $M1$ transition probability to about equal to that of the 58.3-keV transition. The $E2$ component of the 0.56-keV transition (as determined from the appropriate Clebsch-Gordan coefficients and the intensity of the 81.1 transition) would be about $\frac{1}{4}$ of the intensity of the $M1$ component. The total calculated transition intensity for the $M1$ and $E2$ components is $\sim$25% of the alpha decay.

This value may be compared with the intensity of the 80.5-keV transition, 16%. The agreement is fortuitously good considering the approximations involved. There should be a predominantly $M1$ transition of 57.8 keV from the $K=\frac{1}{2}$, $I=\frac{7}{2}$ state to the spin $\frac{5}{2}$ state. If this spin $\frac{7}{2}$ state decays in nearly the same manner as the $K=\frac{7}{2}$, $I=\frac{7}{2}$ state, then the intensity of the 57.8-keV transition would be about $\frac{1}{4}$ of the 58.3-keV transition or below our limits of detection.

The interaction we have proposed between the two spin $\frac{7}{2}$ states at 106 keV would be due to the Coriolis force.[30] Using Nilsson wave functions we calculated the interaction between the two close lying spin $\frac{7}{2}$ members of the Nilsson states $\frac{1}{2}+$ [620↑] and $\frac{7}{2}+$ [613↑] via the various $\Omega=\frac{3}{2}$ and $\Omega=\frac{5}{2}$ states with $N=6$ as intermediaries. The energies of the various states were determined from the Nilsson eigenvalues for $N=6$ with the exception of the $\frac{3}{2}+$ [622↓] state which was arbitrarily placed at 230 keV. The splitting between the two spin $\frac{7}{2}$ states was taken as 0.56 keV. The calculated relative admixture, "a^2", of the two spin $\frac{7}{2}$ states in each other was $\sim$0.7%, in reasonable agreement with the value, 0.2%, inferred from the transition probabilities.

The state at 546 keV de-excites to a $\frac{7}{2}+$ state at 106 keV by cascading $E1$ and $M1$ transitions. It thus should have odd parity and can have a spin between $\frac{3}{2}$ and 11/2 inclusive. The only state in this region which satisfies these requirements is the 11/2− [734↑]. The fact that the 11/2− state lies below the $\frac{9}{2}+$ state in Fig. 7 is not disturbing because its position is determined by the placement of the $j_{15/2}$ state at zero deformation, and this was selected somewhat arbitrarily. It is known, for example, that the $\frac{9}{2}-$ [734↑] state should be about 400 keV higher than the $\frac{7}{2}+$ [624↓] state,[2,3] although it is shown at a lower energy in Fig. 7.

VII. ALPHA DECAY THEORY AND SYSTEMATICS

It has been shown empirically by Prior[31] and theoretically by Mang[21] that for hindered alpha decays between states in which Σ changes sign, the hindrance factors are about 10^3–10^4; an order of magnitude larger than those for which Σ does not change sign. In the latter category, with low hindrance factors, would be the alpha decay from the $\frac{7}{2}+$ [613↑] state to the 11/2− [725↑] state (HF $\sim$25). In the former category with high hindrance factors would be the decay to the $\frac{9}{2}+$ [615↓] state (unobserved, $HF>1000$), and the alpha decay to the $\frac{3}{2}+$ [622↓] state (unobserved).

The alpha decay to the $\frac{1}{2}+$ [620↑] state illustrates both types of hindered decay. As seen in Eq. (3) there are two interfering components in this decay; the major component goes from $K_i=\frac{7}{2}$ ($\frac{7}{2}+$ [613↑]) to $K_f=\frac{1}{2}$ ($\frac{1}{2}+$ [620↑]), with a hindrance factor, HF_L, of 160. Σ does not change sign for this decay, and as expected the hindrance of the first $L=4$ alpha-wave component is low. The second component goes from $K_i=\frac{7}{2}$ ($\frac{7}{2}+$ [613↑]) to $K_f=-\frac{1}{2}$ ($-\frac{1}{2}+$ [620↓]) with an alpha-wave hindrance factor, $HF_L/b_L{}^2$, of 4000. Σ does change sign for this component of the decay and as expected the hindrance factor is higher. Thus, the measured alpha decay hindrance factors for the given state assignments are consistent with the present knowledge of hindered alpha decay.

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Reference Spectrum Method for the Boundary-Condition Model of Nuclear Forces

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The reference spectrum method of Bethe, Brandow, and Petschek for the calculation of the ground-state properties of nuclear matter is applied to the boundary-condition model of Feshbach and Lomon for the two-nucleon interaction. The short-range contribution to the reference G matrix is evaluated analytically, and the formulas needed for evaluation of the outer contribution given. The method is applied to a simple model interaction (boundary condition plus square well acting in S states only) and results compared with those from typical hard-core potentials. The correction terms to the reference approximation are found to be about one MeV per particle. The important region of intermediate state momenta in which the reference spectrum should be fitted, is again found to be near 4 F^{-1}.

I. INTRODUCTION

In a recent paper, Bethe, Brandow, and Petschek[1] have presented a reference spectrum method for calculating the properties of nuclear matter. The method has two aims: firstly, it provides a convenient and transparent method for carrying out the calculations of the Brueckner[2] theory, and secondly, it provides more insight into the role of the two-body interaction in determining the properties of the many-body system. In this respect it follows the line of development of Moszkowski and Scott,[3] and of Gomes, Walecka, and Weisskopf.[4] In all of these papers, the two-body interaction has been considered to contain a repulsive core.

An alternative representation of the two-body interaction is provided by the boundary-condition model (denoted BCM) of Feshbach, Lomon, and collaborators.[5] Here the long-range parts of the force are derived from a potential, which can largely be taken from theory as the one-pion exchange potential and some version of two-pion exchange. The short-range forces, about which experimental evidence is most equivocal, are represented by an energy-independent boundary condition on the wave function at a fixed radius about one-half pion Compton wavelength. The view is that the interaction energy at short radii is very large compared to the bombarding energy used in the study of the forces. This model provides a good fit to phenomenological phase shifts, which at least shows that our knowledge of the form of the two-body force at short radii can be reduced to a small number of boundary-condition parameters. It is therefore of interest to see the predictions of the BCM for nuclear matter, and to compare these with the predictions of hard-core potentials. Some progress in this direction has been made by Lomon and MacMillan.[6] Their method is the direct one of solving the G-matrix equations in momentum space.

In this paper we apply the reference-spectrum method of BBP to the boundary-condition model. This provides us with a simple method of calculating the properties of nuclear matter, and further it allows us to calculate in coordinate space where the BCM is intuitively more meaningful. This development is contained in Sec. II, III, and IV. We adhere closely to the notation of BBP in order to facilitate comparison with their work, and for sake of brevity to avoid rederiving numerous results. In Sec. V and VI, the method is applied to a nonrealistic but simple model interaction.

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